III.—*On the Measurement of the Angle of Aperture of Objectives.*

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IN arranging in the form of a paper, some recent investigations on this yet undecided question, I have no desire to maintain a controversy that has at times appeared as one of personal feeling instead of scientific reasoning. The facts cannot be established by a majority of opinions, but by actual experiments. It is to these principally that I now refer.

Professor Stokes, at the meeting of this Society held in July last, has brought forward a question, and shown in theory that by means of a front lens, with an emergent surface exceeding the hemisphere, a ray may be refracted within the substance of the glass in a direction at right angles to the axis or at an angle of 180° . He then remarks that if the reduction of the surface be to a hemisphere, "the aperture in glass, though reduced from the extreme of 180° , still remains very large."

It is with front lenses having refracting surfaces less than a hemisphere that we have been dealing, and to such Professor Keith's paper on the Tolles $\frac{1}{8}$ refers. It may be stated that we are not seeking for foci within the front lens, or yet on its surface. An immersion lens is not useful for viewing diatoms in balsam only. Every lens that I have seen professing 180° , whether it has an adjustment or not, is expected, and does, in fact, focus upon dry mounted objects. This necessitates a correct focus at a little distance beyond the last surface, from a position which must include a less air angle than 180° , and is consequently within the critical angle of nearly 82° in the crown glass.

The $\frac{1}{8}$ referred to belonging to Mr. Crisp has considerable focal distance in air, and I am confident that when the axial angle is correctly measured it will be proved far short of 180° , and the whole of Professor Keith's calculations concerning it must fall to the ground. The practical obstacles which have hitherto prevented the construction of any objective reaching to an air aperture of 180° still exist.

As much importance, both theoretically and practically, appears to have been attached to the last few degrees of extreme angle of aperture verging upon 180° , I repeat that the value of aperture in theory, considered as a question of rays collected, is palpably as the chord of the arc including the angle, or, in other words, in proportion to the sine of the half angle; for large angles so small is the comparative increase, that the difference between 170° and 180° is only as 99.6 to 100. But setting theory aside in this question as not always working harmoniously with practice, let it be considered, experimentally, what is the probable importance to definition

from rays proceeding at extremely oblique angles from flat surfaces having a raised configuration or structure. Take, for the first example, a piece of coarse-grained canvas or other fabric. Throw light upon or through this obliquely, and examine it at various angles with a shallow magnifier. At visual incidences greater than 45° or 50° no advantage will be gained; on the contrary, there will be a positive loss of definition. The same effect is seen on a different surface consisting of uniform glistening particles, such as bird-seed. To this dissentient will say, "You are comparing observations under a low magnifying power or with none at all to those made with the microscope," but relatively the conditions do not differ. Perhaps the most trying work for a hand magnifier is in examining the polish of glass surfaces in order to ascertain if all the "greys" are worked out. For small lenses a half-inch achromatic is used, held at an angle of about 45° ; at a greater angle the extremely fine specks cannot be discovered. Light is directed at a great obliquity on the surface or is transmitted. Uneducated workmen do not reason about any theory of angles, but adopt the one that gives the best result, simply because they find out at once that it does so.

My argument is that the angular aperture of microscope object-glasses has hitherto been erroneously measured by all the usual means, in which the outer oblique rays extending to the margin of the field of view have been in all cases included, and, in fact, constitute the false measurement. The true angle is the cone of rays diverging from an atom or point in the centre of the field. Other pencils of greater obliquity defining atoms at the margin, are exclusive and independent of the central rays as much as the different objects themselves, yet it is the direction of these exterior rays that we have hitherto been measuring. There seems to be an absence of experiment, or disinclination to admit the evidence of such an error. For one proof of an example wherein angle of direction is erroneously measured as angle of aperture, unscrew and remove the back lenses of high-power objectives, and measure the apparent apertures of the *fronts alone* by any of the usual methods, such as the traversing sector, or by the angle of direction to two distant images. The angle of the front lens alone of a $\frac{1}{3}$ thus measured came out as 83° , simply because the back lens included nearly a hemisphere, which admits lateral rays from a wide angular direction. The aperture of the front of a $\frac{1}{6}$ appeared as 110° , and that of a $\frac{1}{3}$ as 122° , because more rays entered sideways, as the back surface in these last lenses is almost a hemisphere. This exemplifies the absurdity and utter inaccuracy of the usual mode of measuring angle of aperture, as we know that these single lenses have in reality an angle of aperture of a few degrees only.

In order to define more clearly the direction of these outer rays, that cause indications of false aperture, I tried the following expe-

riment. I selected a $\frac{1}{2}$, which worked as an immersion, as this position prevents confusion concerning other points of adjustment. The full aperture, as measured by the sector through a slide with water between that and the front lens, was 120° . The focal distance as immersion was $\cdot 041$. The diameter of transmission on the surfaces of front lens was $\cdot 07$, ascertained by allowing a drop of milk to dry on the front and measuring the diameter of the light spot from parallel rays entering from the back, using a low-power object-glass and micrometer eye-piece for the measurement.

The field of view included a diameter of $\cdot 03$ on a stage micrometer. With the exact focal distance from the front lens as a starting point, it remained to ascertain what were the *apparent* apertures taken through various stops of definite diameter set close to the front, that could only admit the base of a cone of rays from an angle proceeding from the axial focal point up to a known diameter of stop. The arrangement that I use is a form of adjustable slit, consisting of two strips of very thin platinum foil; one piece is cemented on to a slip of thin plate glass, which is made to slide under two staples by a micrometer screw acting against a counter spring. The fixed strip of foil is attached to one of the staples, so that when the screw is quite home the edges meet. The various widths at which the instrument was set were measured under the microscope with an eye-piece micrometer. Having got the desired width, the object-glass to be measured was attached, and the body of the microscope lowered till the slit came in contact with the front lens, a drop of water having been placed over the slit to prevent undue refraction, and obtain more light.

The apparent angles included by these limiting edges or stops were measured by the usual sector method, of rotating the microscope on a turn-table graduated into degrees, and ascertaining the vanishing point of a distant light; or, preferably, by means of an examining lens at the eye-piece, for observing the disappearance from the field of an actual image.

The real or true angles were estimated from the distance of the focal point, up to the known measure of the edges of the stop. Avoiding fractions of degrees, the following table gives the comparative results:—

Working Diameter of Front Lens.	False Aperture.	True Aperture.	Working Diameter of Front Lens.	False Aperture.	True Aperture.
$\cdot 07$	120	89	$\cdot 02$	68	31
$\cdot 06$	118	80	$\cdot 01$	59	16
$\cdot 05$	116	70	$\cdot 005$	53	8
$\cdot 04$	103	58	$\cdot 002$	50	$3\frac{1}{2}$
$\cdot 03$	88	45			

The last item obtained with a stop $\frac{1}{50}$ of an inch in diameter, indicates an aperture of 50° by the usual methods, whereas the true angle is only $3\frac{1}{2}^\circ$, an error being shown in excess of more than fourteen times. The annexed diagram demonstrates the cause of this error. The central angle shown is the true aperture assigned

by the small stop at the base. The oblique angles represent the pencils, including the field of view, and showing light or an image at the eye-piece, up to an angular range of 50° . It is these outside rays which are superadded to angle of aperture taken by the usual method, and which are the cause of erroneous indications greatly in excess of the true angle.

The foregoing is a mere illustration of excessive angles, indicated from oblique pencils, or angle of view alone, irrespective of true angle of aperture, for, of course, limiting stops of known diameter placed on the front lens serve no purpose for measuring full central angles. The difficulty of estimating the degrees of these angles accurately, by such minute measurements as the focal distance, and

working diameter of the front lens, may be avoided by halving this as an unknown quantity, and obtaining the value of the central angle by the differential results, shown between a half-obscured front surface and an entire one. That which is understood (and, in fact, always has been) as a definition of angle of aperture, in a micro object-glass, is a triangle having a base equal to the available diameter of the front lens, and a height equal to the focal length measured therefrom. Now it is clear that if half the front lens is stopped off diametrically close to the surface, only half the base of the triangle must remain, and consequently but half the existing angle or cone of rays in the axial direction. By the sector measurement such is not shown to be the case. The apparent degrees from a half diameter of incident front surface are much in excess of a half quantity, because the rays that form the oblique pencils extend behind and under the half stop, nearly to the margin of the field of view beneath. These are beyond the true axial angle of aperture, and are the cause of the false quantity always measured in excess of the proper angle, the rays including the angle of field must therefore be deducted up to the centre. This the half stop enables us to accomplish. Using the sector measurement the rule is this. *Subtract the degrees shown by half the lens from*

the degrees of the entire lens, and twice the difference is the aperture.

By this simple rule we eliminate the angle of field, which has hitherto been erroneously added to angle of aperture. In order to obscure the half of the front lens, take a small piece of tinfoil with a clean-cut edge, lay this on a glass slip, and smear its upper surface with a bit of dry soap (anything moist will bedew the glass), place this on the microscope stage, and bring the edge in focus under the object-glass, until it exactly bisects the field vertically. Then lower the object-glass on to the foil, which will adhere to the front lens. The microscope body may then be laid horizontally, and the apparent aperture of the open half of the lens measured by the sector, which invariably indicates considerably more than half of the full aperture, taken by the same method. In objectives of large angle, twice this half measurement will amount to absurd and impossible angles.

The application of the above rule will require to be thoroughly investigated before it can be finally adopted. In every case it brings out a reduction on the degrees of aperture indicated by all the modes of measurement in present use. I give its test, on what I have repeatedly stated to be fabulous apertures, viz. 180° . I have one of these objectives—a $\frac{1}{10}$ —yet in ridiculous disproof of such an aperture (which would bring the focal plane on to the surface of the front lens), it is remarkable for its working distance, and will penetrate through the cover of any object in my cabinet, and used with thick covers, either as immersion or dry, its performance is very fine. Taken by the sector, the open aperture, or what I will term angle of field, is 180° . The aperture of half this lens is 115° , twice the difference is 130° ; this represents the degrees of the central pencil, which is the true aperture. I take another example of an immersion lens, in which the maker actually claims “plus 180° .” On trying this by the sector, the light image vanished at a range of 178° , beyond which it was totally obscured. The aperture of half the lens was 114° , twice the difference is 128° .