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On the cause of the remarkable circularity of the orbits of the planets and satellites and on the origin of the planetary system.

By T. J. J. See.

The remarkable circularity of the orbits of the planets of the solar system has been remarked from the earliest ages of Science. This characteristic property among the orbits of the five major planets known to the ancients was the principal circumstance leading to the development of the Ptolemaic System of Astronomy, which continued in use till the time of Copernicus. It was this singular circularity also that so greatly increased the labors of Kepler in discovering and proving the elliptical law of the planetary motions. In the case of the planet Mars, however, an orbit was at length found which was sufficiently eccentric to enable him to demonstrate from the observations of Tycho Brahe that the path was a true ellipse, and not an eccentrically placed circle, as had been supposed since the time of Hipparchus.

The four large satellites of Jupiter discovered by Galilei in 1610 were soon found to exhibit in their orbital motions about that planet the same extreme circularity that was already familiar in the case of the orbits of the major planets about the sun; and the tendency to perfect roundness in the paths of the satellites has so often been confirmed by other discoveries made during the past three centuries that this property has naturally attracted the attention of all philosophic observers. The planet Uranus, added to our system by Herschel in 1781, along with Neptune, discovered from the theoretical researches of Adams and Leverrier in 1846, both preserve the same remarkable circularity of orbit which originally called forth the admiration of the Greek geometers; and the orbits of the satellites of these remote planets have likewise proved to be perfectly circular so far as can be judged from the finest observations made with the largest modern telescopes.

The fact of this wonderful roundness of the orbits of the principal planets and satellites excited the speculative curiosity of Kepler and Newton, but at that early date the cause of this remarkable phenomenon necessarily remained very obscure. In due time it was noticed also with equal surprise by Clairaut, Euler, and Lagrange, but these great geometers have left us no record of their conclusions as to how the property arose. Together with the slight inclination and common direction of motion, the small eccentricity of the orbits was one of the principal points of departure in the celebrated researches of Lagrange, Laplace, and Poisson, on the stability of the solar system, with which the famous mathematicians of the 18th century were so greatly occupied.

Though all these illustrious geometers followed the Greeks in admiring Nature's apparent preference for the circle, no one of them attempted to explain the remarkable roundness of the planetary paths till Laplace promulgated his celebrated Nebular Hypothesis (cf. *Exposition du Système du Monde*, Note VII et dernière, Paris, 1796), and accounted for this circularity of the orbits by a rotation of the central masses which had detached the attendant bodies quite gently and set them revolving in paths of correspondingly small eccentricity.

This explanation of Laplace naturally carried with it the great prestige of the illustrious author of the *Mécanique Céleste*, and has now been very generally accepted by astronomers for more than a century. And yet it may easily be proved to be untenable and altogether devoid of foundation. In fact it will be shown below that the roundness of the orbits of the planets and satellites is to be ascribed to the secular action of the nebular resisting medium formerly pervading our solar system; and that these bodies have never been detached from the much larger central masses which now govern their motions, by acceleration of rotation, as was supposed by Laplace; but have all been captured, or added from without, and have had their orbits reduced in size and rounded up under the secular action of the nebular resisting medium. It may be demonstrated that it was this resisting medium and nothing else which has given the paths of the planets and satellites that beautifully round form which has always been equally admired by the astronomer, the geometer and the natural philosopher.

To establish the truth of this proposition with entire clearness, let us first apply to the bodies of the solar system a criterion based on the mechanical principle of the conservation of areas, which was proposed by Babinet in 1861 (*Comptes Rendus*, Tome 52, p. 481, March 18, 1861). In the paper of Babinet here referred to it is shown that if ω be the sun's angular velocity of rotation with radius r , and ω' and r' the same quantities when the matter of this globe is expanded into a sphere of radius r' ; then by the law of the conservation of areas

$$\omega' r'^2 = \omega r^2. \quad (1)$$

As to the derivation of this formula it is sufficient to recall that the square of the radius is the same as that which appears in the moment of inertia of a rotating mass, and

the product of this by the angular velocity ω gives the moment of momentum, which is always a constant in any free rotating system subjected to no forces except the mutual attraction of its parts. Thus

$$C = \sum m r^2 \omega = \omega \sum m r^2 = \omega' \sum m r'^2. \quad (2)$$

In the case where the matter of the sun is expanded to fill the orbit of the earth, we take $r = 109.5$, and $r' = 23445$, and get $\omega' (23445)^2 = \omega (109.5)^2$.

And the time of the sun's rotation, when the matter of that globe is expanded to the earth's orbit, becomes

$$25^d.3 \frac{\omega}{\omega'} = 25^d.3 \left(\frac{23445}{109.5} \right)^2 = 3192 \text{ years}. \quad (3)$$

And when the matter of the sun is imagined expanded to fill the orbit of Neptune, at a mean distance of 30, we have for the time of rotation of the hypothetical solar nebula

$$25^d.3 \frac{\omega}{\omega'} = 25^d.3 \left(\frac{30 \times 23445}{109.5} \right)^2 = 2888533 \text{ years}. \quad (4)$$

Adopting the system of constants for the solar system employed in the paper in A. N. 3992, with the latest results for the new satellites of Jupiter and Saturn, we find for the entire solar system the following table, which requires no further explanation.

Table showing the application of Babinet's criterion to the planets and satellites, when the Sun and planets are expanded to fill the orbits of the bodies revolving about them.

Solar system.

Planet	R_o The Sun's observed time of Rev.	P_o Observed period of planet	R_c Time of the Sun's rotation calculated by Babinet's cri- terion
Mercury	25.3 days = 0.069267 yrs.	0.24085 yrs.	479 yrs.
Venus		0.61237 "	1673 "
The Earth		1.00000 "	3192 "
Mars		1.88085 "	7424 "
Ceres		4.60345 "	24487 "
Jupiter		11.86 "	86560 "
Saturn		29.46 "	290962 "
Uranus		84.02 "	1176765 "
Neptune		164.78 "	2888533 "

Sub systems.

Planet	Satellite	R_o Adopted rotation of planet	P_o Observed period of satellite	R_c Time of planet's rotation calculated by Babinet's criterion
The Earth	The Moon	1 day	27.32166 days	3632.45 days
Mars	Phobos	24 ^h 62 ^m 29 ^s 7	7.6542 hours	190.62 hours
	Deimos		30.2983 "	1193.52 "
Jupiter	V	9 ^h 9 ^m 28 ^s	11.9563 hours	64.456 hours
	I		1.7698605 days	14.60 days
	II		3.5540942 "	36.900 "
	III		7.1663872 "	93.933 "
	IV		16.7535524 "	290.63 "
	VI		250.618 "	10768.8 "
	VII		265.0 "	11602.4 "
	VIII		930.73 "	61997.2 "
Saturn	Inner edge of ring	10 ^h 64 ^m 1 ^s	0.236 days	0.6228 days
	Outer edge of ring		0.6456 "	2.383 "
	Mimas		0.94242 "	4.2902 "
	Enceladus		1.37022 "	7.0615 "
	Tethys		1.887796 "	10.822 "
	Dione		2.736913 "	17.751 "
	Rhea		4.517500 "	34.620 "

Planet	Satellite	R_0 Adopted rotation of planet	P_0 Observed period of satellite	R_c Time of planets rotation calculated by Babinet's criterion
Saturn	Titan	$10^h 64^m 1$	15.945417 days	186.05 days
	Hyperion		21.277396 "	273.06 "
	Iapetus		79.329375 "	1580.1 "
	Phoebe		546.5 "	20712 "
Uranus	Ariel	$10^h 11^m 12$ (cf. A. N. 3992)	2.520383 days	33.714 days
	Umbriel		4.144181 "	65.435 "
	Titania		8.705897 "	176.05 "
	Oberon		13.463269 "	314.83 "
Neptune	Satellite	$12^h 84^m 81^s 7$ (cf. A. N. 3992)	5.87690 days	141.8 days

It will be seen from a study of the remarkable data given in this table that the hypothetical solar nebula, when it extended to the orbits of the several planets, as imagined by Laplace, could not have rotated with sufficient velocity to detach any of these masses. This inference was already drawn by Babinet in 1861. Indeed he first applied the criterion to the cases of the earth and Neptune. I have developed the table to show moreover that none of the satellites could have been detached from their planets, and thus the argument against the detachment of the planets and satellites is complete and unanswerable.

By the logic of exact data based on a mechanical law of unquestioned validity and without any assumptions as to the law of internal distribution of density except that it remains unchanged, we are thus compelled to admit that the premise adopted by Laplace was false and unjustifiable, when he supposed that the planets were detached from the sun, and the satellites detached from the planets, by acceleration of rotation. Thus this venerable explanation of the roundness of these orbits falls to the ground.

Now the planets and satellites could be formed in but one of two possible ways: 1) They might conceivably have been detached from the central masses which now govern their motions, by acceleration of rotation, as supposed by Laplace. 2) They might have been original nuclei captured in the midst of the solar nebula, and afterwards gradually built up by the agglomeration of more cosmical dust, while at the same time the orbital motion in this resisting medium would have reduced the major axes and eccentricities of their orbits and thus produced the near approach to perfect circularity now observed in our solar system.

We have, however, just proved, by the application of Babinet's criterion, based on the law of areas, that these bodies could not have been detached from the central masses about which they now revolve. Accordingly it follows that they were all captured, and have since had their orbits reduced in size and rounded up under the secular action of the resisting medium formerly pervading the planetary system.

From this unexpected conclusion there is absolutely no escape. For we may prove it by the following reasoning. The effect of a resisting medium in reducing the major axis

and eccentricity of the orbit of the resisted body is fully recognized, and has been known for more than a hundred years. The formulas for the changes of these two important elements may be reduced to the form

$$\delta a = - \frac{2a^2}{p^2(1+m)} [A'v + \text{periodic terms}] \quad (5)$$

$$\delta e = - \frac{2}{p} [Aev + \text{periodic terms}] \quad (6)$$

where p is the parameter of the orbit, a the semi axis major, e the eccentricity, v the true anomaly, and m the mass of the resisted planet, and A and A' constants. As both of these expressions are negative, it follows that under the secular action of a resisting medium the major axis and eccentricity always decrease. In deriving these formulae, however, the density of the resisting medium is supposed to increase towards the center, conformable to what is observed in the nebulae and shown to result from the theory of gases.

Laplace himself has discussed this question with characteristic penetration in the *Mécanique Céleste* (Liv. X, ch. VII, § 19). He shows that when the density of the medium, represented by $\Phi \left(\frac{1}{r} \right)$, increases towards the sun,

the semi axis major and eccentricity always decrease; and finally remarks: "Therefore at the same time that the planet approaches towards the sun, by the effect of the resistance of the medium, the orbit will become more circular". It is surprising that it did not occur to the author of the *Mécanique Céleste* that the roundness of the orbits of the planets and satellites could be explained by a resisting medium quite as easily and simply as by the theory of a rotation which would gently detach these masses and set them revolving in orbits which are nearly circular, especially since the nebular hypothesis itself necessarily implies the existence of such a medium where the planets and satellites now revolve. Laplace merely remarks that if the nebula filled the whole of this space the bodies would encounter such resistance as to cause them to fall into the sun; but in making this statement he overlooked the fact that most of the nebulous matter did go into the sun and planets, and it is from this circumstance that the

central masses became so preponderant, while the attendant bodies are in all cases so very small.

During a recent conference with my friend Professor George Davidson, I mentioned Laplace's proof that a resisting medium had formerly acted against Jupiter's satellites I, II, III, to bring about a near approach to commensurability in their mean motions, and thus enable their mutual attraction to establish a rigorous relationship under the influence of this slowly acting cause. This venerable astronomer justly remarked: »Laplace had the true cause in sight, but he did not carry it far enough to discover the actual process by which the solar system was formed«.

Evidently Laplace had not tested his nebular hypothesis by the criterion based on the conservation of areas, afterwards proposed by Babinet, and it simply did not occur to him that the circularity of the orbits pointed unmistakably to the secular action of a resisting medium. As the very existence of a nebula implies resistance to bodies revolving within it, this oversight is the more remarkable; and unfortunately not only was Laplace's reasoning vitiated, but an equally disastrous effect exerted on all other investigations in Cosmical Evolution for more than a century, because all mathematicians followed the same line of thought, on the false premise that the planets and satellites were detached from the central bodies which now govern their motions.

If in the light of this new theory of the shaping of the orbits under the secular effects of resistance we examine our solar system carefully we shall find many phenomena confirming the former existence of such a medium in our system.

It must suffice here to call attention to but a very few of the numerous survivals of the primordial resisting medium still shown by our system:

1) The rapid motion of Phobos, the inner satellite of Mars, which has been brought down near the planet by resistance, till it now revolves in less than a third of the time of the planet's rotation. It is true that Professor Sir G. H. Darwin explains this motion of Phobos by a tidal retardation of the axial rotation of Mars, but in view of the large part undeniably played by the resisting medium in the formation of our system as a whole this explanation will not hold, though a very small part of the observed effect may be traceable to Tidal Friction.

2) The famous inequality in the motions of the three inner Galilean satellites of Jupiter, which point unmistakably to a resisting medium, as was sagaciously pointed out by Laplace in 1796. His remarks on this subject are as valid and convincing as any which could be made today.

3) The observed rapid motion of the inner ring of Saturn, which greatly exceeds the axial rotation of the planet. The rings evidently were never detached from the planet, but simply survive out of a much larger mass of cosmical dust which has been absorbed in building up the mass of Saturn. All the data in the table relative to Babinet's criterion bear on this same question.

4) The general fact that the satellite orbits are so round, and in general rounder and rounder the nearer we approach the planets, confirms the capture of these bodies in a medium which was denser towards these centers. The

roundness of the satellite orbits shows that resistance was very effective from Mars to Neptune, and therefore no doubt throughout our whole solar system.

5) The retrograde motion of Saturn's satellite Phoebe and Jupiter's VIIIth satellite is likewise to be explained by the capture of these bodies. Whatever may have been their original eccentricities at the time of capture, even retrograde directed bodies could have survived, because the medium against which they revolved was of very slight density at that great distance from the planets. This slight density of the resisting medium at this distance is also indicated by the survival of considerable eccentricities in the orbits of these two satellites. The eccentricity of the orbit of Phoebe is given as 0.22, that of Jupiter VIII as 0.44, which are certainly anomalous enough to excite our suspicion. It is not by chance that retrograde motion in these two cases is associated with the highest eccentricities observed among all the satellites thus far discovered.

6) The orbits of the Asteroids have been gathered into their present positions mainly by the action of Jupiter and of the resisting medium. Originally they were more widely distributed over the whole system than at present; but even now they overlap the orbits of Jupiter and Mars and there may be others of still wider range.

7) The extreme roundness of the orbit of Neptune is a clear indication that this planet moved for a long time against a vast amount of nebulous resistance. Therefore it is very improbable that our planetary system terminates with Neptune. In all probability there are several more planets beyond the present boundary of the system, some of which may yet be discovered.

8) The equatorial accelerations noticed on the globes of the sun and of Jupiter and Saturn, are to be explained by the falling in of matter revolving in vortices about these bodies. As the orbital motion of this matter near these bodies exceeds that depending on the axial rotation, the falling particles necessarily produce an equatorial acceleration. This process may still be going on; at any rate it has been in progress so recently that the effects still continue.

9) The solar system was formed from a spiral nebula, revolving and slowly coiling up under mechanical conditions which were essentially free from hydrostatic pressure. And spiral nebulae themselves arise from the meeting of two or more streams of cosmical dust. The whole system of particles has a sensible moment of momentum about some axis, and thus it begins to whirl about a central point, and gives rise to a vortex. In the actual universe the spiral nebulae are to be counted by the million, and it is evident that they all arise from the automatic winding up of streams of cosmical dust, under the attraction of their mutual gravitation. The two opposite branches of the spiral nebulae, so often shown on photographs, represent the original streams of cosmical dust which are coiling up and forming gigantic spiral systems.

10) When the nebula rotates and the coils wind up in such a way as to leave open spaces between the coils, or at least freedom from sensible hydrostatic pressure, the usual result is the development of a system made up of small bodies, such as the planets compared to the greatly preponderant sun, or the satellites compared to the much

greater planetary masses which control their motions. In the solar system where the conditions are accurately known this is proved to have occurred; and it was repeated so many times always with uniform results giving a large central mass and small attendant bodies that the general law for this condition is clearly established.

11) If the streams so converge that the nebulous mass becomes very concentrated at the center, so as to become a figure of equilibrium under the pressure and attraction of its parts, the nebula may divide into a double star, as I have elsewhere inferred from the researches of Poincaré and Darwin on the figures of equilibrium of rotating masses of fluid.

12) Now both of these forms of development are abundant in the actual universe, and probably almost all of the apparently single stars are surrounded by systems of planets. At least one fifth of all the stars are estimated to be double or multiple. If we assume the chances of the two forms of development to be equal we may calculate from the preponderance of small bodies actually found in the solar system — 8 principal planets, 25 satellites (besides our Moon), and 625 asteroids — that the chances of a nebula devoid of hydrostatic pressure producing small bodies is about 2^{658} to 1, or a decillion decillion (10^{66}) to the sixth power to Unity (10^{66})⁶. This figure is so very large that we shall content ourselves with illustrating a decillion decillion (10^{66}), and for this purpose we avail ourselves of a method employed by Archimedes to illustrate his system of enumeration. Imagine sand so fine that 10000 grains will be contained in the space occupied by a poppy seed, itself about the size of a pin's head; and then conceive a sphere described about our sun with radius of 200000 astronomical units (α Centauri being at a distance of 275000), entirely filled with this fine sand. The number of grains of sand in this sphere of the fixed stars would be a decillion decillion

(10^{66}). All these grains of sand against one is the probability that a nebula devoid of hydrostatic pressure such as that which formed the planets and satellites will lead to the genesis of such small bodies revolving about a greatly predominant central mass.

This is a very brief and inadequate outline of a few of the leading points in a very large investigation with which I am occupied. Most of these results were established over six months ago, and I have had opportunity to discuss them with several experienced astronomers. It is hoped that the whole investigation may appear as Volume II of my Researches on the Evolution of the Stellar Systems; but it seemed advisable to communicate this brief summary of some of the most interesting points without further delay.

The effect of this work will be to give the Theory of the Resisting Medium the highest importance in all researches relating to the history of the Universe. It is very remarkable that the principal secular effects of this cause are exactly opposite to those due to Tidal Friction as investigated by Darwin. For while Tidal Friction usually increases the major axis and eccentricity of an orbit, the resisting medium as regularly decreases both of these elements. In the actual physical universe both causes are at work together, sometimes one influence predominating and then the other. Resisting medium is relatively most effective in a system made up of a large central sun, and small attendant bodies, such as the planets of our solar system; and as the systems of satellites dominated by large planets. Tidal Friction is most effective in systems made up of two large masses, such as the double stars.

It has seemed advisable to call attention to the cause of the roundness of the orbits of the planets and satellites, because it appears likely that the criteria now introduced may go far towards clearing up the mystery which has always surrounded the origin of our solar system.

U. S. Naval Observatory, Mare Island, California, 1909 Jan. 1.

T. J. J. See.

Note. Since communicating the above paper on Jan. 1, 1909, it has seemed advisable to emphasize more strongly the conclusion that there is certainly one, most likely two, and probably three unknown planets beyond Neptune. In December, 1904, I examined the evidence bearing on the place of the planet next beyond Neptune and concluded that this body, which was then designated as Oceanus, was most likely near longitude 200° , and at a distance of 42.25, with a period of 272.2 years. Such a body harmonizes all known data, but the location in longitude is subject to considerable uncertainty. The longitude cannot be much smaller than this, but it might be appreciably larger. The other planets I have placed at distances of 56 and 72 respectively, but they will be much more difficult to discover than that situated

between 42 and 44. In view of the light now thrown upon the mode of formation and constitution of the solar system I would recommend a persistent photographic search of the region of the ecliptic between longitudes 200° and 250° . Different persons will form different estimates of the validity of the grounds on which this extension of our solar system outward has been based, but I am satisfied that time will show the prediction to be well founded. To suppose the planetary system to terminate with an orbit so round as that of Neptune is as absurd as to suppose that Jupiter's system terminates with the orbit of the fourth satellite. The force of this analogy is not appreciably weakened by the fact that Jupiter's satellites constitute a subsystem of our solar system.

U. S. Naval Observatory, Mare Island, California, 1909 Jan. 10.

T. J. J. See.

Longitude of the Washburn Observatory, Madison. The Superintendent of the U. S. C. and G. Survey advised Professor G. C. Comstock that in consequence of recent longitude determinations and a readjustment of the longitude net the best value that can now be assigned to the longitude of the Washburn Observatory is $5^h 57^m 37.896$ west of Greenwich.