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XCIV. *The Rectification of the Spectrum in Relation to the Shift of Ellipses in Displacement Interferometry.* By C. BARUS, Ph.D., LL.D., Hazard Professor of Physics, Brown University, Providence, R.I., U.S.A.*

1. *Introductory.*—In my work of last year † the quantity $\mu - \lambda \frac{d\mu}{d\lambda}$ (μ index of refraction of glass, λ wave-length)

occurred in the reductions, and in the absence of direct data I temporarily made use of the corresponding quantity computed from standard data for a somewhat similar glass. In this way the amount of displacement of the centres of ellipses per cm. of displacement of the micrometer was determined. It must, however, frequently be necessary to find $d\mu/d\lambda$ from a small piece of glass, and I have therefore tested the following method in which a Kohlrausch total reflectometer slides along the graduated rail of Rowland's adjustment for the grating, as modified for the plate grating by M. Barus and myself ‡.

The method is then further modified, in a way that is applicable when a glass grating is given for investigation, as in the immediate problem, but not otherwise. It admits of some interesting applications, as for instance to the refraction of the material of film gratings.

In all cases of solids, the refraction of the more strongly refracting liquid in which the solid is submerged must of course be known for all wave-lengths; but this is the case for carbon disulphide. If the liquid indices are not known throughout, in terms of wave-length, they may be determined by aid of an air-plate, using virtually the same apparatus and method. I shall give examples of all these measurements below.

2. *Apparatus.*—In fig. 1 AA and BB are the two fixed rails of Rowland's adjustment at right angles to each other, R, joining the vertical axes at a and p , is the cross rail, stretching from the slide D which carries the grating g to the slide C which carries the total reflectometer T. Slides C and D move at right angles to each other, and the position x of C is measured along the brass millimetre-scale SS by aid of the vernier v .

* Communicated by the Author.

Abridged from the Report to the Carnegie Institution of Washington, U.S.A.

† Carnegie Publications, No. 149, 1911. "The Production of Elliptic Interferences in Relation to Interferometry," Phil. Mag. [6] xxi. p. 411 (1911).

‡ American Journal, xxxi. pp. 85-95 (1911).

The rail BB is further provided with a slide F for the plane mirror m , by which white light is thrown upon the slit c on the slide S, through the condensing lens L' . L'' is the achromatic collimating objective, and the rod k (which may

Fig. 1.

be grasped in front at h when necessary) is adapted for clamping S and L'' together, at the principal focal distance of the latter. The collimator SL'' does not move with D. The grating g , fixed to a horizontal disk with a spring and three levelling screws, is securely fastened to the table t , which here preferably rotates (not shown) with the end a of the rod R. The grating, in other words, is kept normal to R, so that the angle of diffraction is zero, while the angle of incidence alone varies when C passes from end to end

of AA. If x_1 and x_2 measured on S be the positions of C for identical lines of the spectrum,

$$\lambda/D = (x_2 - x_1)/2R,$$

where D is the grating space and R the length ap .

The lens L''' on R is the objective of a telescope which focusses the coloured image of the slit at the slide C, so that it may there be viewed by an ocular preliminarily. The latter is held in a standard placed on the table t' revolving on a common axis p with this end of R. When the ocular is replaced by the total reflectometer T, of which p is the plate of glass (properly centred) to be examined and q the telescope, the weak lens L''' is superfluous; but as a rule it need not be removed, and is rather advantageous in condensing light when sunlight is not used.

When sunlight is used, the lens L''' may be moved slightly backward, so as to put the coloured images of the slit slightly in front of a cylindrical glass vessel T of the total reflectometer. If the adjustment is properly made the lines are adequately visible in the telescope q , and the index may be found for a given Fraunhofer line without reference to the scale ss . Without sunlight, the scale ss is essential. Naturally the images seen in the telescope are not sharp, inasmuch as the cylindrical mantle of the bottle acts as one of the lenses; but they are clear enough for the purpose. It is usually more satisfactory, however, to ignore the lines and to determine the index of refraction for successive positions x of the carriage C on the rail AA, given by the scale ss .

With the total reflectometer revolvable, as a whole, on its axis, the two symmetrical positions of q relative to R give the double angle of total reflexion, or 2ϕ , for the given liquid (usually carbon disulphide) in which the glass plate p is submerged. The graduated circle of the instrument with which the telescope q is rigidly connected is available for the measurement of angles.

3. *Methods*.—When the air plate is used for the measurements of the indices of refraction of the liquid, the telescope q is directed along the rail R and $2\phi_i$ found by rotating the plate, p , until the two limits or ends of the visible part of the spectrum successively coincide with the cross-hairs. The spectrum is usually nearly homogeneous in colour; but the ends are liable to show interference fringes. Simultaneously the position x of the slide C and the temperature t of the

liquid are read off. Thus if

$$N_t = 1/\sin \phi_t$$

be the index * of the liquid at t° ,

$$N_{20} = N_t(1 + \alpha(t - 20)) = (1 + \alpha(t - 20))/\sin \phi_t$$

is the index at 20° . For carbon disulphide, at mean temperatures, $\alpha = .0008$, nearly.

Again, if the glass plate is now introduced and similar measurements $\phi_{t'}$, t' be made at x , the index of the plate will be

$$n = N_{t'} \sin \phi_{t'} = N_{20} \sin \phi_{t'}/(1 + \alpha(t' - 20)),$$

or

$$n = \sin \phi_{t'}(1 + \alpha(t - t'))/\sin \phi_t^*.$$

Two symmetrical sets of values are thus obtained, one for each of the two first orders of spectra, at x_1 and x_2 , on opposed sides of the undeviated white ray, w . This method does not usually succeed very well; for it presupposes that in case of the air plate and glass plate, which are successively inserted, the total reflexion occurs at identical parts of the spectrum, for identical positions x along the scale s . But supposing the adjustment trustworthy, the wave-length corresponding to any given n is found from the two positions x_1 and x_2 at which the same n occurs, from the equation

$$\lambda = D(x_2 - x_1)/2R.$$

In general it is preferable to use the air plate first to find the indices of refraction of the liquid, if these are not known throughout the spectrum. There will thus be given two symmetrical curves for the two first orders of spectra, separately. Their distance apart $x_2 - x_1$ for the given values of N determines the wave-length, λ , sought; i. e., if N is plotted in terms of x , the horizontal distance apart of the two curves is the $x_2 - x_1$ which determines the wave-length for the index N selected. Hence N is given in terms of λ and should be reduced to 20° . In case of many liquids, in particular carbon disulphide, N is well known in terms of wave-length at the outset, so that the air plate is superfluous.

The observations with the glass plate are made in the same way. The symmetrical curves for $\sin \phi_{t'}$ at t' , reduced to $\sin \phi'$ at 20° , as found from the two first orders of spectra, are plotted in terms of x . Hence their distance apart for two

* In conformity with Kohlrausch's notation N and n are here used as symbols for index of refraction.

equal values of $\sin \phi'$ is $x_2 - x_1$, from which the wave-length λ is computed. This datum in turn gives the appropriate (known) value N of the index of the liquid, all at 20° , so that finally

$$n = N \sin \phi'.$$

4. *Examples. Liquids.*—The circle of the Kohlrausch total reflectometer is only a little over 3 inches in diameter, and the reading is just within $\cdot 1$ degree of arc. The index of carbon disulphide moreover varies, relatively speaking, in marked degree with temperature; *i. e.*, $\cdot 0008$ per degree C. Unless unusual precautions are taken, therefore, one cannot expect the units of the third place to be quite trustworthy, as it corresponds to about $\cdot 07^\circ$ of arc. The apparatus in my possession was an old instrument, and a high order of precision was of secondary interest, and not attempted.

I will first take the case of an air plate with carbon disulphide. A film of air was enclosed between two small plates of microscope cover-glass by cementing them at their edges with glue. In another case, plates of glass $\cdot 3$ cm. thick were similarly prepared. An example of the results is given in the curves fig. 2, which show the value of N_{20} in terms of x .

Figs. 2 & 3.

These curves should be quite symmetrical to the vertical, so that for any height, N_{20} , $(x_1 + x_2)/2$ is constant.

Similarly a given horizontal N_{20} determines the values $x_2 - x_1$ for the computation of the wave-length λ . The film of

the grating had nominally 15,050 lines to the inch, and the oblique rail R was 169.4 cm. long. Hence

$$\lambda = \frac{x_2 - x_1}{15,050 \times 3937 \times 2 \times 169.4} = 4.98 \times 10^{-7} (x_2 - x_1).$$

These values x_2 , x_1 , λ , and N_{20} are given for the successive values of N_{20} used, and fig. 3, which is therefore the corresponding dispersion curve for carbon disulphide at 20° .

5. *Examples. Solid.*—The treatment of a solid medium is similar, except that the total reflexions are observed by reflected light. If the data for carbon disulphide be taken at 20° ,

$$n = N_{20} (1 - \alpha(t - 20)) \sin \phi.$$

Fig. 4 shows the values of the corrected $\sin \phi$ in terms of x . The curves, as they should be, are nearly symmetrical to the vertical, or $(x_1 + x_2)/2$ is nearly constant for the same sine.

If now the distance apart of equal values of the reduced $\sin \phi$ be taken, *i. e.*, if $x_2 - x_1$ be found for suitable increments of $\sin \phi$, the former values determine λ , from which the corresponding N_{20} for carbon disulphide is found in turn.

Figs. 4 & 5.

In this way the data for the values of n to be computed are found, and n may now be expressed in terms of the λ used. If N_{20} for carbon disulphide be taken from Kohlrausch's tables, fig. 5 shows the final dispersion curves, index of refraction n varying with wave-length λ , for the glass of the

grating. The constants of these curves will be computed below and their character will then appear.

6. *Independent Measurements.*—As an example of co-ordinated work, in which the index of the liquid and of the solid are simultaneously found, the following experiment may be cited. The air plate was enclosed between two thin plates of microscope cover-glass. In such a case the limits of total reflexion are determinable both for the reflected rays and for the transmitted rays. The angle ϕ in the former case gives the index of refraction n of the glass in relation to the liquid (carbon disulphide) since

$$n = N \sin \phi;$$

whereas the angle ϕ_0 in the latter case gives the index of the liquid alone, or

$$N = 1/\sin \phi_0.$$

Hence

$$n = \sin \phi / \sin \phi_0,$$

needing no correction for temperature.

In an experiment made at $18^{\circ}.3$ with the plate of microscope glass in question enclosing the air film, $\phi = 68^{\circ}.90$ and $\phi_0 = 38^{\circ}.02$, whence $n = 1.5198$. Using $\phi = 68^{\circ}.90$ as found and the tabulated data for carbon disulphide from Kohlrausch, $N = 1.6291$, $n = 1.5199$, which happens to agree accurately with the preceding n .

7. *Prism (Liquid CS₂) and Submerged Grating.*—The present method for the above purpose makes use of the grating directly. This is enclosed in a suitable closed trough, preferably triangular in form, with windows of plate glass and filled with carbon disulphide. If the grating is mounted with its ruled face coinciding with the axis of rotation, as in Kohlrausch's total reflectometer, the angle of total reflexion may be measured in all parts of the spectrum. The Fraunhofer lines which are simultaneously in focus are used for reference in succession. Parallel rays of sunlight from a collimator are thrown into one face of the liquid prism, nearly normally to its plane. They emerge at the other face. The undeviated rays are ignored, while the telescope is directed toward the first order of diffraction spectra, this emergence also, for the given grating, being not far from normal. Like the first order of spectra, the second and third orders may often be used with advantage, the limit of total reflexion being identical in all spectra on the same side of the undeviated ray.

The limit of total reflexion moves through the spectrum both with the rotation of the prism as a whole (which must

therefore be on an axle), and with the independent rotation of the grating actuated by the alidade. In the definite adjustment this limit must coincide both with the spectrum line λ for which the index, n , is to be found, and with the cross-hairs of the telescope. Three lines, in other words, are to be brought into coincidence; but after a little practice this succeeds fairly well, unless the limit of total reflexion is too faint.

The difference in angular reading on the two sides of the undeviated ray is the double angle of total reflexion, 2ϕ . The adjustment in the first spectrum holds equally for the second and third orders; and it is in the latter that the phenomenon is liable to be particularly clear and the adjustment correspondingly precise. The beam of white light should illuminate the whole grating whose ruled face is very nearly on the axis of rotation of the alidade. Sunlight is essential for these measurements, since the spectrum lines are to be used. This was not of course necessary in the preceding paragraph.

The following example of results obtained in this way may be given

Line.	2ϕ .	t .	n .	n from figs. 5 &c.
D	$138^{\circ}28$	$19^{\circ}0$	1.5210	$\left\{ \begin{array}{l} 1.5215 \\ 1.5206 \end{array} \right.$
E	$136^{\circ}53$	$18^{\circ}7$	1.5244	$\left\{ \begin{array}{l} 1.5240 \\ 1.5250 \end{array} \right.$

which is practical coincidence within the errors of observation, and thus verifies the above work as a whole.

8. *The Dispersion Constants of the Glass of the Grating.*—To reduce these data for practical purposes it will be sufficient within the range and relatively to the precision of observations to assume the simplified form of Cauchy's equation,

$$n = A + B/\lambda^2,$$

if n_E is the index for the E line and λ_E its wave-length,

$$n - n_E = B \left(\frac{1}{\lambda^2} - \frac{1}{\lambda_E^2} \right),$$

from which B may be computed for each pair of values of n and λ . B was found separately from each of the curves (like fig. 8). The values of $n - n_E$ and λ are chosen as near the observations as possible.

From these results the value of the differential coefficient

$$\frac{d\mu}{d\lambda} = -\frac{2B}{\lambda^2}$$

may also be obtained for any wave-length λ . This and the corresponding value * of

$$\mu - \lambda \frac{d\mu}{d\lambda} = \mu + \frac{2B}{\lambda^2}$$

needed in § 9, in the reduction of the observations of the earlier paper † will be computed by accepting the mean value $B \times 10^{10} = .48$.

9. *Computation of the Shift of Centres in Elliptic Interferometry.*—In the preceding report ‡ I showed that the path difference, y , corresponding to the centres of ellipses throughout the spectrum is

$$y = \frac{e}{\cos R} \left(\mu - \lambda \frac{d\mu}{d\lambda} \right), \quad . \quad . \quad . \quad (1)$$

where e is the thickness of the ruled glass plate and μ its index of refraction, for any given colour of wave-length λ . R is the corresponding angle of refraction for light incident at an angle I , or $\sin I = \mu \sin R$.

In the former experiment $I = 45^\circ$. In view of the oblique position of the grating, however, the micrometer motion N of the opaque mirror introduces an additional displacement. Let N be the coordinate for this mirror. Then

$$N = y - e\mu \sin R \tan R. \quad . \quad . \quad . \quad (2)$$

Hence the readings of the micrometer are determined by (subscripts c referring to centres of ellipses),

$$N_c = \frac{e}{\cos R} \left(\mu \cos^2 R - \lambda \frac{d\mu}{d\lambda} \right). \quad . \quad . \quad . \quad (3)$$

Taking the E line as a standard of reference, N_c and the quantities here in question were now computed both for light crown glass ($10^{10}B = .456$, in accordance with the data for μ and λ given in Kohlrausch's tables) and for the glass of the grating actually used ($10^{10}B = .48$), as measured in paragraph 8. Δy_c and ΔN_c refer to distances from the value corresponding to the E line. N_c , y_c , and λ are to be given in centimetres, and the angle of incidence is 45° . The data determine the full lines in fig. 6.

* The symbol μ for index of refraction is resumed.

† Carnegie Publications, No. 149, p. 69 (1911).

‡ Carnegie Publications, No. 149, 44, p. 66 *et seq.* (1911).

Measurements of N_c were now made with the aid of a Fraunhofer micrometer reading to a few ten-thousandths of a centimetre. The opaque mirror was adjustably mounted on a slide and moved normally to itself, in order to put the centres of ellipses successively in coincidence with the B, C, D, &c. lines of the spectrum. At each coincidence a reading was made, care being taken to proceed in one direction only. As there are four spectra more or less in coincidence, it is easily possible to select the wrong line.

Figs. 6 & 7.

By temporarily screening off the reflexion from one mirror, however, and then marking the line desired of the remaining pair by the cross-hairs of the telescope, mistakes are usually avoidable. In the extreme blue and red it was often difficult to see the lines clearly in the superimposed spectra, so that the data for the B and G lines are not quite so trustworthy. In fact the ellipses come out sharpest when but a narrow strip of the grating is used; whereas the resolving power of the grating or general sharpness of lines increases with the breadth of the grating. If the whole grating is used (an intense spot of sunlight focussed on the slit), the adjustment must be very perfect and the surfaces quite true to bring out the ellipses; this is not the case if parallel rays fall on the

slit and the whole divergence in the collimator is due to the diffraction of the slit. With optical plate none of these difficulties would appear.

Apart from drawbacks of this nature, which require patience for their removal, the measurements (shown by little circles in the curves) are quite satisfactory as a whole, as may be seen in fig. 7.

Different values of $(N'_c)_E$ in the two series of fig. 6 are due to the backlash in the micrometer*, as the mirror travelled in opposed directions in these cases. If the observed mean data are compared with the computed data for ΔN_c , the results are practically coincident except at the G line, where the ellipses were too dark to be seen distinctly. Hence equation (3) for N_c may be accepted as correct. If we write it

$$N_c = \frac{e}{\sqrt{\mu^2 - \sin^2 I}} \left((\mu^2 - \sin^2 I) - \mu \lambda \frac{d\mu}{d\lambda} \right), \quad . \quad . \quad (4)$$

and put $m^2 = \mu^2 - \sin^2 I$, $x = \log \mu$, the equation becomes

$$N_c = e \left(m - \frac{dm}{dx} \right). \quad . \quad . \quad . \quad . \quad . \quad (5)$$

On integration and reduction this may be written

$$e\mu \cos R = \lambda (C - \int N_c d\lambda / \lambda^2), \quad . \quad . \quad . \quad (6)$$

where C is a constant of integration. Unfortunately equation (6) is too complicated for the computation of μ , when N_c is observed in terms of λ throughout the spectrum; for the reference to the standard E line gives ($\sin I = \text{constant}$)

$$\sqrt{\mu^2 - \sin^2 I} / \lambda - \sqrt{\mu_E^2 - \sin^2 I} / \lambda_E = -\frac{1}{e} \int_E \Delta N_c \frac{d\lambda}{\lambda}. \quad . \quad (7)$$

If, however, I is very small, or $R=0$,

$$\frac{\mu}{\lambda} - \frac{\mu_E}{\lambda_E} = \frac{1}{e} \int \Delta N_c \frac{d\lambda}{\lambda}. \quad . \quad . \quad . \quad . \quad (8)$$

Let $\mu = B(1/\lambda^2 - 1/\lambda_E^2)$ as above. Then either equation (3) or equation (8) for $I=0$, leads to

$$(\Delta N_c)_0 = (N_c - N_E)_0 = 3eB(1/\lambda^2 - 1/\lambda_E^2). \quad . \quad . \quad (9)$$

This equation is to be tested in the following paragraph.

10. *Continued. Case of $I=0$.*—The limiting values of I in equation (7) are interesting. They correspond to the form of interferometer approached in fig. 8, where gg is the face of

* N' refers to the arbitrary zero on the drum.

the grating, I the incident ray, M and N the two opaque mirrors symmetrically inclined relatively to the grating. Rays R are refracted and rays D are diffracted. In fig. 8

Fig. 8.

Fig. 9.

the spectra on the left of R are also available ; but they are often complicated with the orders of spectra of the undeviated ray before it has been reflected. To avoid these, one should observe between the spectra of the non-reflected ray, or else in the region of non-reflected spectra of the third or higher orders, as these are too faint to interfere with the reflected spectra. They often advantageously supply fixed lines in the spectrum when arc light is used.

To realize the condition of small angle, a collimator I and the mirror N (micrometer movable parallel to itself) were fixed on the edge of the spectrometer-plate diametrically opposite to each other. The mirror M, the grating *g g*, and telescope at R or D were revolvable around the axis of the spectrometer. Grating and mirrors could be clamped. With such an adjustment all values of *I*, in fact, may be tested between 10° and 80° , smaller and larger angles being excluded, because of the lateral dimensions of the apparatus and mounting.

The present method of interference is often exceedingly convenient, inasmuch as the telescope at R is almost in contact with the micrometer at N, which may therefore be manipulated with ease by the observer. A typical form of instrument now completed will be described elsewhere.

The endeavour may now be made to compute B from the

shift of ellipses as suggested in the preceding equation (9) or

$$\Delta N_c = 3eB(1/\lambda^2 - 1/\lambda_E^2).$$

If we compute β in

$$\Delta N_c = \beta(1/\lambda^2 - 1/\lambda_E^2)$$

from the observed value of ΔN_c , the mean value for the B, C, D, F, G lines is found to be $\beta = .975 \times 10^{-10}$. Since $e = .68$ cm. and $B = .48 \times 10^{-10}$ in table 6, $3eB = 10^{-10} \times .979$, which agrees very nicely with the computed value of β just stated.

The full line in fig. 7 contains the data computed for N_c , in case of $I = 17^\circ$ and the glass of the grating ($B \times 10^{10} = .48$). Fig. 7 (upper curve, marked 17°) distributes the observation on the computed curve for ΔN_c , where ΔN_c for the E line is zero.

11. *Case of $I = 90^\circ$ nearly.*—The further case of interest corresponding to fig. 9, where the angle $I = 72^\circ$, and as large as it could be made in the given spectrometer. The spectra were very fine and sharp, but too dark at the violet end and red end.

The computed graph with the observations located with reference to it, is given in fig. 7 on the curve marked 72° . The two curves together indicate the change of sensitiveness on passing from a very small to a very large angle, the slope of the curve, fig. 6, where $I = 45^\circ$ being intermediate. The method of interference given in fig. 9 is useful when observations are made with polarized light (as in the examination of doubly refracting media) and the observer finds it convenient to be near the polarizer at the slit.

12. *Summary.*—I conclude therefore that the observations made at angles of incidence $I = 45^\circ$, 17° , 73° , have verified the equation

$$N_c = e \cos R \left(\mu - \frac{\lambda}{\cos^2 R} \frac{d\mu}{d\lambda} \right) = e \cos R \left(\mu + \frac{2B}{\lambda^2 \cos^2 R} \right)$$

from the shift of the centres of the ellipses satisfactorily. To restore the ellipses from one spectrum line to another is equivalent to a motion of the micrometer over $N_c - N_c' = \Delta N_c$. This constitutes a second method of interferometry of lower sensitiveness; *i. e.*, a coarse adjustment on the number of fringes which pass a given spectrum line. Per fringe or per vanishing ring, obviously, the displacement of the mirror on the micrometer will be a half wave-length of the part of the spectrum in question.

I may add, in conclusion, that I have recently* constructed a displacement interferometer combining the desideratum of lightness and portability with rigidity, and provided with long arms of invariable length (water circulation within), and with sufficient height of mirrors, &c. above the arms to admit of the insertion of bulky objects like a fog-chamber into the interfering beams of light. The adjustment of fig. 8 is particularly useful, the distance M to N being one or more metres. The apparatus is especially adapted for the measurement of the refraction of gases at all temperatures and conversely (in view of the large coefficient) for the measurement of high temperature in terms of the refraction of gases. It is also adapted for the measurement of the pressure variation of the refraction of a gas and conversely for pressure measurement in terms of refraction, particularly in case of the adiabatic transformations of a dry or a wet gas, at all temperatures. For the displacement of ellipses is instantaneous and dependent on the ratio of pressure to absolute temperature. Tests made with the fog-chamber have given results which promise to be fruitful, as will be shown elsewhere.

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Providence, R.I.

XCV. *On Mr. McC. Lewis's Papers on the Internal Pressure of a Liquid.*

To the Editors of the Philosophical Magazine.

GENTLEMEN,—

MR. McC. LEWIS, in his voluminous and interesting papers†, departs from Dupré's proposition, according to which the latent heat of vaporization of a unit of volume is equal to the internal pressure, and concludes in applying the second law of thermodynamics to this case, that "apparently the only accurate relation, the thermodynamic one," is

$$K - l = T \left(\frac{\partial K}{\partial T} \right)_v \quad . \quad . \quad . \quad . \quad . \quad (1)$$

"But," according to Mr. McC. Lewis, "this does not allow us to go further unless some new assumption be made with

* Am. Journ. Science, xxxiii. p. 109 (1912).

† Phil. Mag. xxii. pp. 193-197, and pp. 268-276 (1911). Trans. of Faraday Soc. vii. pp. 94-115 (1911). *Zeitschr. f. phys. Chem.* Bd. lxxviii. pp. 24-38 (1911).