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1916 Trans. Opt. Soc. 17 3

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everyone who knew him at all well to hear of his passing away so suddenly. I have paid my tribute to his memory in the public press, but I should like to say just one word in addition to what has already fallen from your President this evening on the passing away of so great a man of science and so distinguished an optician. He took the greatest interest in all branches of optical science, and he also took an active share in the work which I am going to talk about this evening, especially in the construction of some very large pieces of apparatus which are now in my possession, and gave me most generous advice and criticism and friendly help on all occasions.

ON STRESSES IN TRANSPARENT MATERIALS AS REVEALED BY POLARISED LIGHT.

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Delivered on October 12th, 1916.

THE study of the stresses in materials may be said to have originated with the celebrated discussion by Galileo Galilei, as to the way in which a beam, built into a wall at one end, resists a load which it carries at its outer extremity. In his dialogues, published at Leiden in 1638, Galilei shows a figure of such a beam, and propounds a theory based on the assumption that the material is absolutely rigid. He arrives at the conclusion that, in such a case, there are uniform tensions across the section of the beam, and that these tensions act about the bottom edge as a fulcrum. The question raised by Galilei concerning the state of stress in a beam does not appear to have been subjected to

any kind of experimental test at the time, but the problem served as the starting point of a long discussion of the action of a beam, and gradually an approximately true explanation of the distribution of stress was reached as a result of the work of many different observers.

The results of all these experimental and theoretical enquiries into the internal conditions of materials when subjected to load, formed the foundation of a new science of the strength and elasticity of materials, which at the present day has succeeded in covering a tremendous field of enquiry, although it has by no means exhausted the possibilities of the subject, especially in its relation to the constructional problems of engineers and architects.

In considering the means which are available for the purposes of research and investigation on the strength and properties of materials, it is somewhat remarkable that most of our present experimental knowledge has been obtained by purely mechanical measurements, in which uniformity of stress is necessary over a length, an area, or a volume of a body, and in which the majority of the measurements have been at the surface, the assumption usually being made that the condition of the interior is practically identical. The required conditions of uniformity of stress very much restrict the scope of experimental enquiry, for in the great majority of cases requiring examination the variation of stress in a short distance is enormous, and the stress at one point of a body differs so much from another in its neighbourhood that an average value is useless.

Fortunately, other means are available for examining the nature and kind of distribution of the stress in a body. Among the first to make investigations upon the optical behaviour of transparent materials under stress was Sir David Brewster, who, as early as 1814, found that if a piece of glass is slightly heated it shows effects similar to those which are seen in

some natural crystals when examined under polarised light. He also showed that if a piece of glass is subjected to an external force it exhibits brilliant colour effects so long as the load is applied, but immediately this is removed the colours disappear. The application of these phenomena to practical problems was at once apparent to Brewster, and he suggested that it would be possible to obtain valuable information concerning the stresses in the arched rings of bridges and like structures by such means.

In order to show the effects produced in polarised light by the action of stress in a transparent material, we may conveniently take a simple example such as a strip of transparent material under tension, and arrange the optical apparatus, so that when the strip is unloaded no light is transmitted, the effect of a moderate tension causes the specimen to appear a greyish white, and, as the stress increases, the colour changes by insensible gradations to a lemon yellow, then to a reddish purple, and with a very little increase of stress to a well-defined blue. With a further increase of stress the scale of colours is approximately repeated for twice the intensity of stress, and the relation of colour to stress is found to be approximately that given in the comparative table below. We have, therefore, a recognisable and easily understood colour-scale to mark the intensity of tensional stress in a material.

For simple tension and compression, the relative retardation of the rays which produces the colour effect is proportional to the stress, and to the thickness of the material. In such cases, therefore, the stress intensity may easily be determined by observing the colour bands, bearing in mind that both tension and compression produce similar effects, if changes in the thickness of the material are allowed for. Thus, if we take the case of a transparent beam subjected to a uniform bending moment, a system of colour bands is obtained, distributed as shown

in the photographic illustrations, and by inspection of the colour fringes the distribution across the section can be determined approximately, as shown in the diagram.

TABLE I.

COMPARATIVE TABLE OF TENSION AND COMPRESSION STRESSES CORRESPONDING TO A GIVEN COLOUR.

ORDER.	COLOUR.	STRESS.
I.	Black ...	0
	Grey	3.5
	White	5.5
	Straw	8
	Orange	10
	Brick Red	10.5
	Purple	11
II.	Blue	13
	Yellow	18
	Red	21
	Purple	22

It is there seen that the stress varies uniformly from a maximum tension at the under side to nothing near the centre, and it then changes sign and increases uniformly to a maximum compression stress at the upper side of the beam, in exactly the same manner as occurs in a metal beam subjected to the same kind of loading.

This case indeed affords one of several instances where the results of optical experiments can be compared, not only with mechanical measurements of strain but also with the theory of the distribution of stress in materials, and the experimental determinations for a transparent material show a very good agreement with strain measurements and with the precise theory. We can, therefore, feel very confident in more complicated cases that the stresses in a transparent model are similar to those in a

metal. For example, a beam with a notch cut in it may be taken, as shown, and the effect of the notch is seen to increase the stress in the material very considerably. The distribution is now much more complicated than it is in a simple beam; the neutral axis has moved towards the notch, while the colour effects show that the maximum stress is at least twice as great as that in a beam without a notch.

To apply this kind of experimental work to cases of practical importance, it is desirable that the material used shall possess similar physical characteristics to those possessed by steel, iron, and other materials in general use by engineers.

The physical properties of glass very closely resemble those of cast iron and natural stones, but measurable double refraction can only be obtained when thick plates are stressed to nearly the breaking point of the material. Glass is, moreover, an extremely difficult material to shape, and specimens can only be produced by laborious processes. Other transparent materials, like xylonite, have physical characteristics resembling mild steel and wrought iron, for example, their load-extension diagrams up to fracture are very similar. Xylonite is a preparation of nitro-cellulose, which is not so transparent as glass, and is usually slightly tinted; but it has the very important advantage that specimens may be fashioned quite readily with ordinary wood and metal-working tools, and with reasonable care the contours of the specimen show no signs of residual stress after shaping.

Experimental investigations of the stress distribution in engineering structures and machines, by aid of transparent models, involves the construction of complicated shapes, which are most convenient to make and examine if they are of large size, but Nicol's prisms, for large scale models, are not available. Polarisation of the light is therefore preferably effected

by reflection from black glass when a circularly polarised beam is required, and by refraction through clear glass for plane polarised light.

One of the arrangements designed for such work is shown in the sketch exhibited, in which a parallel beam from an arc lamp is directed on to a black glass reflector at a suitable angle, and is rendered parallel to its original direction by reflection from a second silvered mirror. It then passes through a quarter wave plate, so that the object is illuminated by a circularly polarised beam. The usual inverse arrangement of quarter wave plate and small Nicol's prism, or its equivalent, is used for the analyser. In order to dispose all the apparatus in a line the reflecting box may be doubled, but this offers little practical advantage, except in cases where plane polarised light is used, and it is required to rotate the plane of polarisation to obtain the directions of principal stress.

For this latter purpose it is more convenient to polarise by refraction through thin glass plates arranged in a frame capable of rotation about the axial direction of the beam, and although the polarisation is, in general, less perfect than by reflection, yet it is sufficient for the purpose.

With the arrangements thus described a parallel beam of polarised light may be obtained, which is limited only by the size of the lens system of the arc lamp projector. It is also possible to obtain a very large field of view by employing a diverging beam which is ultimately reduced to a thin pencil of rays in order to pass through the analyser. To effect this a large plano-convex lens may be used, which need not be of a high degree of optical perfection, except in the important particular that the material of which it is composed must be quite free from internal stress.

With such an arrangement a magnified image of the model can be projected on a screen for examination, but if the object is very large the lenses required become so costly that other means

have to be adopted, and a reflecting polariscope may then be employed with advantage.

Measurement of the Stresses.

We must now enquire by what means it is possible to measure the stresses produced in a transparent material by any given system of loading, and it will be apparent that if the stress is merely simple tension or compression its intensity can be read off at once by reference to the colour scale already established. Many cases of this kind occur in practice. If, for example, a simple ring is cut through one side and loaded after the fashion of a hook it is very severely stressed at its principal horizontal section, and the distribution across this section must consist of tension or compression stress only.

We may therefore read off the stress at any point of the section, and obtain curves of distribution such as those now shown, in which the stress intensities for several loads have been determined and plotted.

In this case the experiments agree fairly well with calculations of the stress distribution in a hook of this cross section, and they also show the interesting result that, as the load increases, the neutral line moves away from the tension side. In most cases, however, the stress distribution is more complicated to deal with.

Any case of stress in the plane of a plate can always be represented by two principal stresses at right angles, and if the magnitude and direction of these are determined for all points the stress distribution is solved.

In order to obtain an experimental solution of this problem, it is necessary to enquire into the relation of the optical effect to the principal stress intensities at a point, and it is easy to show this by simple experiments. If, for example, we take two equal tension members and subject them to the same uniform stress intensity the colour effects will be precisely the same for

each, while if they are superposed to interpose a double thickness the colour effect is the same as that produced in a single member under twice the stress.

Neutralised Stress Effects.

Experiments on three or more superposed members readily verify for simple tension and compression that the optical effect is simply proportional to the stress intensity, and also to the thickness of the plate. If, however, two equally stressed tension members of the same thickness are crossed, the common area gives a dark field, showing that the stress effect of one neutralises that of the other. The same dark field is produced if an equally stressed compression member is placed with the direction of stress parallel to that of the tension member, and we may readily verify in all cases that tension and compression stresses in the same direction add their effects, while stresses in directions at right angles subtract them.

The latter result is of chief importance since the stress at any point of a plate can always be represented by two stresses p and q at right angle s , and their optical effect is therefore proportional $p - q$. The value of the stress difference may therefore be determined by matching the optical effect produced at the given point with that produced on a simple tension or compression member, or, better still, by reducing the optical effect to zero by a single tension or compression member set along one of the directions of principal stress, and stressed until a dark field is produced.

The laws which the optical effects obey may be at once utilised for a variety of cases of practical interest. An example is furnished by piping for transmitting fluids under pressure. The action of water, or other fluid, in a pipe can be imitated by applying a uniformly distributed pressure to the interior of a ring such as is shown in the accompanying figure, where the

application of a uniformly applied pressure produces a stress distribution in the circular ring of a perfectly symmetrical character.

The arrangements of the colour bands show that there is a very large stress at the interior surface, diminishing rapidly at first and afterwards more gradually as the outer surface of the pipe is approached. In this case there is known to be a radial compression stress accompanied by a circumferential tension stress, and the optical effects produced at any point are proportional to the algebraic difference of their intensities.

In a thick cylinder of these proportions the radial stress is not large, and its intensity can be determined independently, but the combined effects of both stresses have been measured here, and are compared with the values found from calculation in the diagram now shown.

A fair agreement with the theory is apparent, and it is really closer than the curves indicate, owing to the full pressure recorded on the gauge not being effective.

In this problem a measurable fluid pressure is applied to the cylindrical surface of a ring in such a way that no essential part is obscured from view, and the accompanying slides show the apparatus employed for this purpose.

Fluid pressure of water or other liquid is applied by the action of a small hand pump, the piston of which is actuated by a screw to force oil at any desired gauge pressure into the annular space between two metal discs, which latter are bolted together to hold a retaining ring shaped like a Bramah cup-leather to prevent leakage of the fluid. This latter ring projects slightly beyond the periphery of the discs and carries the transparent ring to be stressed. The cup-leather is itself so thin that a pressure of a few pounds per square inch will burst it, but when the ring is mounted upon it even a pressure of 2,000 pounds per square inch may be applied with safety. A very small per-

centage of the total pressure is absorbed by the cup-leather, and is not exerted upon the ring, and when due allowance is made for this the agreement between experiment and theory is very close.

In cases where calculation is impossible, as often happens in many forms of engineering construction, the magnitude and direction of each principal stress at a point must be determined separately.

Principal Stresses.

A measure of the principal stresses at a point can be obtained, if advantage be taken of the fact that the stress causes a change in the thickness of material proportional to the sum $(p + q)$ of the principal stresses. If, for example, both stresses are tensions, there will be a lateral contraction of $(p + q)/mE$, where E is the modulus of direct elasticity, and m is Poisson's ratio. Both these latter quantities can be determined, and the sum of the stresses can be measured if an extensometer is used of sufficient accuracy to measure the lateral contraction. The values of the physical quantities E and m differ very much for different materials, but for the artificial transparent material used here they are much smaller than for a metal, and the difficulty of this kind of measurement is therefore much lessened.

A fair value of E for xylonite is 300,000 in pound and inch units, while m has a value of about 2.5, so that for each 1,000 lb. of stress intensity, the corresponding lateral contraction for plates of the usual thickness of $\frac{1}{8}$ inch is $\frac{1}{1000}$ of an inch. To measure such a quantity to an accuracy of within 1 or 2 per cent., it is advisable to use an instrument capable of indicating a change of at least one-hundredth of this quantity; such changes have been measured with fair accuracy by using a lateral extensometer capable of detecting a change of about half a millionth of an inch.

Description of Apparatus.

A photograph of one form of such an apparatus is shown attached to a specimen. In this arrangement a frame carries a calibrating screw, the point of which bears against the plate of transparent material, and is immediately opposite to a second piece, the inner end of which is lightly pressed against the plate by a spring, while the outer end presses against the short arm of a lever controlling the angular position of a mirror. Any change which takes place in the thickness of the specimen between the measuring points causes a rotation of the mirror, and this change can be measured by observing the movement of a spot of light which is reflected in the usual manner. The observations can be checked by the calibrating screw, which is provided with a graduated head for this purpose. It will be noticed that the measuring points simply bear against the face of the bar, and do not penetrate it, so that the length over which a measurement is made can be accurately determined.

The whole of the measuring apparatus is moreover supported on a pair of light steel springs attached to an independent clip, so that indentations are avoided at the points of measurement, while errors caused by the weight of the instrument on the measuring points are eliminated.

The extensometer readings give a measure of the sum of the principal stresses at a point in the plate, and their difference is found by an optical measurement. Each stress is therefore determined by measurements which are comparatively easy to carry out, although considerable care is required to obtain accurate values, especially if one quantity is much smaller than the other, since minute errors of observation become a large percentage of the value of the lesser quantity.

Lines of Principal Stress.

Reference has already been made to the fact that any state of stress at a point in a plane may be represented by a pair of stresses at right angles through the point, and whatever may be the character of the stress distribution in a plane, the directions of principal stress can always be represented by two systems of orthogonal curves spaced in a manner which the external loads and boundaries of the plate dictate. If, for example, two symmetrically disposed notches are cut in a tension member, it is clear that equally-spaced tension lines above and below must be crowded together as they pass the narrow neck, and those at the sides will probably come closer together than do those in the centre, thereby indicating a high stress intensity at the middle points of the notches.

Lines of principal stress may be drawn from the data provided by experiment, if advantage is taken of the property possessed by stressed material of causing the two systems of retarded rays to vibrate, one in the direction of the major principal stress, and the other in the direction of the minor principal stress.

Between crossed Nicol's prisms a loaded plate shows, in general, dark bands, which mark the positions of all points where the directions of principal stress correspond to the principal planes of the polariser and analyser, and by varying the angular positions of these latter a series of bands are obtained, each corresponding to definite directions of the axes of stress.

If, for example, the case of a simple tension member is taken with notches in the form of fine cuts in it on each side, dark bands are observed, like those now shown, and these change their positions as the axes of the optical apparatus are rotated. A diagram can be constructed from such observations to show the directions of stress at any place. If lines of stress at some distance away from the slits are

now considered, they will clearly be parallel and perpendicular to the sides of the plate, but as the former set approach the discontinuity, they must bend towards the centre line in order to pass through the narrow neck, since they cannot maintain their continuity in any other way. The observations of the lines of equal inclination show how they are guided, and it is evident that they come close together at the extremities of the slits, and that there is an intense stress at these points.

Other lines of principal stress at right angles to the first set are also indicated by the measurements, and the two systems give a kind of framework diagram, which latter shows the direction of the principal stresses, and therefore completes the solution of the problem. The stress distribution in a plate cut to a required form, and stressed in an arbitrary manner by forces in its own plane, may therefore be determined experimentally.

An Illustrative Experiment.

The complete experimental solution of the stress distribution in a body may be illustrated by an investigation of the action of a rivet near the edge of a rivetted joint, since we can determine the sum $(p+q)$ of the principal stresses, their difference $(p-q)$, and their directions. In this problem we can no longer neglect either principal stress, and it is in general necessary to determine both their directions and magnitudes. If the uniform tension stress in the full section of a plate is represented by equally spaced lines in the direction of stress, we may expect to find alterations in their directions and spacing as they draw near to the discontinuity produced by the rivet, and an optical examination shows that the lines of stress approach one another very closely as they pass around the rivet, and afterwards diverge again if the overlap of the plate is sufficient to permit this.

It is not difficult to explore the whole of a plate stressed in this way, by determining both the sum and difference of the stresses at a sufficient number of points on the lines of stress so found, and some of the measurements, for the cross section passing through the centre of a rivet in a plate, are shown for the case of a plate in which both the overlap and the widths of metal on each side of the rivet are equal to the diameter of the rivet.

They show that the tensile stress at the horizontal cross section reaches a high value, while below the rivet an even greater compression stress is produced. The measurements of radial stress along the sections chosen give marked compression close to the rivet, and it is worthy of note that they are very nearly zero at the outer boundaries of the plate, results which confirm the general accuracy of the measurements. Other measurements of a similar kind show that the action of a rivet produces an intense stress at the hole, sometimes reaching five times the stress in the full plate. In a transparent model this is often accompanied by permanent overstress and local yielding, which tend to equalise the stress in the material.

We have seen that the complete determination of the distribution of stress in a plate subjected to forces in its own plane involves the determination of the directions of the lines of stress, and the measurement of both the sum and the difference of the principal stresses at each point, since these latter cannot in general be measured separately.

It is therefore apparent that this kind of investigation offers great advantages for the measurement of shear stress, since the optical effect is proportional to the difference of the principal stresses, and therefore varies according to the shear stress at a point. The quantitative determination of shear stress is accordingly of a simple kind, since the delicate measurements of the change of thickness in a

plate are not necessary, while a general idea of the distribution of shear stress may be obtained from the picture which the colour effects show.

Shear Tests of Materials.

The variation of stress in a rectangular plate when subjected to nearly pure shear has been examined with some care, and it has been shown that, when the length is greater than 1.75 times the breadth, the shear stress rises to a maximum value near the end of the plate, and has a slightly smaller value at the centre. When, however, the length of the plate is less than 1.75 times the breadth, the shear stress reaches a maximum at the centre, and has a parabolic distribution.

The application of optical methods of investigation to problems which arise in the testing of materials shows that something may possibly be gained by an examination of familiar methods of testing.

Among the cases which have been examined is that of the usual shear test of materials.

Shear tests are usually made upon bars of rectangular or circular cross section gripped in such a manner that shear stress is applied to two similar and equal sections at a finite distance apart, and load is applied until the material is sheared across these sections. In this way a mean value of the shear stress at rupture is obtained, which is of some value, but it affords no indication of the behaviour of the material up to the elastic limit. It seems probable that the assumption of uniform shear stress across the section is not justifiable, and within the elastic limit it is clearly not so, as is immediately obvious from the colour effects now shown, where the stress distribution is due to a shearing force applied to a rectangular section.

Commencing from the top-most edge the colour effects indicate that the stress is a maximum here, and diminishes rapidly as we proceed along the section, until it becomes sensibly

uniform, and then as the lower edge is approached the stress again rises rapidly and reaches its greatest intensity at the lower edge. This increase of stress beyond that at the upper edge is due to the bending of the specimen in the grips, and it indicates that failure will occur here.

Specimens of circular cross section show a different distribution, and it is evident that tests of dissimilar cross sections of the same material are unlikely to give uniform results in shear unless these differences are taken into consideration.

It would not be difficult to supply other examples of the use of optical determinations of stress in models, but the cases already dealt with are probably sufficient to show that results of practical importance can be obtained in this way, which might be difficult, if not impossible, to arrive at by any other methods of investigation used by engineers.

Discussion.

THE PRESIDENT, in proposing a vote of thanks to Professor Coker, said he had not been aware till that evening that the application of polarisation phenomena to quantitative work had been advanced to such a degree as was evident from the very able and interesting paper to which they had just listened. The subject was one in which he had taken a very great interest for many years, and whilst listening to the beautiful exposition given by Professor Coker, he could not prevent his mind from wandering back to the beginning of the previous century, when the first experiments were made in connection with this matter. He believed it was in January, 1815, that Brewster first squeezed a jujube between a pair of crossed Nicols and observed the colours produced. That experiment had resulted in a century's work, which had to-night probably reached its highest quantitative development in the work of Professor Coker.

He might perhaps be allowed to say a few words with regard to the work of Brewster. Brewster was a man who did an enormous amount of good work. He believed he had written something like 300 papers in the course of his life, all of great value, and dealing with a variety of physical subjects. Brewster in his classical paper "On the communication of the structure of doubly refracting crystals to glass, muriate of soda, fluor spar, and other substances by mechanical compression and dilatation," which appeared in the Philosophical Transactions for 1816, page 156, announced the discovery that if homogeneous glass is stressed it develops neutral and polarising axes—the neutral axes being parallel or perpendicular to the direction in which the stressing forces act and the polarising axes being inclined to the neutral axes at an angle of 45° . He then proceeds to describe simple apparatus, by means of which stressed glass could be produced and studied.

That Brewster was very much alive to the possibility of utilising polarising phenomena for the purpose of making quantitative determinations of stress is shown by the following passage from his paper:—

"The tints produced by a polarised light are correct measures of the compressing and dilating forces. . . . The subject, therefore, of the strength of materials and their cohesion will derive new lights from the principles already established. If the arch stones of models are made of glass or any other simply refracting substance, such as gum copal, etc., the intensity and direction of all the forces which are excited by a superincumbent load in different parts of the arch will be rendered visible by exposing the model to polarised light. If different degrees of roughness are given to the touching surfaces of the glass voussoirs, the results may be observed for any degree of friction at the joints. The intensity and direction of the compressing and

dilating forces which are excited in loaded framings of carpentry may be rendered visible in a similar manner."

Brewster goes on further in his paper to show that fluor spar, diamond, horn, gums, caoutchouc, resins, phosphorus, etc., and jellies can be used as substances which when stressed produce polarising colours. He even suggested that a pile of superposed glass bars loaded like a beam could be used for the measurement of forces with great exactness. He pointed out that a letter balance might be made up from a few pieces of glass built together in the form of a beam and placed in a polarising field—the letter to be suspended from the beam and its weight, and therefore the amount of postage to be paid, to be determined by the colour which appeared in the glass beam. (Laughter.) And yet it is sometimes said that scientists are not practical. (Renewed laughter.)

Brewster also suggested that a block of glass embedded in lead or tin could be made to act as a chromatic thermometer. Any change of temperature producing a change of dimension in the metal resulting in the application of varying pressure to the glass, and hence a change of colour in the polariscope.

The study of polarisation phenomena was taken up very keenly by physicists during the first half of last century. England vied with France—Brewster with Biot. This strenuous rivalry between English physicists on the one hand and French physicists on the other resulted in a very rapid advance in our knowledge.

Professor Coker's lecture had been of interest to him from another point of view. At the present time one of the greatest difficulties which they had to contend with in the optical industry was the fact that optical glass was not delivered to the optical manufacturers perfectly homogeneous in structure and perfectly annealed; and many of them, he was sure, would

be grey before the end of the present war unless this problem were solved more readily than appeared probable at present. In listening to what Professor Coker had said that evening, it had struck him that their polarisation tests were not quite so convincing and satisfactory as they were usually assumed to be, because he had told them in effect that equal plates of glass similarly stressed when crossed at right angles between crossed Nicols gave a black field. Well, that meant, of course, that whilst one half of a block of glass might be severely stressed in one direction, the other half might be equally severely stressed, but in such a direction that the two stresses practically acted as equal plates similarly stressed. Under those circumstances it would appear that the optical manufacturer would be quite content that the block of glass was perfectly annealed, whereas as a matter of fact it might be so severely stressed that it would be quite impossible to use it effectively for the manufacture of an optical instrument. He should be very glad, therefore, if Professor Coker could give them some indication as to under what conditions, if any, a piece of badly annealed glass would appear black between crossed Nicols. This was a very important practical problem.

The vote of thanks was carried by acclamation.

PROFESSOR COKER, in acknowledging the vote of thanks, said it was always a pleasure to meet an audience which was interested in scientific investigations, especially when that audience was one which, as on the present occasion, was also one which took an interest in practical problems. He wished also to thank the President for his very kind remarks, as well as for the great interest he had displayed in the work he had been trying to do. Polarisation was a very nice subject to play with, and for a good many years he had been playing with these problems, finding it a delightful way of passing spare time when not actually engaged in teaching.

With regard to the problem which Mr. Cheshire had put to him, he thought it quite possible—he would not say probable—that a case might occur of extremely intense strains being present in glass which nevertheless would pass a satisfactory optical test of the ordinary kind between crossed Nicols. It was quite possible to imagine a condition of things in which a slip of glass had in it very severe stresses so exactly balanced as not to be perceptible under test. But he did not think the case was so bad as might appear from what Mr. Cheshire had said, because on rotating the Nicols, if there were any such severe stresses present in the piece of glass, bands would probably occur which would show the varying conditions all through—although it was quite possible to imagine a case in which that would not happen. Without going into the matter in very great detail, he would go so far as to say that it was quite possible for a piece of bad glass to slip through what might be considered quite a severe test, and yet to have stresses in it which had not been observed owing to the fact that the equal stresses neutralised each other. How to obviate that difficulty he could not say for the moment, although he thought it would be quite possible to devise some test, if one thought about it for a sufficient length of time, which would enable stresses of that kind to be detected. So far he had not thought about the matter; it had not occurred to him. He had on various occasions had optical glass submitted to him for examination, and these specimens he had merely tested by means of crossed Nicols. As a matter of fact he had never needed to go further, because they almost invariably showed stress; in fact, he had hardly ever met with a case where that was not so. (Laughter.) He thought the point raised by Mr. Cheshire was one which ought to be taken into account in the case of the very best optical glass required for special purposes. (Applause.)