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METEOROLOGICAL ACOUSTICS.*

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CHAPTER I.

METEOROLOGICAL EFFECTS ON SOUND.

Introduction.

METEOROLOGICAL acoustics is concerned with every sound of a distinctly meteorological origin, such as the humming of telegraph wires, the "roaring of the mountain," the rumbling of thunder, and the like; and with every effect meteorological conditions have on the travel, distribution, and audibility of sound. In short, it treats of meteorological sounds and of meteorological effects on sound.

Nature of Sound.—The term "sound," as used in the science of acoustics, means either an external disturbance or a sense impression. In the more restricted field of meteorological acoustics, it has only the former of these meanings; the sensation being referred to merely as evidence of the presence and nature of a disturbance in the external medium.

Now, everyone knows: (*a*) by observation, that a tuning fork, bell, violin string, and the like, produce sound only when vibrating;

* Communicated by Dr. E. D. Ball, Assistant Secretary of Agriculture.

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(*b*) by observation, that the hammer is seen to strike, the gun go off, the lightning flash, and innumerable other abrupt causes of sound to occur, before the sound itself is heard, and that this time interval increases with the distance of the observer; (*c*) by experiment, that the sound of a bell, say, mounted on an acoustical insulator within an air pump receiver grows fainter and fainter to extinction as the exhaustion of the receiver is progressively increased.

From these three facts it is obvious that objective sound, or sound in its physical aspect, consists of traveling disturbances of some kind in a material medium. If this medium is the atmosphere (or any other fluid) it is further obvious that the disturbances in question consist of compressions and rarefactions, for the atmosphere has no tendency to recover from distortion of any kind save that of volume alone—no tendency to recover from twist or bend, but a resistance to change of volume that increases with the ratio of that change to the initial volume.

Seeing Sound.—Since sound consists of condensations and rarefactions of the atmosphere traveling outward from the source, and since light is unequally refracted by air of different density, it would seem possible to see, and perhaps even to photograph, the front of a sound pulse. And this has, indeed, often been realized. The outermost sound shell, produced by the firing of a cannon, for instance, has repeatedly been seen sweeping across the sky, and over clouds, like a narrow fleeting shadow. Similarly the initial condensation wave caused by an electric spark has given numerous beautiful and instructive photographs, of which Fig. 1, from Doctor Foley's remarkable collection,¹ showing both the primary wave and its reflection from a flat surface, is an excellent example.

Velocity of Sound.

Velocity of Sound in Still Air.—As stated above, many simple observations show that sound travels through the air with moderate velocity—faster than a mile a minute, and slower than a mile a second. Just what this velocity is might be determined by careful experiment over a large range of temperatures, pressures, humidities, and other factors, and its relation to each empirically determined. But while this has been done to some extent, the collection of an adequate amount of such data would require long

¹ *Phys. Rev.*, 35, p. 373, 1912.

and tedious work and the results in the end would be only empirical and not rational. Hence it seems desirable to logically derive equations that show the exact and necessary relation of the velocity of sound to each of the several factors upon which it depends.

The following method of computing sound velocity is essentially that of Rankine.² It may not be so mathematically elegant as the "velocity potential" method, used by Lord Rayleigh,³ for

FIG. 1.

Photograph of sound waves—incident and reflected. (A. L. Foley).

instance, but it has the great advantage of revealing, rather than concealing, the physical processes involved.

Consider an ordinary sound wave (great explosions which result in additions to the atmosphere, and hence require a modified treatment, are excluded) at some distance from its origin, traveling through the atmosphere. As above explained, this disturbance must consist of longitudinal motions of the air particles in the line of travel of the sound, resulting in alternate compressions and rarefactions. This follows from the fact that the atmosphere

² *Phil. Trans.*, 160, p. 277, 1870.

³ "Theory of Sound," ii, p. 15.

is elastic only to changes of volume and not to twists or bends. Furthermore, it is clear that these sound excursions of the air particles are small—not even leaves visibly quiver as any ordinary sound passes—and the pressure changes comparatively slight, too slight indeed to be felt.

Clearly an atmospheric wave of any form, harmonic or otherwise, can be made to progress unchanged through air at any given speed by applying to each particle a proper external force, directed along the line of travel, in addition to the internal force already acting on it.

Let the external force be X per square centimetre in the direction of propagation, that is, per square centimetre of a plane wave front over which, of course, all particles are in the same vibration phase, and hence subject to exactly equal forces; and let X vary in such manner from phase to phase as to keep the form of the wave unchanged and to give it a uniform velocity v . To further simplify the problem, consider an imaginary tube of unit cross section, parallel to the line of propagation and extending through the wave front slightly into the undisturbed region on one side and more or less into the disturbed region on the other. Let this tube be fixed in position, and let the atmosphere flow past it exactly opposite in direction to that of the sound propagation. That is, let the sound waves or that arbitrary portion of a single one, within the tube, though continuously affecting new material, be absolutely fixed in position and condition.

What, under these conditions, are the relations between the forces involved, internal and external, and the deductions therefrom?

Since the quantity and condition of the air within the tube is constant it is obvious that its momentum is also constant, and also, under the given conditions, wholly along the line of propagation. Hence the momentum entering the front of the tube during a given interval, one second say, is equal to the momentum leaving it during the same interval. This momentum *per second* consists of two parts, a , the mass entering, or leaving, per second, times its velocity, and, b , the pressure upon this mass at the place of entrance or exit, respectively—force is equal to mass times acceleration, which is equal to momentum produced per second in the direction of the force.

Let, now, P be the pressure in the undisturbed atmosphere, and

hence at the front end of the tube; $P \pm p$ the pressure at the rear end of the tube (p being either positive or negative, owing to whether this end of the tube is in the compression or rarefaction portion, respectively, of the wave); v the velocity of entry of air of density ρ into the tube; $v \mp u$, and ρ_1 the velocity and density, respectively, of the air as it leaves the tube; and X the assumed sustaining external pressure, constant at each particular phase, and thus constant at the exit end of the tube.

Hence, the momentum entering the tube of unit cross section per second is, if m is the mass

$$mv + P$$

Similarly, the momentum leaving the tube per second is, since the mass within the tube cannot change,

$$m(v \mp u) + P \pm p + X$$

But the two momenta are equal, therefore

$$mu = \rho vu = p \pm X$$

If, now, v is so chosen that

$$\rho v = p/u, X = 0$$

and the sound moves on without the aid of any external force with the velocity

$$v = p/\rho u \quad (1)$$

As above explained,

$$\rho_1(v \mp u) = \rho v$$

and, if the air remains isothermal,

$$(P \pm p)/\rho_1 = P/\rho$$

Hence, neglecting pu ,

$$u = pv/P$$

and, substituting in (1),

$$v = \sqrt{\frac{P}{\rho}} \quad (\text{Newton})$$

Or, if the compressional changes take place adiabatically,

$$ku = pv/P$$

in which k is the ratio of the specific heat of the air involved at constant pressure to its specific heat at constant volume, and

$$v = \sqrt{\frac{kP}{\rho}} \quad (\text{Laplace})$$

Whether the velocity of sound is in accordance with Newton's equation, or Laplace's, or has, indeed, some distinctly intermediate value (it cannot be outside these theoretical extremes) can, of course, be tested by direct experiment. But here, too, theory is a great help in that it shows⁴ the impossibility of a velocity appreciably different from one or the other of the extremes. The non-mathematical form of this reasoning (Stokes gives both) is substantially as follows:

Let an airtight, frictionless, piston oscillate so slowly in a cylinder with thermally conducting walls that the temperature of the enclosed air remains constant. Clearly, then, at each given position of the piston the pressure is the same during expansion as during compression, and the process is without net loss, or gain, of energy. Similarly, if the reciprocal motion of the piston is so rapid that no heat is lost during compression, nor gained during expansion, the pressure, though different from its former value, is again constant for each given position of the piston, and hence the total external work zero per cycle.

If, however, the speed of the piston is such that some heat is passed, but not enough to maintain a constant temperature, the pressure at each point will be greater during compression than during expansion, and the process can be maintained only at the expense of external work. Under these conditions, therefore, the sound would be damped out far more rapidly than is the case in nature.

Actually, sound waves are so fleeting (only those of the baser tones require more than a hundredth of a second to pass) that the slight pressure changes they produce are all but absolutely adiabatic. Hence, to an extremely close approximation,

$$v = \sqrt{\frac{kP}{\rho}} = \sqrt{\frac{1.40 P}{\rho}} \quad (2)$$

It should be noted here that since neither u , p , nor ρ_1 , the only factors affected by either the amplitude or frequency of the vibrating air, enter into equation (2), therefore the velocity of sound is independent alike of both loudness and pitch—the same for all sounds of whatever origin.

Relation of the Air to Sound Velocity.—From equation (2) it is likewise clear that all meteorological changes that alter the value

⁴ Stokes, *Phil. Mag.*, I, p. 305, 1851.

of k , or the ratio of P to ρ , or both, affect also the velocity of sound; and that it is independent of every other such change.

Changes in pressure, therefore, do not alter the velocity of sound. They obviously do not affect the value of k , and, by proportionately varying ρ , leave the ratio of P to ρ constant. On the other hand, variations in the temperature do affect the velocity of sound. In fact, this velocity is directly proportional to the square root of the absolute temperature, T , as is obvious from the fact that when P and k are constant (k constant implies same composition), ρ is inversely proportional to T . Similarly, changes in the absolute humidity also affect the velocity of sound, since they slightly alter the value of k .

The value of k for dry air is 1.40, very closely. Its value for water vapor is not accurately known for ordinary out-door temperatures, but appears to be 1.30 roughly. Presumably, therefore, in nature

$$k = 1.40 - 0.1 e/P$$

approximately, in which P is the total atmospheric pressure and e that portion of it due to water vapor.

In terms of the reading of the barometer

$$P = (B - .378 w) g \delta$$

in which B is the reduced total height of the barometer, w the reduced height due to the water vapor, g and δ the standard gravity acceleration, and mercury density, respectively.

Finally, if ρ_0 is the density of dry air at 0° C. and when the reduced height of the barometer is 760 mm., then

$$\rho = \rho_0 \frac{(B - .378 w)}{760 (1 + at)}$$

in which a is the coefficient of gas expansion per degree Centigrade at 0° C., and t the temperature, as read on the Centigrade scale.

On substituting these several values in the equation for the velocity of sound it appears that, in terms of centimetres per second,

$$= \sqrt{\frac{76 g B \delta (1 + at) (1.40 - 0.1 w/B)}{\rho_0 (B - 0.378 w)}}.$$

For dry air at 0° C. this reduces to

$$v = \sqrt{\frac{76 g \delta (1.40)}{\rho_0}} = \sqrt{\frac{76 \times 980.665 \times 13.5951 \times 1.40}{0.0012930}},$$

or 331.23 metres per second, which is in close agreement with observations.

The effect of dust, smoke, fog, or any other such foreign substance in the atmosphere, on the velocity of sound is not well known. Presumably, however, the values of both k and ρ , and through them the velocity in question, are altered by all such things. But as the aggregate mass of any such substance is invariably quite small in comparison to that of the atmosphere in which it floats, and since it plays the rôle of a gas to only a limited extent, its effect on this velocity must always be minute, and for most purposes entirely negligible.

Doppler Effect.—Many, if not most of us, have often noticed the change in the pitch of a locomotive whistle, or bell, as the train passed rapidly by us; and also the similar change in pitch of a bell, or other sounding object, as we were carried rapidly past it. This phenomenon, known as the Doppler effect, and dependent on the finite velocity of sound, is easily explained.

Let v be the sound velocity; n the vibration frequency, λ the wave-length, and u the velocity of the sounding body towards, or from, the stationary hearer. If u is zero,

$$\lambda = \frac{v}{n}, \text{ and } n = \frac{v}{\lambda}.$$

If the sounding object is approaching the hearer the resulting wave-length λ_1 and frequency n_1 clearly are given by the relations

$$\lambda_1 = \frac{v-u}{n} = \frac{v}{n_1},$$

and

$$n_1 = \frac{nv}{v-u}.$$

Similarly, if the sounding object is receding from the observer,

$$\lambda_2 = \frac{v+u}{n} = \frac{v}{n_2},$$

and

$$n_2 = \frac{nv}{v+u}$$

Hence

$$n : n_1 : n_2 = 1 : \frac{v}{v-u} : \frac{v}{v+u}.$$

That is, the pitch is highest when the sounding object is approaching, and lowest when it is receding.

If, then, the sounding body were on a train, say, making 47 miles per hour, the pitch, as perceived by a person at rest near the

track, would drop approximately from G (approaching) to F (receding) on the musical scale.

If, however, the hearer is moving with the velocity w and the sounding object is at rest, on approaching the body

$$n_1 = \frac{v+w}{\lambda},$$

and on leaving it,

$$n_2 = \frac{v-w}{\lambda}.$$

Hence, in this case

$$n : n_1 : n_2 = v : v+w : v-w$$

If both object and hearer are in motion the expressions for the pitch ratios are only a little more complex—no new principle is involved.

Sound Perception in the Inverse Order of Sound Production.—Another interesting consequence of the moderate velocity of sound propagation is the occasional perception of sounds in the inverse order of their production. This is especially noticeable in the case of high velocity cannon shells passing close by or over one from a considerable distance. Such projectiles often have double the velocity of sound. Hence the first thing heard is the whine of the shell, then its explosion, perhaps, and last of all the firing of the cannon; or, the explosion may be heard last, if far to the rear. All this is obvious when it is recalled that the shell is continuously producing sound far in advance of the initial wave-front from the cannon.

It is also interesting to note that when the source moves faster than sound, as do cannon shells, the wave-front envelop is a cone whose vertex is at the source, and whose angular opening α is given by the equation

$$\sin \frac{\alpha}{2} = v/s,$$

in which v and s are the velocities of sound and of the shell, respectively.

Velocity of Intense Sounds.—Whenever a sound is produced by a violent explosion, such as by the firing of a cannon, the blast itself drives through the air and gives to the sound a velocity which at first, that is, so long as X in the general velocity equation is real and positive, is in excess of that which applies when X is zero.

This excess velocity, however, occurs only relatively near the

source of the sound, and rapidly decreases with distance from that source. That is, the external force X is soon reduced to a negligible quantity, after which the velocity becomes constant with the value appropriate to $X = 0$.

Sound Reflection.

Reflection by a Rigid Surface.—As above explained, all sounds in the atmosphere are propagated in the form of successive compressions and rarefactions. If, therefore, these air-waves should impinge normally on a wall of stone, or other dense material, it is obvious that they could not pass on unaltered. If the wall is absolutely unyielding, a condition solid and liquid boundaries more or less approach with respect to slight variations of atmospheric pressure, evidently the adjacent molecules cannot oscillate in the line of propagation. The pressure actions then of the wall on the atmosphere are precisely the same as they would be if it and not the air were vibrating with the given period and amplitude. Hence sound-waves incident on an absolutely rigid wall are wholly reflected, and in such a manner that the surface of this wall is a place of minimum (zero) motion—the incident and the reflected wave having always exactly the same phase at the boundary.

Reflection at a Free Surface.—The effect on sound of a medium of the opposite rigidity extreme, that is, infinitely yielding, such as empty space, is also interesting, though not realizable in the case of the atmosphere. Here the incident compression tends to drive the boundary molecules of the first medium away; if, however, they are restrained from flying off, as is the free end of a vibrating rope, say, or the surface of a liquid, this restraint must be of the nature of a tension. In this way the flux of incident energy is reversed in direction—the waves are reflected. Furthermore, since change of pressure is impossible at a free boundary, the incident waves always have exactly opposite phases at the boundary reflecting surface.

Reflection at an Air Interface.—Let the boundary be between masses of air at the same pressure, but of unequal density owing to difference in temperature, or humidity, or both. This case is of especial importance since it includes the problems of internal atmospheric reflections, and of reflections from fogs and clouds.

Here, too, as in so many other acoustical problems, the most general treatment is given in Lord Rayleigh's "Theory of Sound."

But the physical significance of each step in this elegant analysis is, perhaps, not easy to visualise. Hence a more obvious course of reasoning will be followed, based on Fresnel's theory of the reflection of polarized light.⁵

If none of the energy of the incident sound waves is transformed into heat at the boundary (and there is no evidence of any such transformation) clearly all of it must be contained jointly in the reflected and the refracted waves.

The laws of the reflection and refraction of sound in still media

FIG. 2.

Reflection and refraction of sound.

are the same as those of light. They are given below, but assumed here for convenience.

Let, then, the incident, reflected, and refracted waves be as indicated in Fig. 2; let their amplitudes be a , b , and c , respectively; let the velocity in the first medium be v , and in the second v' ; and let the densities of these media be ρ and ρ' , respectively. Then, since the energy in the incident wave is divided between the reflected and the refracted waves without loss, and in each case is proportional to the volume affected, obviously, and to the square of the respective amplitudes as immediately explained below,

$$v \rho a^2 = v \rho b^2 + v' \rho' c^2 \cos r / \cos i$$

in which i is the angle of incidence and r the angle of refraction.

⁵ Fresnel, "Oeuvres," i, pp. 649-653; also Preston, "Theory of Light."

To show, as just assumed, that sound energy is proportional to the square of the amplitude, let f be the restorative, or elastic, force; a the amplitude (extreme displacement of a particle from its position of rest) and w the work of displacement. Now, f is proportional to the compression, that is, to the displacement, x , say. Hence,

$$dw = f dx$$

but

$$f = kx,$$

k being a constant, and

$$w = k \int_0^a x dx = \frac{k}{2} a^2.$$

Now, $v/v' = \sin i / \sin r$; hence,

$$\rho (a^2 - b^2) \sin 2i = \rho' c^2 \sin 2r.$$

Also, as above explained

$$(\rho'/\rho)^{1/2} = v/v' = \sin i / \sin r.$$

Therefore

$$a^2 - b^2 = c^2 \tan i \cot r.$$

Furthermore, if there is no slipping between the media at their interface, and it seems impossible that there can be to an appreciable extent, then the algebraic sum of a and b must equal c , or algebraically

$$a + b = c.$$

Hence,

$$a - b = c \tan i \cot r$$

$$b = -a \frac{\sin(i - r)}{\sin(i + r)} = -a \frac{\{(\rho'/\rho)^{1/2} - \cos i / \cos r\}}{(\rho'/\rho)^{1/2} + \cos i / \cos r}$$

and

$$c = \frac{2a \cos i \sin r}{\sin(i + r)} = \frac{2a \cos i}{(\rho'/\rho)^{1/2} \cos r + \cos i}$$

If, then, sound waves in a clear atmosphere, say, should impinge normally on a fog bank, or a cloud, in which the density of the air at the same pressure is one per cent. less—a possible ratio owing to differences in temperature and humidity—the energies in the incident, reflected, and transmitted (refracted) waves will have to each other the following ratios, approximately:

$$v \rho a^2 : v \rho b^2 : v' \rho' c^2 = 160,000 : 1 : 159,999.$$

In making this calculation the small correction due to differences between the specific heats of the two masses was neglected.

If the angle of incidence is not zero as just assumed, but of considerable size, the reflection obviously is greater, but still small for any angle up to nearly 90° . If, for instance, $i = 80^\circ$ and $\rho'/\rho = 99/100$, as above assumed,

$$v_\rho a^2 : v_\rho b^2 = 100 : 1,$$

roughly. Clearly, then, reflection from fogs and clouds, or at any sharp boundary of sufficient size (greater than wave-length dimensions) between masses of air at the same pressure but of unequal density, varies from a very small value at normal incidence up to total reflection at

$$i = \sin^{-1} (\rho'/\rho)^{1/2}$$

If, as above, $\rho'/\rho = 99/100$,

$$i = 84^\circ 15'$$

nearly.

Hence a small number of successive total reflections—not likely to occur, though conceivable—might greatly change the direction of the sound.

If the change in density is gradual through a distance of many wave-lengths—if the medium has no definite boundary—reflection is practically absent, as the equations show. For instance, there can be no reflection of sound from the gradually attenuating upper layers of the atmosphere, as demonstrated by Lord Rayleigh.⁶ The energy is maintained through increase of amplitude with decrease of density, except as reduced, as it all finally is, by viscosity, to heat.

Laws of Sound Reflection.—By careful experiment it has been determined that sound is reflected according to exactly the same laws as those that apply to light, namely:

1. The normal at any point of the reflected surface and the directions of incidence and of reflection at the foot of this normal all lie in a common plane.

2. If the reflecting surface is flat and its every diameter large in comparison with the wave-length, the reflection is regular; that is, the angle of reflection (angle between the direction of reflection and the normal to the surface at the point of reflection) is equal to the angle of incidence.

3. Surfaces with less than wave-length diameters do not regularly reflect sound, but merely scatter or diffuse it.

⁶ *Phil. Mag.*, 39, p. 173, 1890.

Echo.—It is well known that in the neighborhood of a flat, vertical wall of some size a loudly spoken word, or brief sentence, often is twice intelligibly heard; the first time as coming straight from the speaker, and the second as if coming from some invisible person in the direction of the wall. This latter sound is the familiar echo (nymph of the poets who pined away until only her voice was left) due to reflection by the wall of the incident condensations and rarefactions in the atmosphere. Parallel walls facing each other, the walls of a canyon, for instance, and separated a few hundred feet, may so reflect and re-reflect the same sound as to cause it to be distinctly heard again and again, through a long fading series. If the reflecting wall is quite irregular, or, especially, if there are walls of various shapes and distances on all sides, the multiplicity of echoes becomes jumbled into a continuous, unintelligible reverberation, a phenomenon most pronounced, perhaps, in certain caves—and auditoriums.

There are of course many natural phenomena due to echo, or sound reflection. Some of the more interesting are: (*a*) the discrete single echo; (*b*) the discrete multiple echo; (*c*) the overlapping multiple echo—reverberation; (*d*) the diffuse echo; due to scattering of the sound by many relatively small objects; (*e*) the harmonic echo; due to the greater scattering of an overtone than of the fundamental by small objects—the scattering of sound, like that of light, being inversely proportional to the wave-length; (*f*) the musical echo; due to reflection from, or scattering by, a series of objects, such as palings or stairs, spaced at uniformly increasing distances from the source.

Obviously a single sound impulse such as the crack of a whip, a clap of the hand, and the like, in front of a flight of steps, or just beyond and a little to one side of a row of palings, will be scattered by the successive steps, or palings, respectively, in a series of regularly spaced similar pulses. Hence the “echo” thus produced of any sound pulse is a musical note, whose pitch depends not on the parent sound, but on the spacing of the palings or width of stair treads, as the case may be, and position of the observer.

If s is the spacing of the pickets, or distance from centre of one to the centre of the next, along the path of propagation of the incident sound; and if the origin of the parent sound impulse is nearly in line with the row of pickets, then the distance between the

sound impulses, or wave-length l , due to any few successive pickets, is given by the equation

$$l = s (1 + \cos \theta)$$

in which θ is the angle, at the pickets in question, between the origin and observer.

An observer therefore at the source, which, as above explained, is beyond the row of pickets and a little to one side, hears a distinctly musical echo of each sharp sound pulse; since at his place the value of $\cos \theta$ is very nearly the same, unity, for all the pickets, and hence the pitch substantially constant. If, however, the observer is off from the middle of a long straight row of palings, and the source of the impulse is located as before, $\cos \theta$ will vary from nearly -1 to nearly $+1$, and the echo heard sweep down from the inaudible through a considerable range of the shriller notes.

Sound Refraction.

Sound Refraction in Still Air of Uniform Temperature.—The direction of propagation of sound is abruptly changed whenever it passes obliquely from one to another mass of air at the same pressure but of different density. This is owing to the fact that the velocity depends on the density, as above explained.

If both masses are quiet and equally humid, the velocity of propagation is inversely proportional to the square root of the density. If then i is the angle of incidence in the air whose density is ρ and r the angle of refraction in the air whose density is ρ' , the change in direction, as indicated in Fig. 2, is the difference between i and the corresponding value of r as given by the equation

$$\sin r = \sin i (\rho/\rho')^{\frac{1}{2}}$$

The laws of refraction in this case are identical with those of light, namely:

1. The normal, at any point of the refracting surface and the directions of incidence and of refraction at the foot of this normal all lie in a common plane.

2. The sine of the angle of refraction (angle between the direction of refraction and the normal to the surface at the point of refraction) is to the sine of the angle of incidence as the velocity in the refracting medium is to the velocity in the incident medium; or inversely as the square roots of the densities of the re-

spective media, or directly as the square roots of their absolute temperatures.

Spread of Sound.—Since the velocity of sound in an atmosphere of homogeneous composition is, as previously explained, directly proportional to the square root of the absolute temperature, and since the atmosphere, when at rest, frequently is horizontally stratified with respect to temperature, it follows that sound cannot always spread in precisely the same manner.

Special cases are interesting:

(a) Temperature uniform, air quiet and of constant composition. Under these conditions, which, however, never exist, sound would spread uniformly in every direction.

(b) Temperature decreasing uniformly with elevation, air quiet and of constant composition. These conditions often are approximately fulfilled through a considerable depth of the atmosphere. They evidently give an oblate ellipsoidal sound shell, with the shorter axis, axis of rotation, vertical. If the origin is on a horizontal surface, the wave front tends to incline upward, owing to the greater velocity in the lowest and warmest air, and hence, the sound to become faint, or even lost, at points on this surface.

(c) Temperature increasing upwards (temperature inversion), air quiet and of constant composition—conditions that obtain on a typical frost night.

Let the surface be an indefinite plane, such as the surface of a large frozen lake, and let the temperature be 0° C. at the surface and increase rapidly with elevation to a maximum of 6° C. at, say, 20 meters elevation, and let a sound be produced near the surface.

The velocity of sound, since it varies as the square root of the absolute temperature, will therefore be about $1/100$ greater at the top of this inversion layer (layer through which temperature increases with elevation) than at the surface. Hence total reflection will occur for all sound leaving the source between the angles i and $\frac{\pi}{2}$ to the vertical; i being determined by the equation

$$\sin i = \frac{100}{101} \sin 90^{\circ},$$

from which $i = 82^{\circ}$, about.

Hence the whole volume of sound leaving the source S , Fig. 3, within a horizontal toroidal wedge of angle θ (16° in this case) is confined, by refraction above and reflection below, to the surface

layer of 20 metres depth. At times this inversion layer is much shallower, but whatever its thickness it is, acoustically, a "speaking disk" in which the intensity of sound is inversely proportional, roughly, to the first power only of the distance. This explains why sounds often carry surprisingly far on still, clear nights.

As a further illustration of this phenomenon let the temperature inversion occur over a river whose banks are free of trees and steep, or canyon-like. Here not only the vertically spreading sound is constrained to a shallow layer, as above explained, but also the horizontally spreading portion is roughly confined to the course of the river by reflection from the banks. That is, the river is one wall of a gigantic "speaking tube" through which sounds occasionally are heard at amazing distances.

(d) Temperature rapidly decreasing upward for a few metres, air quiet and of constant composition. Under these un-

stable conditions sounds travel appreciably faster near the surface than at some distance above it. Hence, sounds originating within or but little above this relatively high temperature surface layer may pass entirely over one's head, even as close as 100 metres to the source, owing to the greater velocities of the lower portions of the sound waves, in proportion to the square roots of the absolute temperatures, and the consequent upward inclination and propagation of these portions of the wave fronts.

Under the given conditions in the lower air and with an approximately constant temperature through the next layer above, a sound front may be shaped substantially as shown at ABC , Fig. 4, and if so it must later develop into the front $A'B'C'$. Owing to refraction CC' will not be quite straight, but may be so drawn to represent the general results.

Clearly, then, under the given conditions a person in region 1 will hear only a faint sound, if any at all; at 2 he will hear two identical sounds, probably more or less out of phase, one from the direction of the actual source as given by the undistorted wave front $A'B'$, and the other apparently from some point more or

less below the actual source, as indicated by the front $B'C'$. At 3 only one sound is heard, namely, the undisturbed original, and, of course, in its true direction.

The phenomena here described are identical with those of the inferior optical mirage, and are due to the same meteorological conditions. They constitute, therefore, the *inferior acoustical mirage*. Indeed a series of sounding objects arranged vertically might, in position 2, if one were equipped with a suitable sound-pointer, be heard both right side up and upside down.

(e) Temperature uniform throughout a shallow layer, then increasing with elevation over several meters to a uniform higher

FIG. 4.

temperature above, air quiet and of constant composition—conditions identical with those that give the superior optical mirage.

The necessary acoustical consequences of this assumed, and sometimes actual, temperature distribution, may be easily inferred by the aid of Fig. 5. After the sound whose origin is at only a moderate elevation, has traveled for some distance its wave fronts have some such shape as that indicated by $ABCDE$, in the figure, owing to the relation of sound velocity to temperature, as explained above. Farther on this front obviously transforms into $A'B'C'D'E'$, of the figure, from which it appears that: In position 1 only a single sound is heard, and in the direction of the source; in position 2, three identical sounds are possible, one in the correct direction, another apparently from a slightly higher point, and inverted if, as explained under d , that be possible, and a third, not inverted, apparently from a still higher point; in position

3, a single sound, apparently from a point distinctly above the true source and, finally, in position 4, also a single sound and again, as in position 1, in the direction of the source.

Since these phenomena are closely analogous to those of the superior optical mirage, and have identical causes, it is suggested that they be called the *superior acoustical mirage*.

Sound Refraction by Straightaway Winds.—Everyone knows, of course, that, in general, winds do not prevent sounds from being heard. Obviously, however, the vibrating portions of the air are carried along with the wind, or, rather, are a portion of it. Clearly, then, the effect of a known wind on the path of a sound

FIG. 5.

Superior acoustical mirage.

ray can be computed with all desired accuracy. Thus, following Barton⁷ substantially:

Let AB , Fig. 6, be the boundary between two horizontal layers of air; let the wind velocity in the lower layer be u_1 , and that of the upper u_2 , both in the same direction, and $u_2 > u_1$; let θ_1 and θ_2 be the inclinations of the wave fronts to the horizontal in the lower and upper layers, respectively; let ϕ_1 and ϕ_2 be the corresponding angles between the sound paths in the two layers and the vertical, and let the velocity v of sound in still air be independent of altitude—true, on the average, to about one part in a hundred between bottom and top of the lowest kilometer, and correspondingly nearer true in this region for less depths.

Draw CB' at right angles to the wave front AC , or along a

⁷ "Text-book of Sound," Macmillan, 1908, pp. 99-106.

$\operatorname{cosec} \theta$ in the above equation less than unity provides a case of total reflection.

Obviously, the actual path of a sound ray can be traced through the various layers when their speeds and directions of motion, temperatures, and other acoustical factors, are all given. Thus, from the figure, in which QN is perpendicular to AB , it is evident that

$$\frac{AN}{QN} = \frac{A'N}{QN} + \frac{AA'}{QA' \cos \theta_2},$$

and, therefore, that in every layer

$$\tan \varphi = \tan \theta + \frac{u}{v} \sec \theta.$$

If u increases continuously and uniformly with elevation, and if the temperature and composition of the air are the same along

the sound path, then it is possible to find an equation for the path, at least in the vertical plane through the origin. But as the conditions specified never strictly obtain, the solution is scarcely more than an abstract mathematical exercise, and therefore will not be given here.

From the above discussion of the effect of the bodily movement of air on the velocity and direction of travel of sound, it is obvious that its leeward and windward distributions may be radically different.

Let the centre of the distorted circle above S , Fig. 7, be the place of origin of a sound, and let the wind be blowing from right to left, as indicated, and increasing with elevation. Obviously, the velocity of the sound, and the consequent distances between the successive compressions and rarefactions will be greatest on the leeward side and least to the windward. The pitch, however, if source and observer are both stationary, will be the same. Furthermore, since the propagation is normal to the wave fronts, the sound is lifted to the windward as indicated in the figure, and thus be-

comes faint, if not quite inaudible, to one on or near the surface in that direction. On the other hand, it is driven down to leeward, in which direction it consequently is heard much farther and louder. All of which are matters of great importance in fog and other sound signaling. Indeed it is even possible for the lifted sound to windward to strike a shore cliff, or other object, and be reflected back in such manner that the echo alone is heard, and the friendly, warning signal thereby transformed into a fiendish lure to danger.

From the figure it is also evident that the sounds are much farther and better heard to the windward from elevated sources than from those near the surface—hence one of the great advantages in having bells in high steeples rather than near the ground. Likewise, sounds to windward, especially if originating

FIG. 8.

Sound crossing a mountain to leeward.

near the surface, often are best heard at elevated points; better in the "crow's nest," for instance, than on the deck.

Another interesting phenomenon in this connection is the carrying of sounds from the windward to the leeward side of a mountain where they sometimes are so distinctly heard as to appear but a little way off. The wind, increasing in speed with distance from the surface, flows parallel, roughly, to the mountain sides—up one side and down the other. Hence the forward portion of a wave front tends more and more towards the vertical as it climbs the windward side of the mountain, crosses the ridge upright (if the wind is suitable) and then focuses onto the leeward valley beyond, all as diagrammatically illustrated in Fig. 8, in which *S* is the source of the sound and *F* its diffuse leeward focus, determined by the wind currents, as indicated in the figure by the progressive positions of the wave front.

The acoustical effects of oppositely directed winds in adjacent layers are worth considering, because such opposing winds frequently occur, for instance, when, in middle latitudes, the surface

wind is from the east; in places of strong land and sea breezes; and in all trade wind regions.

The general effect in question is quite obvious from Fig. 7. That is, a sound made near the surface, by the firing of a large cannon, say, is first lifted on the windward side as indicated, and later, on getting into the oppositely directed upper current, tipped forward and downward, as readily conceived, though not indicated in the figure. In this way exceptionally loud sounds occasionally are heard a hundred miles or more from the source, and over extensive areas beyond a broad zone of silence.

Locating by Sound.

Sound Pointing.—It is well known that a very small difference in the phase of a sound as it enters the two ears generally is quite perceptible.⁸ If then one could actually, or virtually by some artificial aid, stretch his ears far apart, and listen in at widely separated portions of the wave front he obviously could determine the direction of the sound arrival with great precision. And this has been done⁹ by the aid of long, gently tapering, hollow cones, mounted some distance apart with their axes parallel, quickly adjustable in direction, and supplied, at their smaller ends, with aural tubes.

On correcting for refraction, due essentially to wind and differences in temperature (often impossible to do very accurately), the direction of the source, or, at least, its direction when the received sound was emitted, is determined.

Sound Ranging.—Not only is it possible to determine the direction of the source of a sound, but, by the aid of three listening stations, its exact location as well. In this way the positions of many enemy cannon were located during the world war.¹⁰ The details of sound ranging, as the exact location of an object by sound is called, are numerous and their variations many. Nevertheless, the underlying principles are very simple.

Let A, B, C , Fig. 9, be three listening stations and O the place of origin, to be determined, of the sound pulse—place of the firing cannon, say. Let v be the velocity of sound in still air appropriate to the current temperature and humidity. Corrections can easily be made for any observed wind, but for simplicity it will be assumed

⁸ Stewart, *Phys. Rev.*, 15, p. 432, 1920.

⁹ Stewart, *Phys. Rev.*, 14, p. 166, 1919.

¹⁰ Trowbridge, *Jour. Franklin Inst.*, 189, p. 133, 1920.

that there is no wind, and hence no wind corrections. Let the sound pulse originating at O , be received at A , B , and C at the times t_1 , t_2 , and t_3 , respectively.

Then, at the time t_1 the pulse is at A , and distant r_1 , or $(t_2 - t_1)v$, from B , and r_2 or $(t_3 - t_1)v$ from C . Hence, O is the centre of that circle whose circumference passes through A and is tangent to a circle of radius r_1 about B and also to another circle of radius r_2 about C .

Another method of locating O is to note that it is on the loci of three several points, the difference of the distances of one of which from A and B is r_1 , a constant; of another from A and C is r_2 , a constant; and of the third from B and C is $r_2 - r_1$, likewise

FIG. 9.

Sound ranging, or locating sound source.

a constant. That is, it is at the intersection of any two, and hence all three, of the hyperbolas whose foci are A, B ; A, C ; and B, C , respectively, conditioned by $OB - OA = r_1$; $OC - OA = r_2$; and $OC - OB = r_2 - r_1$, respectively.

Dissipation of Sound.—In an absolutely quiet and homogeneous medium the intensity of sound must vary inversely as the square of the distance from the source, except in so far as the mechanical energy of the waves is slowly dissipated into heat. In the ordinary atmosphere, however, at least near the surface, the intensity of sound decreases far more rapidly.¹¹ A portion of the excess in the rate of decrease of the intensity over that required by the inverse square law is due, no doubt, to internal reflections.

¹¹ Stewart, *Phys. Rev.*, 14, p. 376, 1919.

and refractions incident to temperature irregularities. However, the dissipation of sound generally is more pronounced the stronger the wind. Hence, since winds smooth out horizontal temperature irregularities and establish an approximately adiabatic cooling with altitude (1° C. per 103 metres) it follows that, at least when there is considerable wind, some other factor than reflection and refraction of the type just mentioned must be effective in dissipating sound. This factor appears to be the ordinary turbulence, or eddy motions, of the lower air.

Since eddy motion is very irregular, both as to speed and direction, and since the velocity of only a moderate wind is several per cent. of that of sound (22 miles per hour equals about three per cent. of the velocity of sound) it follows that a sound front is greatly distorted on passing through such gusty winds as are common near the surface of the earth. These distortions obviously alter the direction of propagation—normal to the wave front—and on the whole increase the dispersion of the sound and rate of its enfeeblement.

To visualize the effect of eddies on sound a little differently, let an instantaneous distribution of atmospheric compressions and rarefactions, constituting a train of sound waves, be fixed in position except as acted on by mass movements of the air. Such inequalities would quickly be so intermingled by any moderately strong and turbulent wind as to practically obliterate the sound waves, and this same oblitative process is quite as operative, presumably, on travelling, as it would be on stationary waves. The time actually available for such operation is, of course, very brief, but it is sufficiently long, apparently, to permit a great reduction of the sound intensity.

Sounds to and from Elevated Points.—Balloonists, mountain climbers, and others, who have visited quiet places of considerable altitude, have reported two interesting sound observations:

(a) The balloonist, for instance, tells us that the higher-pitched sounds on the surface of the earth are heard to a greater altitude than lower ones of equal initial loudness. A chorus of spring frogs (*hylas*), for example, or the excited cackling of a flock of chickens, is heard after the human voice of at least equal loudness has faded away. This is in spite of the fact that the sounds of shorter wave-length are more rapidly dissipated by

atmospheric irregularities than are those of longer wave-length, and due to the greater sensitivity, within certain limits, of the ear to the higher-pitched notes¹²—to our keener perception here, as also in the case of earthquake shocks, of acceleration than of amplitude.

(b) The balloonist may hear and understand persons on the ground while they cannot hear and understand him. This fact is so striking that it has repeatedly evoked wonderment and comments. Its explanation, however, is not, perhaps, immediately obvious. Probably several factors contribute to the final result, among them:

1. Since the density of the atmosphere is less at the level of the balloon than on the ground, it follows that the voice of the balloonist will be correspondingly weaker (carry less energy) than that of a person on the surface. At an altitude of half a mile, probably about the limit at which vocal communication is possible, the density of the atmosphere is still nine-tenths, roughly, that near the ground. Hence, the density factor, while in the right direction, is not sufficient to account for the commonly observed differences in audibility.

2. The pilot hears both the direct and the reflected portions of the voice below him (whether reflected by the horizontal surface or by objects on it) in substantially the same direction, while the ground observer hears the balloonist's direct voice from overhead, its chief reflection from below, and minor reflections more or less from all around horizontally. Just what the resultant will be in each case is not obvious, but it would seem that with respect to this factor also the balloonist has the better listening position.

3. The balloonist, even when up only half a kilometre, is in a region prevaillingly of utter silence, where any occasional sound is most easily perceived. On the other hand, his surface interlocutor is immersed in a flood of sound, where the limit of audibility is soon reached. That this difference in noise environment is an important factor in the cause of the phenomenon under consideration is evident from the fact that the balloonist often can hear the earth echo of his own voice when he is unable to arrest the attention of persons below him.

¹² Sabine, *Contributions, Phys. Lab. Harvard Univ.*, No. 8, 1919. Stewart, *Phys. Rev.*, 14, p. 376, 1914.