

MgO.	Per cent.	Sand	Condi- tion of cupel.	Lead Gms.	Silver Mgs.	Silver loss. Per cent.	Surface.
80	20	coarse	soft	10	35-45	2.5	slightly cracked
80	20	coarse	hard	10	35-45	2.6	slightly cracked
80	20	fine	soft	10	35-45	2.3	deep cracks
80	20	fine	hard	10	35-45	2.5	deep cracks
85	15	coarse	soft	10	35-45	3.2	badly pitted
85	15	coarse	hard	10	35-45	2.2	badly pitted and cracked
85	15	fine	soft	10	35-45	3.3	slightly cracked
85	15	fine	hard	10	35-45	2.5	badly cracked

None of the magnesium oxide cupels have properties either as to loss of silver or as to hardness after cupellation, which warrant the substitution of that substance for other cupel material.

#### SUMMARY.

Under similar conditions, with about 10 grams of lead and 40 mgs. of silver, the average percentage losses in different cupels were as follows:

	Silver loss. Per cent.
Morganite.....	1.99
Casseite.....	3.09
Brownite.....	2.89
Bone ash.....	2.36
Cement.....	3.38
Equal parts cement and bone ash.....	2.95

It was furthermore determined that cupels made of different grades and sizes of bone ash give the same percentage loss of silver within the limits of experimental error.

UNIVERSITY OF ARIZONA,  
TUCSON.

#### HEAT RADIATION.

By HAROLD P. GURNEY.

Received Aug. 9, 1911.

Heat transmission by radiation is utilized in the abstraction of heat from the zone of combustion in furnaces, and it is the principal restrictive factor in the maintenance of high temperature. Chemical and metallurgical industries abound with instances where application of the well established laws of heat radiation and conduction would reveal important information with regard to economy of operation or design. Although heat radiation is rarely unaccompanied by conduction and convection of heat, it may well be treated separately, at first, as it is subject to different laws.

The laws of heat radiation differ in very essential respects from the laws of heat conduction. Heat always flows from high to low temperature, but the rate of flow of radiant heat is not proportional to the temperature difference. When heat is transmitted by conduction, the thermal pressure causing the flow of heat may be considered as proportional to the absolute temperature, consequently the rate of flow per unit of temperature drop is a constant, because the differential of the thermal pressure with respect to the temperature is a constant. The pressure of radiant heat is proportional to the fourth power of the absolute temperature, consequently the rate of flow per unit of temperature drop at any temperature is proportional to the cube of the absolute temperature.

Heat of conduction flows through solids, its rate of transit depending on the heat-transferring properties of the solid, its density, its specific heat, and its thermometric conductivity. Heat of radiation passes through space between surfaces, its rate of transit depending on the nature and disposition of the surfaces between which it passes and not on the space itself, that is, not on the nature of the gas or vapor filling that space. This is not absolutely correct, as there is no gas which is completely diathermanous, that is, which does not absorb radiant heat; but for air, products of combustion, and most gases of common occurrence, the absorption of radiant heat is negligible over short distances.

The fourth-power law of heat radiation was first advanced by Stefan<sup>1</sup> who found it to accord with the results of experimental researches of Dulong and Petit, de la Provostaye and Desains, and Draper and Tyndall. Boltzmann later demonstrated mathematically from thermodynamic principles that such a law should hold.

A surface of  $A$  square feet at a temperature of  $\Theta_1$  degrees absolute (equals  $\theta_1$  degrees Fahrenheit plus 460.7) radiates  $R$  heat units<sup>2</sup> to another surface whose absolute temperature is  $\Theta_{11}$  in a length of time  $T$  hours. The mean solid angle subtended by the latter surface with respect to the surface in question is  $\varphi$  hemispheres. The net coefficient of emission and absorption between two surfaces is  $E$ . According to the Stefan-Boltzmann radiation law, the following relation exists:

$$R = 0.16TA\varphi E \left[ \frac{\Theta_1^4}{0.01} - \frac{\Theta_{11}^4}{0.01} \right]$$

The constant in the above expression has been variously assigned to values ranging from 0.136 to 0.190.<sup>3</sup> It was originally given as 0.152, but late work<sup>4</sup> by Bauer and Moulin places it at 0.160, and it has ordinarily been quoted at 0.160.

The coefficient of absorption of a surface is the ratio of the amount of radiant heat absorbed to the amount of heat incident on the surface. Lampblack absorbs practically all heat rays impinging on its surface and reflects none; its coefficient of absorption, then, is unity. The coefficient of emission is the ratio of the heat actually radiated to the heat an ideal black body would radiate, and it is the same in value as the coefficient of absorption. The net coefficient between two surfaces is very nearly the product of the coefficients of both surfaces, as the heat transmitted will be diminished both in emission and in absorption.

$\varphi$  is the mean of the solid angles subtended by one surface with respect to each elementary area of the other surface. A unit solid angle is bounded by a hemisphere and this is the most usual technical case. Rules cannot be given for evaluating  $\varphi$  under all circumstances; but as this matter is seldom given but scant consideration, it will here be taken up more in detail.

<sup>1</sup> Wiener, *Ber.*, 1872.

<sup>2</sup> British thermal unit.

<sup>3</sup> C. Féry, *Compt. rend.*, 1909.

<sup>4</sup> E. Bauer and M. Moulin, *Journal Physique*, 1910.

## VALUES OF E.

Lampblack.....	1.00	Sheet lead.....	0.18
Interior of furnaces.....	0.90-0.95	Polished steel.....	0.17
New cast iron.....	0.87	Polished sheet iron.....	0.12
Common steam pipe.....	0.85	Polished brass.....	0.07
Glass.....	0.80	Polished tin.....	0.06
Ordinary sheet iron.....	0.76	Polished copper.....	0.06
Polished zinc.....	0.19	Polished silver.....	0.03
Lampblack <sup>1</sup> .....	1.00	Mica.....	0.80
Paper.....	0.98	Graphite.....	0.75
Resin.....	0.96	Tarnished lead.....	0.45
Sealing wax.....	0.95	Mercury.....	0.20
Crown glass.....	0.90	Polished lead.....	0.19
India ink.....	0.88	Polished iron.....	0.12
Ice.....	0.85	Tin plate.....	0.12
Red lead.....	0.80	Gold, silver, copper.....	0.12
Lampblack <sup>2</sup> .....	1.00	Building stone.....	0.90
Paper.....	0.94	Sawdust.....	0.88
Silk.....	0.92	Powdered charcoal.....	0.85
Wool.....	0.92	Powdered chalk.....	0.83
Oil paint.....	0.92	Glass.....	0.73
Calico.....	0.91	Zinc.....	0.56
Fine sand.....	0.90	Tin.....	0.05
Wood.....	0.90	Polished copper.....	0.04
Plaster.....	0.90	Polished silver.....	0.03

As far as radiation is concerned, a perfectly plane or even surface will radiate just as much heat as a rough, corrugated, or irregular surface of the same extent as far as boundaries are concerned. For a rough surface, the true value of  $\phi$  would be less, and the true value of  $A$  greater than for a smooth surface of the same extent. With heat radiation, then, all surfaces may be treated as though they were smooth, an assumption which would be impermissible with heat conduction and convection.

In Fig. 1, the plane  $A_{11}$  subtends a mean solid angle of  $\phi_1$  hemispheres with respect to  $A_1$  and it is desirable to express  $\phi_1$  in terms of quantities which may be more easily estimated than solid angles.

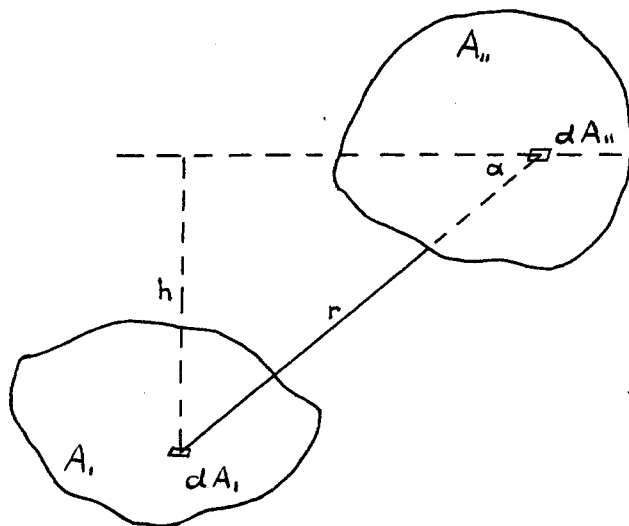


Fig. 1.

Both  $A_1$  and  $A_{11}$  may be divided into elementary areas  $dA_1$  and  $dA_{11}$  whose distances apart are  $r$ , and where the lines  $r$  connecting these make angles  $\alpha$  with the surface of  $A_{11}$  at  $dA_{11}$ . Then, if  $\phi$  is the solid angle subtended by  $dA_{11}$  with respect to  $dA_1$ , its value is

$$\phi = \frac{dA_{11} \sin \alpha}{2\pi r^2}.$$

<sup>1</sup> Leslie, Watts' "Dictionary of Chemistry."

<sup>2</sup> Ser, *Physique industrielle*.

By definition, the mean solid angle  $\phi_1$  is

$$\phi_1 = \frac{\int \phi dA_1}{\int dA_1} = \frac{\iint \frac{dA_1 dA_{11}}{2\pi} \cdot \frac{\sin \alpha}{r^2}}{A_1} = \frac{A_{11}}{2\pi} \left( \frac{\sin \alpha}{r^2} \right)_m.$$

Here  $\left( \frac{\sin \alpha}{r^2} \right)_m$  is the mean of all values of  $\frac{\sin \alpha}{r^2}$  between  $dA_1$  and  $dA_{11}$ . For regular surfaces this could be obtained exactly by calculus, but in general practice, it will be close enough to obtain it by estimation, or by averaging a representative series of values of  $\frac{\sin \alpha}{r^2}$  between evenly distributed points on the two surfaces.

When the surfaces are planes and parallel, then  $\sin \alpha = \frac{h}{r}$  where  $h$  is the perpendicular distance between the planes. Then,

$$\phi_1 = \frac{h A_{11}}{2\pi} \cdot \frac{1}{r_m^3}.$$

$r_m$  is the mean distance from any point in plane  $A_1$  to any point in  $A_{11}$  and may generally be taken as the mean between the maximum and minimum distances between the surfaces. Also,

$$\phi_{11} = \frac{h A_1}{2\pi} \cdot \frac{1}{r_m^3};$$

hence,

$$\frac{\phi_1}{\phi_{11}} = \frac{A_{11}}{A_1}$$

or

$$A_1 \phi_1 = A_{11} \phi_{11}.$$

The fundamental law of heat conduction is expressed in the following relation, where  $Q$  is the heat units transmitted in  $T$  hours through a wall whose conductance is  $G$  units and under a temperature drop of  $\Delta$  degrees:

$$\frac{Q}{T} = \Delta G$$

A similar though arbitrary relation will be assumed to hold for the heat units  $R$  radiated in  $T$  hours under a temperature drop  $\Delta$  degrees from or to a surface whose radiant conductance is  $C$ ,

$$\frac{R}{T} = \Delta C.$$

$H$  is the total heat transmitted where radiation and conduction occur simultaneously:

$$H = Q + R = \Delta T(C + G)$$

And just as the conductivity  $g = \frac{G}{A}$  where  $A$  is the area of the wall or surface in square feet, so the radiant conductivity  $c = \frac{C}{A}$ .

In referring to actual experiments, it is convenient to refer to a value  $K = \frac{H}{\Delta T} = C + G$ , and to a value

$$k = \frac{K}{A} = c + g.$$

In the light of the Stefan-Boltzmann radiation law, it will be of advantage to investigate the significance of  $c$ ,

$$c = \frac{\cdot R}{\Delta AT},$$

and

$$R = 0.16TA\varphi E \left[ \overline{0.01\Theta_1}^4 - \overline{0.01\Theta_{11}}^4 \right].$$

The ratio of  $\Theta_{11}$  to  $\Theta_1$  will hereafter be referred to as  $a$ . Then,

$$R = 0.16TA\varphi E 0.01\Theta_1(1 - a^4),$$

but

$$(1 - a^4) = (1 - a)(1 + a + a^2 + a^3)$$

and

$$0.01\Theta_1(1 - a) = 0.01\Theta_1 - 0.01\Theta_{11} = 0.01\Delta;$$

hence,

$$R = 0.0064TA\varphi E 0.01\Theta_1\Delta \left[ \frac{1}{4}(1 + a + a^2 + a^3) \right]$$

and

$$c = 0.0064\varphi E 0.01\Theta_1 \left[ \frac{1}{4}(1 + a + a^2 + a^3) \right].$$

In the following plot (Fig. 2),  $1 - a^4$ ,  $1/4(1 + a + a^2 + a^3)$ , and  $a$  as ordinates are plotted against  $a$  as abscissae. It can readily be seen that  $a$  does not vary greatly from  $1/4(1 + a + a^2 + a^3)$  where  $a > 0.3$ . Since  $a = \frac{0.01\Theta_{11}}{0.01\Theta_1}$  an approximate expression for  $c$  is

$$c = 0.006\varphi E 0.01\Theta_1 0.01\Theta_{11}.$$

The usual case of combined heat radiation and conduction is where heat passes from a surface at temperature  ${}^{\circ}\theta_1$  through a space of approximately the same temperature, a conducting wall of thickness

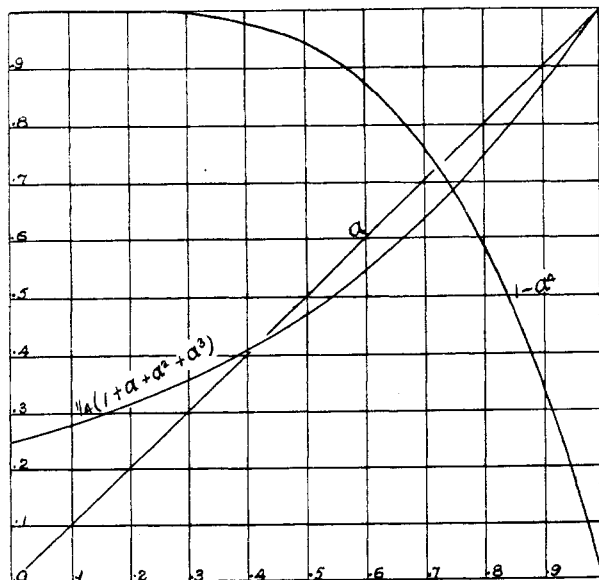


Fig. 2.

$\delta$  inches whose outside temperatures are  ${}^{\circ}\theta_{11}$  and  ${}^{\circ}\theta_1$ , and into a fluid of temperature  ${}^{\circ}\theta_{11}$ . The internal resistivity of the wall is  $\rho$ . At  ${}^{\circ}\theta_{11}$  the conductivity is  $\gamma_1$  and the radiant conductivity is  $c_1$ , and at  ${}^{\circ}\theta_1$  the boundary resistivity is  $\zeta_{11}$ . Both  ${}^{\circ}\theta_1$ ,  ${}^{\circ}\theta_{11}$ , and  $\Delta = {}^{\circ}\theta_1 - {}^{\circ}\theta_{11}$  are known, but  ${}^{\circ}\theta_{11}$

and  ${}^{\circ}\theta_1$  are not known.  $H$  heat units are transmitted in  $T$  hours (see Fig. 3).

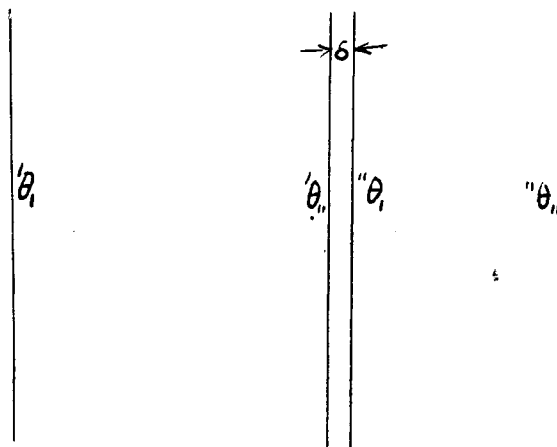


Fig. 3.

Then,

$$\frac{H}{T} = \frac{\Delta A}{\frac{1}{\gamma_1 + c_1} + \delta\rho + \zeta_{11}}.$$

Very commonly, the quantities  $\delta\rho$  and  $\zeta_{11}$  may be neglected in comparison with  $\frac{1}{\gamma_1 + c_1}$

and

$$\frac{H}{T} = \Delta A(\gamma_1 + c_1).$$

Since the temperature drops are proportional to the included resistivities, then,

$$\frac{{}^{\circ}\theta_1 - {}^{\circ}\theta_{11}}{\frac{1}{\gamma_1 + c_1}} = \frac{{}^{\circ}\theta_{11} - {}^{\circ}\theta_1}{\delta\rho} =$$

$$\frac{{}^{\circ}\theta_1 - {}^{\circ}\theta_{11}}{\zeta_{11}} = \frac{\Delta}{\frac{1}{\gamma_1 + c_1} + \delta\rho + \zeta_{11}}.$$

Generally  $\delta\rho$  may be neglected especially for metallic walls, but if  $\delta_1\rho_1$  representing the thickness of material like boiler scale multiplied into its internal resistivity, it cannot be neglected:

$${}^{\circ}\theta_1 - {}^{\circ}\theta_{11} = \zeta_{11}\Delta(\gamma_1 + c_1)$$

$${}^{\circ}\theta_{11} - {}^{\circ}\theta_1 = \delta_1\rho_1\Delta(\gamma_1 + c_1)$$

$${}^{\circ}\theta_{11} - {}^{\circ}\theta_1 = \frac{\delta_1\rho_1\Delta}{\frac{1}{\gamma_1 + c_1} + \delta_1\rho_1} = \frac{\delta_1\rho_1(\gamma_1 + c_1)}{1 + \delta_1\rho_1(\gamma_1 + c_1)}\Delta.$$

A few samples will demonstrate the practical application of these formulae. In a steam boiler the temperature of the steam is  $250^{\circ}$  [ $=710^{\circ}$  absolute], the temperature of the grate is  $1200^{\circ}$  [ $=1660^{\circ}$  absolute], and the thickness of the boiler plate is  $1/4$  inch. It is now desired to find the temperature difference between the water side and the fire side of the plates exposed to direct radiation, where the value of  $\rho = 0.003$ ,  $\gamma_1 = 3$  and  $\phi = 0.70$ :

$$0.01\Theta_1 = 16.6 \quad 0.01\Theta_{11} = 7.1$$

—2

$$c_1 = 0.006 \times 0.70 \times 16.6 \times 7.1 = 8.2$$

$$c_1 + \gamma_1 = 11.2$$

$$t_{011} - t_{01} = 950 \times 1/4 \times 0.003 \times 11.2 = 8^\circ$$

If instead, copper plate had been used of  $1/2$  inch thickness and 0.003 internal resistivity, then,

$$t_{011} - t_{01} = 950 \times 1/2 \times 0.003 \times 11.2 = 1.6^\circ.$$

It is here assumed that the copper plate has been sooted over, and  $E$  is around 0.95 instead of 0.05 were the copper polished.

These temperature drops of 1 per cent. and  $1/6$  per cent. of the total temperature drop are absolutely negligible in comparison with the inaccuracy of data and assumptions. Blechynden<sup>1</sup> in experiments on the heat transmission of iron and copper plates exposed to combustion found no appreciable difference.

With boiler scale, it is quite different. Under the same conditions as before, it will be assumed that the heating surface is coated with a boiler scale of  $1/2$  inch thickness and of internal resistivity 0.07, a fair value.

$$t_{011} - t_{01} = 950 \times \frac{1/2 \times 0.07 \times 11.2}{1 + 1/2 \times 0.07 \times 11.2} = 270^\circ,$$

$$t_{011} = 520^\circ [= 980^\circ \text{ absolute}].$$

If the temperature of the boiler plate exposed to the fire of absolute temperature 1660° is 980° absolute instead of 710°, a new value for  $c$  should be solved, for

$$c = 0.006 \times 0.70 \times 16.6 \times 9.8 = 11.4$$

$$\gamma = 3 \quad \gamma + c = 14.4$$

$$t_{011} - t_{01} = 950 \times \frac{1/2 \times 0.07 \times 14.4}{1 + 1/2 \times 0.07 \times 14.4} = 320^\circ,$$

$$t_{011} = 570^\circ.$$

As the decrease in heat transmission is proportional to the ratio of the resistance of the boiler scale to the total resistance, this means a decrease of 33 per cent. on the heat transmitted were no new scale present.

Under the same conditions the temperature drop  $t_{011} - t_{01}$  from the water side of the plate to the mean temperature of the water may well be investigated. Here  $\zeta_{11}$  ranges from 0.001 to 0.01, but in common practice with good circulation,  $\zeta_{11}$  would not much exceed 0.002. This would mean a temperature drop of 20° in 950° of total temperature drop, that is, only 2 per cent. Had a sluggish liquid like sulphuric acid been evaporating in a platinum or cast iron still, the value of  $\zeta_{11}$  would probably exceed 0.01 and 0.02 and would impose a large temperature drop on the boundary between the metal and the liquid.

The common problem of determining the loss of heat from steam pipe may readily be handled by this method of analysis. A steam pipe of 80 feet length and 3 inches diameter is covered with a 1-inch packing of asbestos. The temperature of the steam is 275°, and of the outside air and surroundings 80° [= 540° absolute]; and it is desired to find the hourly condensation of steam in the pipe. Other data to be given is  $\rho_1 = 1.1$  for asbestos,  $\zeta_{11} = 0.0004$  from steam to metal, a very low value,  $\gamma = 0.45$  from the surface of the asbestos to air,  $\phi = 1$ , and  $0.0064E = 0.005$ :

<sup>1</sup> Engineer, 1896.

$$\frac{H}{T} = \frac{\Delta}{\frac{\zeta_{11}}{A_{11}} + \frac{\delta_1 \rho_1}{A} + \frac{1}{A_1(\gamma_1 + c_1)}}$$

$$A_{11} = \frac{80 \times 3 \times \pi}{12} = 63 \quad A = \frac{80 \times 4 \times \pi}{12} = 84$$

$$A_1 = \frac{80 \times 5 \times \pi}{12} = 105$$

$$-3$$

$$\delta_1 \rho_1 = 1.1 \quad c_1 = 0.005 \times 5.4 = 0.8 \quad \gamma_1 + c_1 = 1.25 \quad \Delta = 195$$

The value of  $c_1$  here is incorrect. On this assumption, the total resistance is 0.0206 while the outside surface resistance is 0.0076 which means a temperature drop of 72°, an outside surface temperature of 152° which is 613° absolute. Hence  $a = \frac{541}{613} = 0.88$  and  $1/4(1 + a + a^2 + a^3) = 0.84$ .

$$-3$$

$$c_1 = 0.005 \times 6.13 \times 0.84 = 0.97$$

$$\frac{H}{T} = \frac{195}{0.000006 + 0.013 + 0.0067} = 9900 \text{ B. t. u.}$$

= 10.8 pounds of steam condensed per hour.

A simple practical application of the combined conduction and radiation of heat is the investigation of the resistivities of air spaces of furnace walls at high and low temperature, and the resistivities of the same thicknesses of fire-brick.

In a furnace wall where the mean temperature is 1000°, an air space of 2 inches is left open to act as a non-conductor of heat. The value of 0.0064E will be assumed at 0.004 and the value of  $\gamma$  for either surface at 0.4, consequently the resultant conductivity of both surfaces in series with respect to the flow of heat is  $1/2\gamma = 0.2$ :

$$c = 0.004 \times 10 + 4.6 = 12.5$$

$$c + 1/2\gamma = 12.7$$

$$\frac{\Delta AT}{H} = \frac{1}{12.7} = 0.08$$

Had the air space been filled in with fire brick with  $\rho = 0.14$ , then,

$$\frac{\Delta AT}{Q} = \delta \rho = 2 \times 0.14 = 0.28.$$

This shows that it would be better to have the space filled in with fire brick, as it would offer greater resistance to the flow of heat. Had the space been  $1/4$  inch in thickness,  $\frac{\Delta AT}{H} = 0.08$  as before, but

$\frac{\Delta AT}{Q} = 0.04$ , in which instance, the air space would offer the greater resistance. Had the mean temperature of the space been 100° instead of 1000°, then,

$$c = 0.004 \times 1 + 4.6 = 0.7$$

$$c + 1/2\gamma = 0.9$$

$$\frac{\Delta AT}{H} = \frac{1}{0.9} = 1.1$$

and

$$\frac{\Delta AT}{Q} = 0.28.$$

In this case the air space would offer more resistance. This has been well brought out both by experiment and by calculation by Ray and Kreislinger<sup>1</sup> in "The Flow of Heat through Furnace Walls." Hence a practical criterion for air spaces for insulating purposes is

$$\delta < \frac{1}{\rho(0.2 + c)}.$$

The effectiveness of the heating surface of boilers may as readily be investigated by application of the laws of heat radiation and conduction as by the use of empirical formulae derived from experiment, since in the latter, factors are often omitted that are taken into account by the former. The amount of heat absorbed from hot gases in fire tubes is a problem involving the laws of heat conduction and convection alone, the solution of which has been previously outlined. The problem of determining the heat absorbed directly from the furnace grate of a boiler depends on the assumption of the absolute temperature of the surface of the fuel. This depends on the heating value of the fuel, the rate of fuel and air supply, and to a certain extent, on the velocity of reactions involved. It is probably simpler to assume an absolute grate temperature and work backward to determine what combinations of these factors will satisfy this assumption than it is to determine the temperature of combustion from these factors, but that lies outside the scope of this paper.

The calculation of the heat-transmitting capacity of the heating surface of the combustion chamber of internally fired boilers presents no difficulties. All areas are known. The temperature of the heating surface is the same approximately as the temperature of the steam. The grate is at an assumed or known absolute temperature and it radiates heat in all directions to a surface of known temperature. The heat absorbed by conduction may be calculated from the assumed or calculated velocity of the gases of combustion.

In the case of an externally fired boiler, the conditions are not so simple. The area  $A_1$  of the grate surface, the area  $A_{11}$  of the boiler-heating surface opposite, and the area  $A$  of the furnace walls included between these are all known. The value of  $\phi$ , the mean solid angle subtended by  $A_{11}$  with respect to  $A_1$ , may be determined by estimating  $r_m$  the mean distance between the surfaces. The temperature  $\theta_1$  of the surface of the fuel is assumed, and the temperature of the boiler surface  $\theta_{11}$  is known, but the temperature of the furnace walls  $\theta$  is not known. Now it is clear that heat passes from the fuel bed to the boiler surface in four different ways: (1) heat is radiated from the grate at an absolute temperature  $\theta_1$  to the boiler surface at an absolute temperature  $\theta_{11}$ ; (2) heat is conducted from the hot products of combustion at a temperature somewhat less than  $\theta$ , to the boiler surface at  $\theta_{11}$  by conduction; (3) heat is radiated from the fuel surface to the furnace walls at an unknown temperature  $\theta - \theta_1$  and  $\theta_{11}$  and from thence radiated to the boiler surface  $A_{11}$ ; and (4) heat is conducted into the furnace walls and thence radiated to

the boiler surface. There is always a small flow of heat through the furnace walls, so the effect of (4) on  $\theta$  may be neglected. The first step is to determine the mean absolute temperature  $\theta$  of the furnace walls. The heat radiated from the fuel surface to the furnace walls equals the heat radiated from these to the boiler. It will be assumed that  $0.16E = 0.14$ .

$$0.14A_1(1 - \phi_1) \left[ \frac{0.01\theta_1^4}{\phi_1} - \frac{0.01\theta^4}{\phi_1} \right] = 0.14A_{11}(1 - \phi_{11}) \left[ \frac{0.01\theta^4}{\phi_{11}} - \frac{0.01\theta_{11}^4}{\phi_{11}} \right]$$

$$\frac{A_1(1 - \phi_1)}{A_{11}(1 - \phi_{11})} = \frac{1 - \phi_1^2}{A_{11} - \phi_1} = N = \frac{\frac{0.01\theta^4}{\phi_1} - \frac{0.01\theta_{11}^4}{\phi_{11}}}{\frac{0.01\theta^4}{\phi_{11}} - \frac{0.01\theta_{11}^4}{\phi_{11}}}$$

Since  $\phi$ ,  $A_1$ , and  $A_{11}$  are known, the value  $N$  is known and it commonly varies from 1 to 0.5.

$$0.01\theta = \sqrt[4]{\frac{N \cdot 0.01\theta_{11}^4 + 0.01\theta_{11}^4}{N + 1}}$$

If  $a = \theta_{11}/\theta_1$  and  $a_1 = \theta/\theta_1$ ,  $R$  can now be solved, for

$$R = 0.14A_1 \frac{0.01\theta_1^4}{\Delta} [\phi_1(1 - a^4) + (1 - \phi_1)(1 - a_1^4)].$$

With respect to  $A_{11}$ , the value  $c$  may be obtained by dividing  $R$  by  $A_{11}\Delta T$ , where  $\Delta = \theta_1 - \theta_{11}$ .

$$c = 0.14 \frac{A_1}{A_{11}} \frac{0.01\theta_1^4}{\Delta} [\phi_1(1 - a^4) + (1 - \phi_1)(1 - a_1^4)]$$

and

$$k = c + g = \frac{H}{\Delta T}$$

A common formula for  $g$  in terms of the pounds of coal  $B$  burned per square foot of grate surface per hour is

$$g = 0.4 + 0.9\sqrt{B}.$$

The classical researches of Blechynden are often quoted to show that heat transfer from hot furnace gases is proportional to the square of the temperature drop. Formulas of Rankin, Werner, and others assume that heat transfer at high temperatures is proportional to the square of the temperature difference.

It would not be out of place to present here some experimental data obtained by Blechynden<sup>3</sup> on this point. The furnace was cylindrical in shape of 12 1/2 inches internal diameter, above which was supported a cylindrical vessel of 10 inches diameter and 12 inches height. The furnace was heated by five jets of illuminating gas mixed with air, and these played upon a layer of asbestos so as to provide an even heat. The vessel rested upon a conical mantle provided with four exit pipes to carry away the products of combustion, of which the temperature was taken by means of a pyrometer, as was also the temperature just above the asbestos grate. The three moduli obtained by Blechynden may be expressed in terms of the

<sup>1</sup> Bull. 8, Bureau of Mines.

<sup>2</sup>  $A_1\phi_1 = A_{11}\phi_{11}$ .

<sup>3</sup> Proc. Inst. Nav. Arch., 1894; Engineer, 1893.

temperature  $\theta_1$  of the asbestos grate,  $\theta$  of the heating gases, and  $\theta_{11} = 212$ , the temperature of the vessel. H, A and T have the usual significance.

$$\mu = \frac{H}{AT(\theta - \theta_{11})} \quad \mu' = \frac{H}{AT(\theta - \theta_{11})(\theta_1 - \theta_{11})} \quad \mu'' = \frac{H}{AT(\theta_1 - \theta_{11})}$$

Hence,

$$\mu'' = \mu' \left( \frac{\mu'}{\mu} \right).$$

Plate.	Thickness.	$\mu$	$\mu'$	$\frac{\mu'}{\mu}$	$\mu''$
A.....	1 1/4"	0.0155	..	..	..
A.....	3/4"	0.0176	..	..	..
A.....	9/16"	0.0212	..	..	..
A.....	1/4"	0.0230	..	..	..
A.....	1/8"	0.0239	..	..	..
B.....	1 5/32"	0.0239	..	..	..
B.....	3/8"	0.0245	..	..	..
B.....	1/4"	0.0257	..	..	..
B.....	3/32"	0.0261	0.0206	0.79	0.0163
C.....	1 3/16"	0.0182	..	..	..
D.....	1/2"	0.0237	0.0174	0.74	0.0129
E.....	1 1/4"	0.0142	0.0097	0.68	0.0066
E.....	3/4"	0.0191	0.0142	0.74	0.0105
Average.....		0.0213	0.0155	0.74	0.0115

Another set of experiments were carried out at the Physikalisch-Technischen Reichanstalt in 1895-96 in the same kind of apparatus. The temperatures  $\theta$  were taken with a pyrometer at 1 1/2 inches below the center of the heating surface.

$\theta$ .	$\theta - \theta_{11}$ .	$\frac{H}{AT}$	$\frac{H}{AT(\theta - \theta_{11})}$	$\mu' = \frac{H}{AT(\theta - \theta_{11})^2}$
Plate thickness = 1 3/16".				
705	493	4400	8.92	0.0181
812	600	6050	10.08	0.0168
875	663	7610	11.47	0.0173
896	684	7970	11.65	0.0168
912	700	8820	12.60	0.0180
1042	830	11020	13.28	0.0160
1162	950	13760	13.96	0.0147
1209	997	15200	15.25	0.0153
1245	1033	17410	16.85	0.0163
Average,				0.0166
Plate thickness = 1 3/32".				
655	443	3200	7.23	0.0163
763	551	4800	8.71	0.0158
903	691	7500	10.85	0.0157
1117	905	12200	13.48	0.0149
Average,				0.0153
Plate thickness = 5/16".				
586	374	2370	6.32	0.0169
768	556	4980	8.95	0.0161
931	725	8730	12.04	0.0166
962	750	8280	11.03	0.0147
1064	852	11900	13.97	0.0164
Average,				0.0161
Plate thickness = 3/16".				
607	395	2540	6.44	0.0163
784	572	4775	8.35	0.0146
930	718	7720	10.77	0.0150
1122	910	13730	15.10	0.0166
Average,				0.0156

Since the temperature  $\theta$  employed in these latter experiments is probably intermediate between the temperature of the asbestos grate and the temperature of the outgoing products of combustion, the mean value  $\mu'$  here obtained is 0.0159, checks very close to  $\mu'$  obtained by Blechynden 0.0155. If 0.0159 is multi-

plied by the mean value of  $\frac{\mu'}{\mu}$ , 0.0118 is obtained where Blechynden obtained 0.0115.

It is very clear from these experiments that the heat transmitted at high temperatures varies nearly as the square of the temperature difference. That this is in accord with the laws of heat radiation and conduction can be shown by applying the formulas on page 811 to a special case of heat transfer into an externally fired boiler from the combustion chamber

where  $\frac{A_{11}}{A_1} = 1.2$ ,  $\phi_1 = 0.6$  and consequently  $N = 0.67$ .

The temperature of the steam is 300° and the temperature at the grate varies from 1400° to 1000°. The value of  $g = 2$ .

$\theta_1$ .	0.01 $\theta_1$ .	0.01 $\theta_{11}$ .	$\Delta$ .	0.01 $\theta$ .	$\alpha$ .	$\alpha_1$ .	$c$ .	$k$ .	$\mu'' = \frac{k}{\Delta}$ .
1400	18.6	7.6	1100	14.95	0.41	0.80	10.4	12.4	0.0113
1300	17.6	7.6	1000	14.20	0.43	0.81	9.0	11.0	0.0110
1200	16.6	7.6	900	13.40	0.46	0.81	7.8	9.8	0.0109
1100	15.6	7.6	800	12.65	0.49	0.81	6.8	8.8	0.0110
1000	14.6	7.6	700	11.95	0.52	0.82	5.9	7.9	0.0113
Average,									0.0111

The value of  $\mu'' = 0.0111$  checks very closely to the value 0.0115 obtained by Blechynden, and 0.0118 from the experiments later conducted. This is purely accidental. What is of importance is that results obtained by experimental investigation can be rationally duplicated by systematic application of the laws of heat radiation and conduction.

#### THE EFFECT OF ADDED FATTY AND OTHER OILS UPON THE CARBONIZATION OF MINERAL LUBRICATING OILS.<sup>1</sup>

By C. E. WATERS.

Received September 9, 1911.

In a paper on "The Behavior of High-Boiling Mineral Oils when Heated in the Air,"<sup>2</sup> the author called attention to the fact that two straight mineral oils under investigation yielded more "carbonized" matter, insoluble in petroleum ether, when they were heated in brass tubes than when they were heated in glass tubes of the same dimensions. The reverse was true of a third oil having a saponification number indicating the presence of about 0.5 per cent. of fatty oil. This naturally suggested the advisability of determining the amount of carbonization of a straight mineral oil and of the same oil with known amounts of other constituents, such as lard oil, rosin oil, tallow, etc., added. The results of a series of such determinations are given in the present paper.

The oil selected was an engine oil flashing at 140° in the Pensky-Martens closed cup apparatus. For the first tests there were seven samples as follows:

No. 1. The straight mineral oil. When 10 grams, diluted with 50 cc. of petroleum ether, were allowed to stand over night, it yielded only traces of precipitate. These proportions of oil and solvent are the same as adopted in the carbonization tests.

No. 2. The mineral oil heated with "ivory" soap.

<sup>1</sup> Published by permission of the Director of the Bureau of Standards.

<sup>2</sup> Bull. Bur. Standards, 7, 365 (1911); THIS JOURNAL, 3, 233 (1911).