THE QUALITY OF THE TWENTY-FOUR-INCH OBJECTIVE OF THE SPROUL TELESCOPE, AS DETERMINED BY HARTMANN TESTS.*

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THE telescope for the Sproul Observatory was completed in the summer of 1911 and mounted in the observatory the following December. The objective of the telescope, which was built by the John A. Brashear Co., Ltd., is 24 inches in diameter, and has a focal length of approximately 433 inches. The lens was made from a Crown disc furnished by the Parra-Mantois firm, of Paris, and a flint disc furnished by Schott and Genossen, of Jena. In February of 1912, the writers began a test of the lens, using the method of extra-focal images devised by Hartmann,¹ and applied by him, and also by others, to numerous objectives, the most notable of which is perhaps the test of the 80 cm. objective of the Potsdam refractor. The method of making this test has been frequently described² and the reader will recall its essential features. The objective is covered with an opaque screen in which a suitable number of circular openings have been made, through which light from a star (or other source) can pass. These openings serve to isolate such parts of the lens as the observer chooses to test.

If the focus of the telescope is at F^1 , and if the bundles of rays that come from a pair of holes intersect a plane P_1 , within the focus, at the points B and C respectively, one can measure $BC = e_1$ (say). If also the bundles of rays that come from the

* Communicated by Professor Miller.

¹ Hartmann: *Zeitschrift für Instrumentenkunde*, vol. 24.

Plaskett: *Astrophysical Journal*, vol. 25.

Fox: *Astrophysical Journal*, vol. 27.

² Opera cited: Pub. des Astrophysikalischen Obs. zu Potsdam, Nr. 46.

same pair of holes intersect a plane P_2 , without the focus at the points D and E , one can measure the distance $DE = e_2$ (say). If, further, he knows the distance between P_1 and P_2 , he can by solving similar triangles calculate the distance of F , the intersection of the beams from any pair of openings, diametrically situated on the same zone, from the plane P_2 by the formula—

$$A_2 - A = \frac{e_2}{e_1 + e_2} (A_2 - A_1) \quad (1)$$

where A_1 , A_2 and A are respectively the distances of P_1 and P_2 and F from the optical centre of the objective. This formula, including the notation, is practically the same as the one which was deduced by Hartmann, which we modified slightly in order that we might compute $A_2 - A$, the distance of F from the plane P_2 , instead of its distance from the objective as Hartmann did.

FIG. 1.

The distances e_1 and e_2 may be measured with a micrometer, or, having let the rays fall on a photographic plate, the distance between the images may be measured on the plate. The latter is the more convenient way. Now, the quantity $A - A_2$ will depend chiefly upon the objective, but it will be influenced by the color of light being used, and also upon the character of the plate. One might, as Hartmann did in many of his tests, render the rays of light monochromatic by using the spectroscope. This we did not do, our object being rather to determine the quality of the objective under the same circumstances that the telescope is being used. The telescope at present is used chiefly for making photographs for the determination of stellar parallax. The objective is corrected for visual rays, and in order to use the flattest part of the field we photograph through a ray filter, made for us by

Wallace, which is opaque to the violet end of the spectrum and nearly so for the red end. The filter is made of optical glass and is placed very close to the plate. We use Cramer's Instantaneous Isochromatic Plates. The photographs for making the extra-focal test of the objective, which is discussed in what follows, were made in this way.

EARLIER TESTS.

Our first attempt was made on February 22, 1912. Six exposures were made on Capella, when it was near the meridian, through a screen containing 44 circular holes, each 33 mm. in diameter, so arranged as to cover nine zones of the objective. Six sets of plates were made, measured, and reduced. Hartmann's characteristic quantity ³ T , which measures the quality of a lens, calculated from these plates was small ($=.27$). But the measures showed certain discrepancies among the focal distances of different parts and zones of the lens that, although not excessive, were absent when the objective was in the optical room.

The objective is prevented from sliding in its cell by the pressure of a spring on one side of it, and Mr. McDowell, of the Brashear Company, suspected that these discrepancies might be due to this pressure. Later, June 5, 1912, we reduced this pressure and repeated the extra focal test, using Arcturus when it was near the meridian. The screen used in this test contained 78 circular openings each 25 mm. in diameter, so arranged that we could investigate ten zones. We found that, although the exposures on Capella were made when the temperature was low and with the telescope east of the pier, while those on Arcturus were made when the temperature was high and the telescope west of the pier, those parts of the objective that showed short (or long) focus in one of the tests showed the focus short (or long) in the other test also, though in the second case they were numerically smaller.

The extra-focal images, at least, are not good, and the smaller the openings the worse they are. But the larger the opening the smaller the number that can be put into the screen, and the less exhaustive the test is. Moreover, the plates must be put far enough out of focus so that images in the pattern, resulting from

³ *Zeitschrift für Instrumentenkunde*, Bd. 24.

intercepting the beams from the separate holes, may not overlap each other. Having these natural precautions in mind, it so happened that when we made the first screen for photographing Capella we chose values for the radius of the circular opening and the distance of the plates from the focus in such a way that a most peculiar condition resulted. Between each two adjacent images on a diameter there was found a spurious image elongated in a direction perpendicular to the radius upon which the openings lay. We afterwards increased the radius of the openings and changed the extra-focal differences somewhat, thereby eliminating these spurious images entirely.

FIG. 2.

However, we were curious to know the relation that must exist between these various magnitudes in order that the spurious images might appear, or rather would not appear. Miss Marion R. Stearne, then a Senior in Swarthmore College, investigated the matter and found that these spurious images are interference effects and deduced a formula by which one might construct a screen, giving extra-focal patterns from which these spurious images are absent. Miss Stearne found that these spurious images were due to systems of circular maxima and minima (those resulting from the diffraction pattern of a single aperture) which are crossed by alternate dark and bright bands, which are arcs of hyperbolæ and which differ very little from straight lines which are perpendicular to the line joining the centres of the images of the star from two adjacent apertures as in the figure

—and are due to interference of the beams coming from the two apertures.

Below follows a *résumé* of Miss Stearne's work:

Let i = intensity of the illumination at any point of a plane by light coming through a single circular aperture.

Let I = intensity of illumination of any point of a plane by light coming through two circular apertures, of equal radii.

c = distance between the centres of the apertures.

ϕ = angle of diffraction.

λ = wave-length of light.

r = radius of the apertures.

It is proved in physical optics that if light comes through a single circular aperture, there results a system of concentric circular maxima and minima of intensity of illumination of a plane. The minima of intensity are all zero. The centres of these circular maxima and minima coincide with the centre of the images, and their angular radii, θ , as seen from the optical centre is (m an integer):

$$\theta = \frac{m\lambda}{r}$$

Let us call these the first system of maxima and minima. It is further proved in physical optics that

$$I = 4i \sin^2 \left(\frac{\pi c}{\lambda} \sin \phi \right).$$

If we now fix our attention on a particular maximum of the first system, then i is constant.

If we put $\frac{dI}{d\phi} = 0$, one finds either

$$\cos \phi = 0$$

OR

$$\sin \left(\frac{2\pi c}{\lambda} \sin \phi \right) = 0.$$

And since $\frac{\pi}{2}$ is not an admissible value,

$$\sin \phi = \frac{n\lambda}{2c}.$$

When n is odd, $\frac{d^2I}{d\phi^2}$ is negative, and hence for these values I is a maximum.

When n is even, $\frac{d^2I}{d\phi^2}$ is positive and hence for these values I is a minimum. That is, there results a system of alternately dark and bright bands, which are straight lines perpendicular

to the line joining the centres of the apertures. Accordingly, in addition to the precautions mentioned on page 467 it becomes necessary to eliminate these bands that arise from the interference of the beams that come from two adjacent apertures. Since ϕ is small we may put $\sin \phi = \phi = \frac{n\lambda}{2c}$; that is, the distances of these bands from a line perpendicular to one joining the centres of the images are inversely proportional to the distance between the centres of the apertures, while the radii of the first system of circular apertures vary inversely as the radius r of the apertures. We can therefore eliminate these to any degree of approximation either by properly choosing r and c , or by placing the plates far enough from the focus; the latter method has the objection that the far-out-of-focus images are not measurable.

Let θ = the angular radius of the n th maximum of the circular system, and if we put,

$$m_n = \frac{\pi r \sin \theta}{\lambda} = \frac{\pi r \theta}{\lambda}$$

$$\theta = \frac{m_n \lambda}{\pi r}.$$

It can be shown from the geometric properties of image formation that the interference from the beams through two adjacent apertures can be eliminated to the $\left(\frac{n}{2}\right)$ th band, if we put

$$\frac{m_n \lambda}{\pi r} > \frac{2n\lambda}{2c}.$$

Also if—

f = focal length of the objective,

x = distance of the plate from the focal plane,

h = distance from the axis of the objective to the extra-focal image.

Then,

$$\frac{f-x}{h} < \frac{2c}{n\lambda} = \frac{r}{\lambda};$$

$$\frac{x}{h} = \frac{2f}{c}$$

$$\frac{f}{h} < \frac{r}{\lambda} + \frac{2f}{c}$$

$$x = \frac{2fh}{c}$$

That is, if one is testing an objective of focal length, f , having chosen two of the quantities r , n , c or x , arbitrarily, he may determine the remaining ones from the preceding equations and inequalities, so that the spurious images mentioned above are absent.

THE FINAL TEST.

In 1913, we decided to repeat the Hartmann test of the objective, after having again reduced the pressure of the spring referred to on page 467. We constructed screens meeting approximately the foregoing specifications, choosing $n = 9$, and $r = 18$ mm.; hence since for our objective $f = 11,010$ mm. approximately, $c = 81$ mm., and $x = 90$ mm. approximately.

Because the photographic eyepiece of the telescope made it convenient to do so, instead of $x = 90$ mm., we chose to separate the planes P_1 and P_2 by 200.5 mm. The resulting images were the best we had obtained.

In order to cover the lens very completely, we decided to use three screens.

Screen No. 1. Contained 32 holes, arranged on 8 zones; the radii of the zones on this screen were 2, 5, 6.5, 8, 8.75, 9.5, 10.25, and 11 inches.

Screen No. 2. Contained 36 holes, arranged on 9 zones; the radii of these zones were 2.25, 4.25, 5.25, 7.25, 8.375, 9.175, 9.875, 10.625, and 11.375 inches.

Screen No. 3. Contained 14 holes, arranged on 7 zones; the radii of these zones were 1.5, 3, 3.75, 4.5, 5.875, and 7.625 inches.

On January 9, 1913, Capella, when near the meridian, was photographed through each of these screens, in the following manner:

Each screen was used in two positions. Screen No. 1 was set with a given diameter parallel to the equator, and five exposures made. It was then rotated through 45 degrees about an axis through the centre of the lens perpendicular to the plane of the screen and five additional exposures made on another plate. Screen No. 1 was then removed and Screen No. 2 placed over the lens, and, five exposures through it having been made, it was turned 45 degrees and five additional exposures made on another plate. Screen No. 2 was then removed and Screen No. 3 put in its place, and, five exposures having been made, it was

turned through 90 degrees. Integrating these results, this was equivalent to photographing through a single screen containing 164 holes, arranged on 24 zones. There were 8 holes on each of 17 zones and 4 holes on each of 7 zones. These holes were distributed on diameters making angles of $22\frac{1}{2}$ degrees with each other. Fig. 3 shows this integrated screen.

FIG. 3.

It will be noted (see Fig. 3) that some of the holes partially overlap each other. The numbers in the holes show the number of *hundredths of a millimetre* that that particular part of the lens was in error. The heavy circles represent the positions in which the lens focused too short, and the light circles where it focused too long. The arrangement of the holes is also exhibited in Table I.

We used every precaution possible to eliminate local errors. The screens had been in the dome room for several hours pre-

vious to making the photographs in order that they might be of the same temperature as the lens. The screens were constructed carefully and oriented so that the centre of the screen was superimposed on the centre of the objective. Each set of five exposures was made on the same plate. The first exposure was in the centre of the plate and the remaining four grouped symmetrically around the centre one, displaced just enough so that their patterns did not overlap each other. The plates were then measured on a measuring engine, one set each by Prof. S. G. Barton and the writers. The results are exhibited in Table I. It will be noted that Dr. Barton measured only those plates which were made before the screens were turned, through 45 and 90 degrees respectively. The agreement between his measures and those of the writers' are satisfactorily accordant.

If one, by means of formula (1), compute A , the distance from a fixed plane, of the juncture of the rays coming from two holes, he will find, except in the case of a perfect objective, that

$$A = f(\phi, r, \lambda).$$

If when r and λ are constant, A shows a dependence on ϕ , the lens is astigmatic. If the astigmatism is regular and small we may assume, as Hartmann does, that there is one meridional section, making an angle of ϕ_0 with some arbitrarily chosen zero meridional section for which A is a minimum, and another making an angle of $\phi_0 + 90^\circ$ for which A is a maximum; and further that, following Hartmann's notation, if ${}_\phi A$ denote the value of A in a meridional section making an angle ϕ with the zero section, then ${}_\phi A$ may be computed by the formula,

$${}_\phi A = {}_{\phi_0} A + a \sin^2 (\phi - \phi_0) \quad (2)$$

where a is the numerical difference between the greatest and least value of A for any given zone. Hence,

$$\frac{1}{2} ({}_\phi A + {}_{\phi+90^\circ} A) = {}_{\phi_0} A + \frac{a}{2}$$

If one computes ${}_\phi A$ for all meridional sections of any zone of radius r , for which he has observational data (in our case this number varies from 2 to 4), and if he let A^r be the mean of all the values of ${}_\phi A$ thus obtained for any one zone, it is evident that

$$A^r = {}_{\phi_0} A + \frac{a}{2}$$

Put now,

$$R = {}_{\phi_0} A - A^r$$

then,

$$R = -\frac{a}{2} \cos 2(\phi - \phi_0) \quad (3)$$

Thus, every two meridional sections give two equations for the determination of a and ϕ_0 . Where there are more than two meridional sections, as is true for most of the zones that we measured, normal equations may be formed and solved. This is the method described by Hartmann and applied by him to the 80 cm. refractor of the Potsdam Observatory. In making the test of the Potsdam refractor, Hartmann constructed the screen so that two zones had openings on more than two diameters and solving the normal equations, formed from the measures of these zones, he found that they gave fairly accordant results for those two zones; that is, the value of a , and ϕ_0 obtained from the solutions for the two zones did not differ widely. We had openings on more than two diameters in 17 zones. We formed the normal equations for each of these zones, and solved them for the quantities a and ϕ_0 . The value of ϕ_0 for all these zones showed little if any tendency to collect around any meridional section. The results of the computation are found in column 15 of Table I.

Having obtained a and ϕ_0 for any one zone, we computed R' by substituting in equation (3) the value of ϕ_0 and a found by computation. These quantities R are the computed astigmatisms. The results thus obtained show clearly that there is no regular astigmatism in the lens.

Table I is a summary of the arrangement of the openings in the screen, and the results of the measures. For the sake of ready reference, we give here a description of the table. All linear magnitudes are given in millimetres. Column 1 contains the radii of the zones, column 2 the angles that the meridional section made with a meridional section parallel to the equator, measured in a counter clock-wise direction as seen from the eye end of the telescope. Columns 3, 4, and 5 contain the quantities ϕA^r , A^r , and R as determined by Marriott, where ϕA^r is the distance of the focus from the plane P_2 (Fig. 1), *i.e.* from the plane of the photographic plate which was exposed farthest from the objective, as determined by the juncture of the rays from two openings, in a zone of radius r , and in a plane making an angle ϕ with a plane parallel to the equator, A^r the means of the ϕA^r in any zone and R are the astigmatisms. Columns 6, 7, and 8 contain

the same quantities as determined by Miller, columns 9, 10 and 11 the values as determined by Barton, column 12 contains A^r_m the means of A^r , as determined by Marriott and Miller, column 13 the mean of the R 's as determined by the same observers, columns 14 and 15 the value of a and ϕ_0 as computed by equation (3), column 16 the computed astigmatisms, column 17 the value $\Delta = R - R'$. Even a casual glance at Table I will show that in the region of the zone $r = 5$ inches, the lens focuses shorter than at any other place.

The curves in Fig. 4 are the curves of zonal error; the abscissa of any point is the radius of zone, and the corresponding

ordinate is the quantity A^r diminished by a constant and multiplied by 25. So that the difference of any two ordinates is 25 times the difference in the focal length of two zones; that is, the errors are magnified 25 times. The actual difference between the greatest and least focal length of any two zones is less than 1.1 mm. The curves EF , CD , and AB are respectively the graphs of the measures made by Miller and Marriott, and the mean of those measures.

We computed the diameters, d_i , of the circles of confusion for each of the 24 zones in that plane, whose distance from the plane P_2 is A_0 where A_0 is determined by Hartmann's formula:

$$A_0 = \frac{r_k A_k + r_h A_h}{r_h + r_k},$$

where h and k are determined by the condition that no diameter of confusion shall be greater than d where

$$d = 2r \frac{(A_s - A_0)}{F},$$

TABLE

r	Φ	ϕA_r	A_r	R_1	ϕA_r	A_r	R_2
288.92	22.5°	101.62	101.13	-.49	101.70	101.18	-.52
	67.5	101.00		+.12	101.00		+.18
	112.5	101.03		+.10	101.07		+.11
	157.5	100.85		+.27	100.94		+.24
279.40	0.0	100.99	101.25	+.26	100.90	101.20	+.29
	45.0	101.14		+.10	101.15		+.04
	90.0	101.52		-.27	101.43		-.24
	135.0	101.34		-.09	101.29		-.09
269.87	0.0	101.59	101.23	-.36	101.63	101.18	-.45
	45.0	101.10		+.13	100.80		+.38
	90.0	101.07		+.15	101.20		-.02
	135.0	101.14		+.09	101.09		+.09
260.35	22.5	101.72	101.34	-.38	101.70	101.16	-.54
	67.5	101.44		-.10	101.15		+.01
	112.5	101.32		+.02	101.01		+.15
	157.5	100.87		+.46	100.78		+.38
250.82	22.5	101.67	101.49	-.12	101.22	101.27	+.06
	67.5	101.48		+.01	101.33		-.06
	112.5	101.63		-.14	101.48		-.21
	157.5	101.25		+.24	101.07		+.21
241.30	0.0	101.88	101.27	-.61	101.85	101.22	-.63
	45.0	100.93		+.34	100.87		+.31
	90.0	101.20		+.07	101.44		-.22
	135.0	101.05		+.21	100.71		+.51
233.04	0.0	101.90	101.56	-.34	101.60	101.43	-.18
	45.0	101.32		+.23	101.34		+.09
	90.0	101.76		-.20	101.54		-.12
	135.0	101.24		+.32	101.22		+.21
222.25	22.5	101.50	101.51	+.01	101.42	101.37	-.05
	67.5	101.48		+.03	101.36		+.01
	112.5	101.30		+.21	101.15		+.22
	157.5	101.75		-.24	101.56		-.19
212.75	22.5	101.65	101.48	-.17	101.57	101.48	-.09
	67.5	101.53		-.06	101.51		-.03
	112.5	101.31		+.16	101.43		+.06
	157.5	101.41		+.07	101.43		+.06
203.20	0.0	101.56	101.54	-.02	101.43	101.52	+.09
	45.0	101.14		+.39	101.19		+.32
	90.0	101.55		-.01	101.58		-.07
	135.0	101.90		-.36	101.86		-.35
193.67	45.0	101.52	101.64	+.11	101.49	101.57	+.08
	135.0	101.75		-.11	101.65		-.08
184.15	0.0	101.68	101.67	-.01	101.66	101.67	+.01
	45.0	101.41		+.26	101.37		+.29
	90.0	102.08		-.40	101.92		-.26
	135.0	101.52		+.15	101.71		-.05
174.62	0.0	102.07	101.86	-.21	102.34	101.99	-.35
	90.0	101.65		+.21	101.65		+.35

I.

ϕA^r	A^r	R_3	A^r_n	R	a	Φ_0	R'	Δ
101.01	100.88	-.13	101.15	-.51 +.15 +.10 +.26	.62	27°36'	-.30 -.05 +.30 +.05	-.21 +.20 -.20 +.21
100.74		+.13		+.27 +.07 -.25 -.09			+.26 +.08 -.26 -.08	+.01 -.01 +.01 -.01
100.92	101.30	+.38	101.22	+.27 +.07 -.25 -.09	.54	98°45'	+.26 +.08 -.26 -.08	+.01 -.01 +.01 -.01
101.69		-.38		+.27 +.07 -.25 -.09			+.26 +.08 -.26 -.08	+.01 -.01 +.01 -.01
101.03	100.90	-.13	101.20	-.41 +.25 +.07 +.09	.50	170°45'	-.24 +.08 +.24 -.08	-.17 +.17 -.17 +.17
100.76		+.13		-.46 -.05 +.09 +.42			-.27 -.23 +.27 +.23	-.19 +.18 -.18 +.19
101.14	101.00	-.14	101.25	-.46 -.05 +.09 +.42	.72	42°45'	-.27 -.23 +.27 +.23	-.19 +.18 -.18 +.19
100.85		+.14		-.03 -.02 -.17 +.22			+.07 -.12 -.07 +.12	-.10 +.10 -.10 +.10
101.24	101.33	+.09	101.38	-.03 -.02 -.17 +.22	.28	82°37'	+.07 -.12 -.07 +.12	-.10 +.10 -.10 +.10
101.42		-.09		-.62 +.34 -.08 +.36			-.26 -.01 +.26 +.01	-.36 +.35 -.34 +.35
100.78	100.85	+.07	101.24	-.62 +.34 -.08 +.36	.55	1°03'	-.26 -.01 +.26 +.01	-.36 +.35 -.34 +.35
101.91		-.07		-.26 +.16 -.16 +.26			-.05 -.05 +.05 +.05	-.21 +.21 -.21 +.21
101.11	101.38	+.27	101.49	-.26 +.16 -.16 +.26	.14	22°30'	-.05 -.05 +.05 +.05	-.21 +.21 -.21 +.21
101.64		-.27		-.02 +.02 +.21 -.21			-.11 +.11 +.11 -.11	+.09 -.09 +.10 -.10
101.05	100.90	-.15	101.44	-.02 +.02 +.21 -.21	.32	180°00'	-.11 +.11 +.11 -.11	+.09 -.09 +.10 -.10
100.76		+.15		-.13 -.04 +.11 +.06			-.12 -.05 +.12 +.05	-.01 +.01 -.01 +.01
101.48	101.42	-.06	101.48	-.13 -.04 +.11 +.06	.26	33°48'	-.12 -.05 +.12 +.05	-.01 +.01 -.01 +.01
101.37		+.06		+.03 +.36 -.04 -.35			+.00 +.35 -.00 -.35	+.03 +.01 -.04 -.00
101.43	101.49	+.06	101.53	+.03 +.36 -.04 -.35	.71	134°36'	+.00 +.35 -.00 -.35	+.03 +.01 -.04 -.00
101.56		-.06		+.09 -.09 -.00 +.27 +.27 -.33 +.06			+.09 -.09 -.00 +.17 +.11 -.17 -.11	-.00 -.00 -.00 +.17 +.16 -.16 +.17
102.02	101.77	-.25	101.60	+.09 -.09 -.00 +.27 +.27 -.33 +.06	.18	135°00'	+.09 -.09 -.00 +.17 +.11 -.17 -.11	-.00 -.00 -.00 +.17 +.16 -.16 +.17
101.53		+.25		+.09 -.09 -.00 +.27 +.27 -.33 +.06			+.09 -.09 -.00 +.17 +.11 -.17 -.11	-.00 -.00 -.00 +.17 +.16 -.16 +.17
101.54	101.50	-.04	101.67	+.09 -.09 -.00 +.27 +.27 -.33 +.06	.39	106°14'	+.09 -.09 -.00 +.17 +.11 -.17 -.11	-.00 -.00 -.00 +.17 +.16 -.16 +.17
101.47		+.04		+.09 -.09 -.00 +.27 +.27 -.33 +.06			+.09 -.09 -.00 +.17 +.11 -.17 -.11	-.00 -.00 -.00 +.17 +.16 -.16 +.17
102.20	101.90	-.30	101.93	+.09 -.09 -.00 +.27 +.27 -.33 +.06	.54	0°00'	+.09 -.09 -.00 +.17 +.11 -.17 -.11	-.00 -.00 -.00 +.17 +.16 -.16 +.17
101.61		+.30		+.09 -.09 -.00 +.27 +.27 -.33 +.06			+.09 -.09 -.00 +.17 +.11 -.17 -.11	-.00 -.00 -.00 +.17 +.16 -.16 +.17

TABLE

r	Φ	ϕA^r	A^r	R_1	ϕA	A^r	R_2
165.10	0.0°	101.15	101.47	+32	101.25	101.46	+21
	45.0	101.62		-.15	101.57		-.12
	90.0	102.05		-.57	101.97		-.52
	135.0	101.07		+40	101.03		+43
149.22	0.0	101.97	101.86	-.11	102.60	102.14	-.46
	90.0	101.75		+11	101.68		+46
133.35	22.5	101.54	101.99	+45	101.28	101.75	+47
	67.5	101.38		+61	101.06		+69
	112.5	102.50		-.50	102.30		-.55
	157.5	102.55		-.55	102.35		-.60
127.00	0.0	102.45	102.05	-.40	102.47	102.09	-.38
	45.0	102.05		+00	101.94		+16
	90.0	101.20		+85	101.13		+96
	135.0	102.51		-.46	102.84		-.75
116.84	45.0	101.87	101.77	-.10	102.07	102.02	-.06
	135.0	101.66		+10	101.96		+06
107.95	22.5	102.71	102.12	-.59	102.86	102.30	-.56
	67.5	102.80		-.68	102.97		-.67
	112.5	101.47		+65	101.76		+54
	157.5	101.50		+62	101.62		+68
95.25	45.0	101.91	101.72	-.19	101.69	101.66	-.04
	135.0	101.52		+19	101.62		+04
76.20	0.0	102.34	101.98	-.36	102.17	101.93	-.24
	90.0	101.62		+36	101.69		+24
57.15	22.5	100.84	101.30	+46	100.84	101.45	+61
	67.5	100.98		+32	101.27		+18
	112.5	102.07		-.77	101.99		-.54
	157.5	101.31		-.01	101.71		-.26
50.80	0.0	100.94	101.27	+33	101.92	101.19	+27
	45.0	101.86		-.58	101.78		-.59
	90.0	100.61		+66	100.35		+84
	135.0	101.68		-.41	101.69		-.50
38.10	0.0	100.68	101.04	+35	100.84	101.18	+34
	90.0	101.38		-.35	101.52		-.34

(All linear magnitudes in this

I.—Continued

$\phi A'$	A'	R_3	A'_n	R	a	Φ_0	R'	Δ
100.75	100.95	+.20	101.46	+.26 -.13	.96	72°59'	+.39 -.26	-.13 +.13
100.16		-.20		-.54 +.41			-.39 +.26	-.15 +.15
102.08	101.86	-.22	102.00	-.28 +.28	.56	0°00'	-.28 +.28	-.00 +.00
101.64		+.22		+.46 +.65			+.48 +.60	-.02 +.05
101.40	101.55	+.15	101.87	-.53 -.58	1.57	138°05'	-.48 -.60	-.05 +.02
101.71		-.15		-.38 +.08			-.64 +.34	+.26 -.26
102.43	101.63	-.80	102.07	+.08 +.90	1.45	165°50'	+.64 -.34	+.26 -.26
100.83		+.80		-.60 -.08			-.08 +.08	-.00 +.00
102.67	102.55	-.12	101.89	+.08 +.57	.16	45°00'	-.54 -.61	-.03 -.06
102.44		+.12		-.67 +.59			+.54 +.61	+.05 +.04
101.62	101.40	-.12	102.21	-.65 -.11	1.75	46°50'	-.11 +.11	-.00 +.00
101.19		+.12		+.11 -.30			+.30 +.30	-.00 +.00
102.43	102.08	-.35	101.68	+.30 +.53	.60	0°00'	-.30 +.30	-.00 +.00
101.72		+.35		+.53 +.25			+.59 +.19	-.06 +.06
102.43	101.85	-.58	101.95	-.65 -.13	1.24	127°25'	-.59 -.19	-.06 +.06
101.27		+.58		+.30 +.30			+.59 +.19	-.06 +.06
101.50	101.03	-.47	101.37	+.30 +.59	.46	8°00'	-.22 -.06	+.52 -.53
100.56		+.47		+.75 -.46			+.22 +.06	+.53 -.52
99.94	99.90	-.04	101.23	+.34 -.34	.68	0°00'	+.34 -.34	+.00 -.00
99.87		+.04						
101.49	100.77	-.72	101.11					
100.06		+.72						

table are given in millimetres.)

where s takes the values h and k , and F is determined by the Gauss-Hartmann method and is found to be 11,010.9 mm. (433.5 inches).

TABLE II.

1	2	3	4	1	2	3	4
r	d	d	d	r	d	d	d
288.92	.133	.158	.143	174.62	.129	.139	.138
279.40	.073	.146	.106	165.10	.016	.015	.002
269.87	.080	.147	.111	149.22	.110	.055	.135
260.35	.030	.150	.085	133.35	.127	.052	.092
250.82	.033	.097	.028	127.00	.132	.122	.130
241.30	.055	.116	.083	116.84	.069	.097	.084
233.04	.057	.032	.015	107.95	.130	.141	.135
222.25	.036	.050	.003	95.25	.048	.022	.037
212.75	.024	.008	.010	76.20	.071	.052	.064
203.20	.043	.001	.026	57.15	.010	.053	.066
193.67	.071	.019	.047	50.80	.011	.026	.010
184.15	.076	.046	.066	38.10	.075	.020	.021

Table II gives the diameters of the circles of confusion for each of the 24 zones in the plane at a distance A_0 from P_2 . These diameters are expressed in *hundred-thousandths of the focal length*. The maxima of these diameters of confusion as determined by Marriott and Miller are respectively .133 and .158, while the maximum of the diameters as determined by using the mean of the measures of Marriott and Miller is .143. This we have called the "Resultant" of the two sets of measures. These quantities might be taken as the characteristic quantity that determines the quality of the objective since Hartmann's quantity " T " is the weighted mean.

It is our custom, in actual practice, to determine the photographic focus by making a series of exposures, changing the focus by one one-hundredth of an inch between successive exposures. We determined the focus one evening recently by the Hartmann method, using a screen that contained eight holes, and also in the usual way, by making a series of exposures. The focuses determined in these two ways were exactly coincident.

Being curious to know the separating power of the objective, we tried it on some close doubles on a few nights when seeing seemed steady. The components of the closest double that we separated were 0.26" apart. But we do not regard this as the final word on the separating power of this objective.

From every standpoint the lens is an excellent one. The Hartmann test shows that the focal discrepancies, the diameters of confusion, and T , the weighted diameter of confusion, are all extremely small.

Hartmann's characteristic quantity " T " was computed by his formula

$$T = \frac{100,000}{F} \frac{\Sigma rd}{\Sigma r} = \frac{200,000 \sum_{n=1}^{24} r^2 (A_n - A_0)}{F^2 \Sigma r},$$

where F is the focal length of the objective.

We found:

$T = .068$ from Marriott's measures.

$T = .086$ from Miller's measures.

$T = .070$ from the "resultant" of the measures of Miller and Marriott.

We also, following Fox's ⁴ suggestion, determined T by making Σrd a minimum. The quantity Σrd was computed by varying by a few hundredths of a millimetre, the quantity A_0 , from that determined by the Hartmann formula and computing the corresponding values of Σrd .

TABLE III.

1	2	3	4	5	6	7	8
Marriott	A_0	101.55	101.50	101.45	101.41	101.35	101.30
	Σrd	554.71	539.53	521.32	530.50	565.94	606.80
	T	.073	.069	.067	.068	.072	.078
Miller	A_0	101.51	101.49	101.45	101.40	101.35	101.30
	Σrd	673.14	667.42	603.39	600.17	607.25	623.24
	T	.086	.085	.077	.077	.078	.080
Resultant	A_0	101.50	101.45	101.40	101.35	101.30	101.25
	Σrd	574.48	549.10	547.28	550.07	573.37	603.21
	T	.073	.070	.070	.070	.073	.077

The results are exhibited in Table III. The columns 3, 4, 5, 6, 7, and 8 contain the assumed values of A_0 , the value of T , and of Σrd , computed with assumed values of A_0 . The value of T , set in bold figures, is obtained by the Hartmann formula.

⁴ *Astrophysical Journal*, vol. 27.