ART. XXX.—*On vibrating Water-falls*; by ELIAS LOOMIS, Professor of Natural Philosophy and Astronomy in Yale College.

IN the year 1843, I published in this Journal, vol. xlv, p. 360, a notice of several water-falls which at times exhibited very strong vibratory motion. I am not aware that any one else had ever published anything on this subject previous to that time. In that article, I proposed an explanation of these vibrations, which was naturally suggested by the only case which I had myself had an opportunity to investigate; viz., that the dam itself was the vibrating body, and that the vibrations were analogous to those of a stretched cord. The attention of several other observers has since been called to the same subject, and Prof. Snell has endeavored to show that the vibrating body is the column of air behind the sheet of water, and that the time of vibration depends upon the length of this column. I have thus been led to examine the subject anew, and have been compelled to modify the views which I first published. I have made a large number of observations on several different water-falls, and have obtained numerous observations made by other persons. These observations have been made chiefly at three places, viz: at South Natick, Mass.; at Holyoke, Mass.; and at Lawrence, Mass.

Observations at South Natick, Mass.

In August, 1859, Mr. William Edwards communicated to me the results of numerous observations which he had made upon a water-fall at South Natick, Mass., about 18 miles west of Boston. The dam over which the water fell, consisted of two separate portions. One of them, the northern, was 98 feet long and 9 feet high, built of hewn timbers generally about 10 inches square, framed together, and banked up with earth to within

three feet of the edge. Mr. Edwards, who resided near the dam, had determined that

When the height	10 inches,	the number	280 per minute.
of the water	8 "	of	300 "
on the edge	7 "	vibrations	335 "
of the dam was	5 "	was	over 400 "

I at once perceived that these numbers indicated a general law; viz., that *the time of one vibration is nearly equal to the time in which a heavy body would fall through a space equal to the depth of water on the dam.* This will appear from the following table, in which column first shows the depth of water on the edge of the dam; column second shows the time of one vibration expressed in fractions of a second; and column third shows the time which a heavy body would require to fall through the

spaces in column first, and is computed by the formula $t = \sqrt{\frac{2s}{g}}$.

Column fourth shows the quotients obtained by dividing the numbers in column second by those in column third.

Depth of water.	Time of one vibration.	Time of falling through depth.	Ratio.
10 inches.	0 ^s .214	0 ^s .228	0.940
8 "	0.200	0.204	0.982
7 "	0.179	0.190	0.940
5 "	less than 0.150	0.161	less than 0.932

In April, 1861, a portion of this dam, about 70 feet in length, was carried away, and was rebuilt during the following summer. The new part of the dam is of the same height as the old one, and is boarded up tight in front at a slope of about 75°; but the old part, being about 30 feet in length, is open in front as formerly. In the annexed figure,

AB represents the height of the dam, AC the slope inclined against the stream, and AD shows how the new part is boarded up on the lower side. Formerly, the space behind the sheet of water was entirely free, back to AC, with the exception of the interruption caused by the beams which supported the

dam. Now when the water is only 5 inches deep, the descending sheet strikes AD within about 12 inches of the bottom. There is a square stone abutment on the south side of the sheet, and it is boarded up with planks on the north side, so that the air behind the sheet of water is almost completely enclosed.

The south dam is 100 feet long, and 9 feet high, and boarded up in front at a slope such that with a depth of 8 inches of water,

the descending sheet strikes the sloping boards nearly in the middle of their height. The south dam is quite old and its edge is rough, so that the vibrations are never very regular or strong. The north dam is the one upon which Mr. Edwards' observations have all been made.

In the spring of 1863, Mr. Edwards, at my request, made a more extended series of observations, measuring accurately the depth of water on the edge of the dam, and counting the corresponding number of vibrations. The depth of water was measured by inserting a small rod 4 inches from the edge of the dam. Mr. Edwards states that, at a distance of 20 rods above the dam, the water was $1\frac{1}{2}$ inches higher than at the point where he measured it. The following table shows the results of Mr. Edwards' observations in 1863, and is deduced from numerous trials between April 23 and May 4.

Depth of water.	Time of one vibration.	Time of falling through depth.	Ratio.
9.21 inches.	0.220	0.218	1.007
8.12 "	0.192	0.205	.937
7.00 "	0.177	0.190	.930
6.31 "	0.164	0.181	.906
5.06 "	0.138	0.162	.850

These results differ sensibly from those deduced from Mr. Edwards' first series of observations. The difference may be ascribed in part to the greater accuracy of the last observations, since fractions of an inch were disregarded in the first measurements. Perhaps also something is due to the modification in the structure of the dam. I am however inclined to think that the influence of the latter cause is slight, and that the discrepancies are mainly due to the unavoidable imperfection of such observations. Any one who shall attempt to repeat these observations, instead of complaining that Mr. Edwards' results are not perfect, will probably be surprised at the precision which he has actually attained.

The result, then, which we deduce from these observations at South Natick, is that *the time of one vibration is a little less than the time in which a heavy body would fall through a space equal to the depth of water on the dam*; and this deficiency increases as the depth of water diminishes.

Observations at Holyoke, Mass.

The dam across the Connecticut river at Holyoke, Mass., is 1017 feet long and 30 feet high. It is formed of square timbers, inclined 22 degrees to the horizon, having one end bolted to the rock, and the other resting upon a timber frame-work. From the crest of the dam, the water descends along an apron about four feet in length, which slopes downwards at an angle

of 22 degrees. Professor Snell has observed the vibrations at six different times, and with the following average results (this Journal, vol. xxviii, p. 229).

When the water	{ 1 foot in height,	the number	{ $257\frac{1}{2}$ per minute.
on the edge	{ 2 feet "	of vibra-	{ $137\frac{2}{3}$ "
of the dam is	{ 5 feet "	tions is	{ 82 "

I have spent considerable time at Holyoke, for the purpose of determining the number of vibrations corresponding to different depths of water. I visited the place in August, 1859, but there were only 6 inches of water on the crest of the dam, and the vibrations were not noticeable. I obtained the engineer's plan

2.

to 76 inches. I also made a few observations in July, 1861.

To indicate the depth of water, there is a gauge, graduated to feet and inches, placed 60 feet from the crest of the dam,

3.

and firmly secured to the stone abutment. The annexed plan will show the position of the gauge with reference to the dam. The figure represents the stone abutment on the west side of the dam, together with a small portion of the dam. The position of the gauge is indicated by the letter G. At each of my observations, I recorded the depth of water by this gauge; and when the water was not more than two feet deep, I measured it directly on the crest by means of a pole. When the depth was

too great to allow this mode of measurement, I measured the difference of level between the surface of the water at the gauge,

and on the crest, by comparing it with the top line of the abutment, which appeared to have been levelled with considerable care. In this manner it was found that when the depth, as shown by the gauge, was $97\frac{1}{2}$ inches, the depth on the crest was 15 inches less. The following table exhibits the average of a considerable number of comparisons.

Depth by gauge.	Depth on crest.	Difference.
14.75 inches.	12.25 inches.	2.5 inches.
16.5 "	13.5 "	3 "
23 "	19 "	4 "
97.5 "	82.5 "	15 "

For depths less than two feet, the depression of water on the crest of the dam is 0.175 of the depth by the gauge; but for a depth of 8 feet, the depression is 0.154 of the depth by the gauge. From these results, I have deduced the depth of water on the crest from the indications of the gauge.

On the 10th of April, 1861, the vibratory motion of the descending sheet of water was through a space of one or two feet, and was synchronous from one end of the dam to the other. Through each of the abutments, and on a line with the edge of the dam, is a passage way, $2\frac{1}{2}$ feet wide and 7 feet high. Through these passages, the air frequently flows with great force. On the 10th of April, at each vibration of the sheet of water, the air rushed through with great violence towards the water-fall. There was however no *outward* motion, but simply a lull between the vibrations.

April 11th, 1861, the motion of the sheet was greatest near the middle, but was very noticeable throughout the whole length. There was no wind of any consequence. Depth of water by the gauge 4 ft. $0\frac{1}{2}$ inch.

April 12th, the vibrations were exceedingly strong, with a fresh breeze from the south. Gauge 4 ft. $3\frac{1}{2}$ in.

April 13th, in the morning, the vibrations were sometimes very strong, and then soon afterward moderate, with frequent variations, the wind blowing in gusts from the south with a drizzling rain. Gauge 4 ft. $5\frac{1}{4}$ in.

April 13th, toward evening, the vibrations were very strong for 6 or 8 times, then feeble for about as many more; and so on, alternately strong and feeble. Near the middle of the sheet, the vibrations were tolerably regular; but towards either extremity, it appeared inclined to stop, and seemed to be forced into vibration by the power of the middle portion of the sheet. Gauge 4 ft. $9\frac{1}{2}$ in.

April 14th, 1861, the vibrations had ceased entirely. Gauge 6 ft. 8 inches.

April 15th, the sheet of water was extremely smooth, without the slightest sign of vibrations. Through the passage way in

the abutment on the east side of the river, the air flowed in a strong and steady breeze without pulsations. There was a slight breeze from the northwest. Gauge 8 ft. $2\frac{3}{4}$ inches.

April 16th, the sheet of water was tolerably smooth without the slightest appearance of periodical vibrations. Gauge 7 ft. $10\frac{1}{2}$ inches.

Thus it will be seen that when the gauge indicated 4 ft. $9\frac{1}{2}$ inches, the vibrations were becoming irregular; and when the water rose to 6 ft. 8 in. the vibrations ceased entirely. As this rise of the water occurred in the night, it was impossible to determine the exact depth of water at which the vibrations ceased.

The following table presents a summary of all my observations at Holyoke.

Prof. Loomis' observations at Holyoke.

Date.	Depth of water by gauge.	No. of vibrations per minute.	Temperature of water.	Date.	Depth of water by gauge.	No. of vibrations per minute.	Temperature of water.
1859, Aug. 16	6 inches.	0		1860, July 7	28 inches.	135.4	
1861, July 30	14 "	257.0	$73\frac{1}{2}^{\circ}$	1861, Apr. 10	47 "	81.6	$43\frac{1}{2}^{\circ}$
" July 31	$14\frac{1}{2}$ "	256.6		" Apr. 11	$48\frac{1}{2}$ "	81.2	
1860, July 10	16 "	254.9	73	" Apr. 12	$50\frac{3}{4}$ "	80.8	43
" July 9	17 "	249.2		" Apr. 12	$51\frac{1}{2}$ "	80.9	
" July 9	18 "	256.0		" Apr. 13	54 "	80.0	
" July 8	23 "	134.4		" Apr. 13	$57\frac{1}{2}$ "	78.6	
" July 8	25 "	134.8		" Apr. 14	80 "	0	43

Mr. Joseph P. Buckland, a graduate of Yale College, who resides at Holyoke, has at my request made a series of observations, of which the following is a summary.

Date.	Depth by gauge.	Vibrations per minute.	Date.	Depth by gauge.	Vibrations per minute.
1862	ft. in.		1862	ft. in.	
Sept. 22	1 $10\frac{1}{2}$	308 nearly.	Sept. 10	2 $5\frac{1}{2}$	128
" 21	1 $10\frac{1}{4}$	288 or 290	" 6	2 6	133
" 9	1 $11\frac{1}{2}$	296 or 298	" 4	2 7	128
Aug. 31	2 0	280	" 5	2 $7\frac{1}{2}$	134
Sept. 1	2 0	280	" 5	2 $7\frac{3}{4}$	133
" 9	2 $\frac{1}{2}$	292	" 2	2 $8\frac{1}{2}$	133
" 8	2 $2\frac{1}{2}$	273	" 3	2 $10\frac{1}{2}$	130

It will be perceived that my observations, for depths from 14 to 18 inches, agree pretty well with Prof. Snell's, as stated on page 355; but Mr. Buckland differs materially from both of us. No one, without trying it, can properly appreciate the difficulty of estimating the number of these vibrations when they exceed four per second; and I am of opinion that Mr. Buckland has estimated the velocities for depths from 22 to 26 inches, considerably too great. For depths a little greater than two feet, we all agree reasonably well.

It must be considered as sufficiently established by these observations, that the sheet of water at Holyoke exhibits three

different rates of vibration, viz., about 256, 135, and 81 vibrations per minute, corresponding to depths of 16, 28 and 56 inches of water on the dam. The change from the first to the second rate of vibration, takes place when the depth of water is from 23 to 26 inches; and the change from the second rate to the third, takes place when the depth is from 35 to 47 inches. The vibrations are not noticed when the depth of water is less than about 12 inches, and they also disappear when the depth is as great as 80 inches.

Moreover, for either of these modes of vibration, the number of vibrations per minute is not absolutely constant. On account of the discrepancies in the observations, this proposition may not be considered as fully established for the two more rapid modes of vibration; but for the slower mode of vibration the proof is unequivocal. The observations from April 10th to 13th, 1861, were very numerous, and were made with great care, and the vibrations were so slow that they could be counted with almost as great precision as those of an ordinary seconds' pendulum. The number of vibrations per minute diminishes as the depth of water increases from 47 to 57 inches. This effect is of the same kind as that observed at South Natick, but the quantity of the effect is much less.

In order to reduce, as far as possible, the influence of the errors of observation, and to diminish the labor of calculation, I have divided my observations into four groups, and taken the average of the depths of water, as well as of the times of vibration. The first group includes the observed depths less than 20 inches; group second includes the observed depths from 20 to 40 inches; group third, the depths from 40 to 51 inches; and group fourth, the depths from 51 to 60 inches. The results are as follows:

Depth of water by gauge.	No. of vibrations per minute.	Temperature of the water.	Observed time of one vibration.
15.90 inches.	254.74	73°	0.8236
25.33 "	134.87	73	.445
48.75 "	81.20	43	.739
54.33 "	79.83	43	.752

It is evident, from the motion of the air through the passage way in the abutment at the east end of the dam, that the air behind the sheet of water vibrates simultaneously with the sheet. Is it possible to explain the preceding observations on the supposition that the time of one vibration depends simply upon this column of air? The column of air behind the sheet of water is 1017 feet long; its height April 12, 1861, was about 28 feet, and its average thickness was perhaps 8 feet. The temperature of the air behind the sheet is assumed to have been that of the water, viz: 43°. The velocity of sound at this temperature is 1098 feet per second.

A column of air 1017 feet long, and having a very small diameter, would make one vibration in $\left(\frac{1017}{1098}\right)0^s.926$. According to experiments with organ pipes, the length of the column in this formula should be increased by some function of the breadth, generally about twice its breadth. If we increase the length of the column of air by the sum of the height and thickness, we shall have for the computed time of one vibration $\left(\frac{1053}{1098}\right)0^s.959$. Either of these quantities exceeds the time actually observed, by about one-fourth of the latter quantity.

It appears, from experiments with organ pipes of different forms and with different embouchures, that the time of one vibration varies considerably according to the mode in which the impulse is communicated to the column of air; but I do not know of any established principle which will explain how a column of air, having the dimensions of that at Holyoke, should make one vibration in three-fourths of a second, whether it vibrates as one mass, or as two separate portions. The change in the number of vibrations, when the depth of water increases from 48 to 54 inches, indicates that the time of one vibration does not depend solely upon the dimensions of the column of air; and this motion has the appearance of a constrained vibration, depending upon two causes which separately would produce vibration in unequal times. I have therefore computed the time of one vibration, first upon the supposition that the air is the only vibrating body, and secondly upon the supposition that the vibrations originate in the sheet of water, according to the law indicated by the observations at South Natick.

Depth by gauge.	Depth on crest = D.	Observed time of vibration.	Vibration of air computed.	Time of falling through D.
15.90	13.12	0 ^s .236	0 ^s .232	0 ^s .261
25.33	20.90	.445	.465	.329
48.75	40.63	.739	.959	.459
54.33	45.28	.752	.959	.484

In the preceding table, column I shows the depth of water in inches according to the gauge; column II shows the depth on the crest of the dam, determined according to the principles explained on page 356; column III shows the observed time of one vibration; column IV shows the time of one vibration of a column of air 1017 feet long, computed in the manner explained above; the air in the first case being supposed to be divided into four vibrating portions, and in the second case into two portions; and column V shows the time in which a heavy body would fall through the spaces given in column II.

It will be noticed that the observed time of vibration in each case falls between the times computed by these two methods.

Observations at Lawrence, Mass.

The dam at Lawrence is 900 feet long, and has an average height of 34 feet. It is in the form of an arc of a circle whose radius is 7433 feet; and the versed sine of the arc is therefore nearly 14 feet. It is built of hewn stone, consisting of large blocks, accurately fitted to each other.

Sixty-five feet from the south side of the river, is a fish way 30 feet wide, which effectually divides the falling sheet of water, cutting off 95 feet of the dam. The portion of the sheet of water which is entirely free, cannot exceed 805 feet in length.

Stout bars of iron about three feet long, are inserted in holes drilled in the top stones, at intervals of 3 or 4 feet, and these support a row of planks called flash boards, which raise the water 18 inches higher than the stone work of the dam. These planks are put up in June, and remain until they are carried off by the spring or winter freshets. Until these planks are carried away, the vibrations are never noticed.

I have obtained the coöperation of an excellent observer at Lawrence, viz: Mr. Benjamin Coolidge, engineer of the Essex Manufacturing Company, who has watched the dam for several years, and has made careful observations at my request. The following is a summary of his results:

In the spring of 1860, there was no freshet, and no vibrations were noticed upon the sheet of water.

In the spring of 1861, the vibrations were observed as follows:

March 5th. 100 vibrations in $37\frac{1}{2}$ seconds. Depth of water 3·63 feet.

March 11th. 100 vibrations in $37\frac{1}{2}$ seconds. Depth of water 3·41 feet.

April 1st. 100 vibrations in $37\frac{1}{2}$ seconds. Depth of water 3·12 feet.

The flash boards were on the north end of the dam, for about 60 feet; thence for 40 feet mostly gone. At the south end of the 40 feet, the vibrations began and continued to the fish weir, which gives about 700 feet of vibrating fall. The vibrations made a distinct noise that could be heard pulsating above the roar of the fall.

April 16th. 100 vibrations in 51 seconds. The depth of water on the edge of the crest-stone was 5 feet. Perhaps there were some 50 feet more of vibrating over-fall than on the 1st of April.

April 20th. 160 vibrations per minute. Water on the crest-stone 3·08 feet. Temperature of water 44° F. Temperature of air 46°. As near as I could judge, the length of the vibrating sheet was very nearly 780 feet, and very much the strongest in the middle of the sheet. The rush of air at the north end of the sheet (the only one open to observation) was very strong;

and followed the vibration of the sheet, but with a continuous inward draft.

In the spring of 1862, the following observations were made.

April 18. 162 vibrations per minute. Depth of water 5 feet. Thermometer 44° . These vibrations were for 200 feet at north end of dam, alternating with vibrations in middle of dam.

April 19. $134\frac{1}{2}$ vibrations per minute. Depth of water 5 feet. Therm. 42° .

April 21. 136 vibrations per minute. Depth of water 5 feet. Therm. 42° .

April 26. $156\frac{1}{2}$ vibrations per minute. Depth of water 3.18 feet. Therm. 43° .

May 5. 150 vibrations per minute. Depth of water 3.25 feet. Therm. 49° .

May 6. 154 vibrations per minute. Depth of water 2.95 feet. Therm. 50° deg.

April 18, 1863, Mr. Coolidge observed the number of vibrations to be 122 per minute, and the depth of water $5\frac{1}{2}$ feet measured on the crest of the dam.

The following table presents a summary of

Mr. Coolidge's observations at Lawrence, Mass.

Date.	Depth of water.	Observed time of one vibration.	Date.	Depth of water.	Observed time of one vibration
1862, May 6	2.95 feet.	0 ^s .390	1861, March 5	3.63 feet.	0 ^s .375
1861, April 20	3.08 "	.375	" April 16	5.00 "	.510
" April 13	3.12 "	.372	1862, April 18	5.00 "	.370
1862, April 26	3.18 "	.383	" April 19	5.00 "	.446
" May 5	3.25 "	.400	" April 21	5.00 "	.441
1861, March 11	3.41 "	.375	1863, April 18	5.50 "	.492

The first seven of these observations agree with each other about as well as such observations usually do. The mean of these seven observations gives a depth of 3.23 feet, and the corresponding time of vibration is 0^s.381.

The observations of April 16, 1861, and April 18, 1862, differ very widely. I can only ascribe the discordance to the effect of flash boards which had not been carried away, and which created openings in the descending sheet of water. I have therefore thought best to take the mean of the last five observations, which gives the time of one vibration 0^s.452, corresponding to a depth of 5.10 feet.

I visited Lawrence in August, 1859, and again in August, 1861, but the depth of water at these times was not sufficient to produce vibration. In April, 1863, I had the good fortune to witness the vibrations with considerable force. I found it very difficult to count the number of vibrations satisfactorily. For a few seconds, the pulsations would be very marked, and then they would often become so faint as to be noticed with difficulty, so that it was generally impossible to follow them for an entire

minute very satisfactorily. The descending sheet of water was very uneven, being marked by numerous ridges, probably caused by the flash boards having been only partially carried away. I spent two days in almost constant attempts to count the vibrations, and my result was an average of 148.68 vibrations per minute, corresponding to a depth of 4.41 feet water by the gauge. If we reduce this depth according to the rule determined at Holyoke, it would give a depth of 3.68 feet on the crest of the dam; but according to Mr. Coolidge's measurements, it would indicate a depth of exactly 3 feet on the crest of the dam.

Comparing all these observations, they seem to favor the conclusion that the time of one vibration increases as the depth of water on the dam increases; but the discrepancies are so great that even this conclusion might be called in question, if it were not confirmed by the observations at South Natick. If we take an indiscriminate mean of all the observations, we find the average time of one vibration to be 0^s.409, corresponding to a depth of water on the crest of the dam amounting to 4 feet, and a temperature of the water 44°.

A column of air 805 feet in length, 30 feet high, and 7 feet wide, according to the principles stated on page 359, would make one vibration in 0^s.766; or if it vibrated in two portions, the time would be 0^s.383. A heavy body would fall through a space of 4 feet in 0^s.499. The observed time is intermediate between the times computed by these two methods.

It seems impossible to explain all the preceding observations upon the supposition that the air behind the sheet of water is the sole vibrating body, following the laws which have been established in the case of organ pipes. The observations at South Natick naturally suggest the idea that the vibrations originate in the sheet of water itself; the time of one vibration being nearly proportional to the square root of the depth of water on the dam.

Is there any known principle which will enable us to explain vibrations of this description?

It has been suggested that such vibrations may originate in the unequal velocity of the upper and lower strata of the sheet of water which pours over the dam. If there were no friction, the lower stratum should have a velocity equal to that which a heavy body would acquire in falling through a space equal to the height of the water above the dam. The upper stratum has only the velocity of the stream, combined with that which is imparted by the motion of the lower stratum. The lower stratum (if it could move independently) would describe a parabolic path, which could be computed from the depth of water on the dam. But the upper stratum acts as a load, forcing the lower stratum to describe a path more nearly vertical. The descending sheet of water is therefore in a condition of equilibrium, resulting from

the superior momentum of the under part of the sheet, balanced by the weight of the upper part. Suppose now that from any external cause (for example a puff of air) the sheet is forced back beyond the position of equilibrium, the instant this force ceases to act, the momentum of the under part of the sheet forces the entire mass forward, and its inertia carries it beyond the position of equilibrium. The weight of the upper part of the sheet forces it again backward; and thus it has been supposed there might result a series of vibratory movements.

It seems difficult, however, upon this theory, to account for the nearly isochronous, long continued and powerful vibrations which we actually observe.

It has been suggested that this vibratory motion originates in the column of air behind the sheet of water, while the water serves merely as a load, to retard the velocity of these vibrations. The column of air behind the sheet of water at South Natick, is of such dimensions that it should make one vibration in about one-tenth of a second. When the depth of water is 5 inches, the observed time of vibration is $0^s.038$ greater than the computed time of an air vibration; but with a depth of 10 inches the observed time is $0^s.114$ greater than the computed time.

When an elastic string is loaded, the time of one vibration varies as the square root of the load. At South Natick, when the depth of water increases from 5 to 10 inches, the increase in the time of a vibration seems to be even more rapid than the increase in the depth of water.

At Holyoke, when the depth of water is 45 inches (if we suppose the air to vibrate as one mass) the observed time of vibration is less than the computed time for a column of air without any load, which appears impossible according to this theory. If we suppose the column of air to be divided into two vibrating segments, the time of an air vibration is $0^s.480$, while the observed time is $0^s.272$ greater than this.

When the depth of water is 40 inches, the observed time is $0^s.259$ greater than that of an air vibration.

When the depth of water is 21 inches, (if we suppose there are only two vibrating segments), the time of an air vibration is $0^s.465$, while the observed time is a little less than this; which seems impossible upon the present theory. If we suppose there are three vibrating segments, the time of an air vibration is $0^s.310$; while the observed time is $0^s.135$ greater than this.

When the depth of water is 13 inches, (if we suppose there are four vibrating segments) the computed time of an air vibration, is $0^s.232$, while the observed time is only $0^s.004$ greater than this. If we suppose five vibrating segments, the time of an air vibration is $0^s.186$, while the observed time is $0^s.050$ greater than this. If we suppose six vibrating segments, the

observed time is 0^s.081 greater than the computed time of an air vibration.

At Lawrence, (if we suppose there are only two vibrating segments,) the time of an air vibration is 0^s.383, while the observed time exceeds this by only 0^s.026. If we suppose there are three vibrating segments, the time of an air vibration is 0^s.255, and the observed time exceeds this by 0^s.154. If we suppose there are four vibrating segments, the observed time is 0^s.218 greater than the computed time of an air vibration. If we suppose there are six vibrating segments, the observed time is 0^s.281 greater than the computed time of an air vibration.

If now we compare the results obtained at these three localities, we shall find that we may make such a supposition with regard to the number of vibrating segments into which the column of air divides itself, that the apparent retarding effect of the water shall be nearly proportional to the thickness of the sheet. The following table presents a summary of these results.

	Depth of water = D.	Observed time of one vibration.	Number of vibrating segments supposed.	Computed time of an air vibration.	Retardation produced by water = R.	Quotients. $\frac{R}{D}$
South Natick	5.06 inches,	0 ^s .138	1	0 ^s .100	0 ^s .038	.0075
	10.00 "	0 ^s .214	1	0 ^s .100	0 ^s .114	.0114
	13.12 "	0 ^s .236	6	0 ^s .155	0 ^s .081	.0062
Holyoke	20.90 "	0 ^s .445	3	0 ^s .310	0 ^s .135	.0064
	40.63 "	0 ^s .739	2	0 ^s .480	0 ^s .259	.0064
	45.28 "	0 ^s .752	2	0 ^s .480	0 ^s .272	.0060
Lawrence	48.00 "	0 ^s .409	4	0 ^s .191	0 ^s .218	.0045
	48.00 "	0 ^s .409	6	0 ^s .128	0 ^s .281	.0059

Column fourth shows the computed time of an air vibration, when the number of vibrating segments is assumed as in column third; column fifth shows the difference between the numbers in columns second and fourth, being the retarding effect of the sheet of water according to the theory under examination; and column sixth shows the quotients obtained by dividing the numbers in column fifth, by the numbers in column first. These quotients, with one exception, are nearly constant, indicating that the retarding effect of the sheet of water is nearly proportional to its thickness. The exception to this general rule which occurs at South Natick, may perhaps be ascribed to the small height of the fall, in consequence of which, especially when the water is high, the descending sheet acquires somewhat of the rigidity of a solid body.

The number of vibrating segments of the column of air, is the principal unknown quantity which renders our conclusions so very uncertain. I do not know of any theory which will enable us to compute the precise influence of a sheet of water of given dimensions; but at present, it seems probable that the vibratory motion originates in the column of air behind the sheet of water,

and that the descending sheet serves merely as a load, to retard the velocity of these vibrations.

This theory will be fully established or disproved, when we are able to compute the effect produced upon the vibrations of a column of air, by a descending sheet of water of given dimensions.

Why are not these vibrations noticed wherever water pours over a dam?

It is believed that most water-falls exhibit some degree of vibratory motion at certain stages of water; but in order that these vibrations may be powerful and long-continued, the edge of the dam must be horizontal, and quite smooth; otherwise the thickness of the descending sheet will not be uniform; and the sheet will swell into ridges in some places, while other parts become thin. The sheet will divide in some places before reaching the bottom of the fall, and this leaves an opening in the enclosure which contains the column of vibrating air.

This is probably the reason why many water-falls never exhibit this phenomenon in a palpable manner; and why in only a few cases is the vibration so powerful as to cause any annoyance.

Why is a particular height of water requisite to produce these vibrations?

The descending sheet of water must have a thickness of several inches; otherwise it is divided by the action of the air, and the column of air ceases to be enclosed on all sides. With a fall of nine feet, as at South Natick, a thickness of 4 or 5 inches is requisite; and with a fall of 30 feet, as at Holyoke, a thickness of nearly a foot is requisite. At Lawrence, with a fall of 34 feet, the vibrations are not noticed when the depth of water is much less than 3 feet; but this seems to be owing to the inequalities on the top of the dam, resulting from the iron bars being bent over by the spring freshets, and confining some of the flash boards to the top of the dam.

The vibrations cease almost entirely when the water exceeds a certain height, because the thickness of the sheet becomes too great in comparison with its height, and there being some cohesion between the particles of the liquid, the sheet partakes somewhat of the rigidity of a solid body. In order to produce a strong effect, the thickness of the sheet must not exceed about one-sixth or one-eighth of the height of the fall. At South Natick, with a fall of 9 feet, which is somewhat diminished by the back water at the time of a freshet, the vibrations cease when the depth of water much exceeds ten inches. At Holyoke, with a fall of 30 feet, which is also diminished by the back water at the time of a freshet, the vibrations cease when the depth of water much exceeds 5 feet. At Lawrence also, where the fall is a little greater than at Holyoke, the vibrations cease when the depth of water on the crest of the dam much exceeds five feet.