XVII.—On “*Optical Tube-length*”; an *Unconsidered Element in the Theory of the Microscope.*

By FRANK CRISP, V.P.L.S., Sec. R.M.S.

(*Read 14th November, 1883.*)

It is not a little strange that at this late period in the development of the Microscope, an element of capital importance both from a theoretical and a practical point of view should have been left entirely unconsidered, and indeed unknown; and the fact that it is so, illustrates the disadvantages which English-speaking microscopists have always been under in having no text-book dealing with the theory of the Microscope.

In a letter written more than a year ago in reference to the Table of Magnifying Powers published in the Journal, Professor Abbe called my attention to the erroneous notions which prevailed on the subject of the magnifying power of the Microscope, and which he had been the first to clear up,* and I ought then to have published the explanation now given here, but the pressure of other engagements diverted my attention, and I confined myself to explaining the matter verbally to those who attended the meetings. Finding, however, that the Committee on Eye-pieces of the American Society of Microscopists have been misled by the Table in question, it is obviously desirable not to delay the explanation any longer.

Microscopists have always recognized that the length of the tube of the Microscope is a factor in determining the amplification of the image, that the amplification is generally greater with a 10 in. tube than with one of 6 in.; and that we obtain an increase of power by pulling out the draw-tube. Here, however, all exact notions as to the function of the tube-length have practically stopped, so much so that there has not been any agreement even as to how the length of the tube is to be measured, whether from the front or back lens of the objective to the field lens, the diaphragm, or the eye lens of the eye-piece.

In particular, no view of tube-length has been held which would explain the following apparently paradoxical statements:—

That two objectives of precisely the same focal length used with the same tube and the same eye-piece may nevertheless give different magnifying powers.

That two objectives of different focal lengths used with the same

* Professor Abbe also communicated it to Dr. Dippel, by whom it was embodied in the last edition of ‘*Das Mikroskop*,’ 1882.

tube and eye-piece will not give magnifying powers in proportion to their focal lengths; thus a 1-2 in. will not necessarily give double the power of a 1 in.

Conversely, two eye-pieces will not amplify in proportion to their focal lengths, though used with the same tube and objective.

Indeed, the true magnifying powers may differ from the powers which would be obtained on the ordinary assumptions by more than 100 per cent., and Prof. Abbe records the existence of objectives (of somewhat exceptional construction it is true) which exhibit this paradoxical behaviour: that one of longer focal length amplifies much more than one of shorter focal length; that one gives the same amplification with a long and a short tube, and that one gives a higher amplification with a short tube than with a long one.

What then is the explanation of these paradoxes?

The explanation is not to be found in any question of the length of the objective or eye-piece, or the character of their respective settings, but depends upon the fact that hitherto microscopists have regarded the outside only of the tube and have left out of consideration the optical action which goes on within it.

To properly understand the matter it will be necessary to consider the principles on which the action of the Microscope in regard to magnifying power is founded.

The magnifying power of a lens depends of course upon its focal length and varies inversely with it; the ordinary mode of obtaining the power being to divide the distance of distinct vision l (assumed as 10 in.) by the focal length, or expressing it by a formula $N = \frac{l}{f}$. Thus if the focal length f of an objective is 1-8th in. $10 \div \frac{1}{8} = 80$. The same applies to the action of the Microscope as a whole, that is with eye-piece and objective combined; when we have determined its focal length we similarly obtain its magnifying power.*

We have therefore to ascertain the proper mode of determining the focal length f of the entire Microscope, having given the focal length f^1 of the objective and the focal length f^2 of the eye-piece.

The usual assumption hitherto has been that f is determined by multiplying f^1 and f^2 together and dividing by the length of the tube 10 in., or

$$f = \frac{f^1 f^2}{10},$$

* The quotient obtained by dividing 10 in. by the focal length gives the linear amplification of an image—real or virtual—which is projected by an objective to a distance of 10 in. *from its posterior focus*, and not from the objective, as has been so commonly assumed.

so that if we had an objective of 1-8th in. and an eye-piece of 2 in. the focal length of the Microscope

$$f = \frac{\frac{1}{8} \times 2}{10} = \frac{1}{40}.$$

A Microscope of a focal length of 1-40th in. would magnify 400 times, so that if this method of arriving at the focal length of the Microscope were correct, we should only have to multiply the power of the (1-8th in.) objective (80) by that of the (2 in.) eye-piece (5) to have the total magnifying power (400), the brass tube being assumed to be constant at 10 in.

The fallacy of this method lies in the fact that the true formula is not

$$f = \frac{f^1 f^2}{10}, \quad \text{but} \quad f = \frac{f^1 f^2}{\Delta},$$

Δ being the distance between the posterior principal focal plane of the objective, and the anterior principal focal plane of the eye-piece, or, as Professor Abbe terms it, the rational or *optical tube-length*, in contradistinction to the mechanical or physical length.*

The accompanying fig. 154, where F^1 is the posterior focal plane of the objective and F^2 the anterior focal plane of the eye-piece, will illustrate this more clearly.

As Δ is the divisor of the fraction which represents the focal length, the latter is of course larger or smaller according as Δ is smaller or larger, that is, it varies inversely as Δ ; and as the magnifying power is inversely to the focal length, the magnifying power varies directly as Δ , which is therefore seen to be a fundamental factor of microscopic amplification.

We can now see how it is that two objectives of the same focal length may yet give different magnifying powers with the same tube and eye-piece. By the different methods of construction adopted by their makers, the focal plane of the one objective may be further off the back lens than is the case with the other. The distance Δ between the focal

planes of the objective and eye-piece will be correspondingly

* The principal focal planes are the planes passing through the point on the axis in which parallel rays coming from the *opposite* side of the lens are brought to a focus. "Anterior" and "posterior" are used in reference to the direction in which the rays come to the observer.

FIG. 154.

diminished, and the focal length of the whole Microscope increased. The magnifying power will therefore be diminished.

Again, take the case of two objectives of say 1-8th in. and 1 in. focal length used with the same eye-piece (2 in.) and tube. If the distance Δ remained constant, say 10 in., the total focal length would vary with that of the objectives,

$$f = \frac{\frac{1}{8} \times 2}{10} = \frac{1}{40}, \quad \text{or} \quad f = \frac{1 \times 2}{10} = \frac{1}{5}.$$

But the posterior focal planes of the two objectives, instead of coinciding, may have different positions, every variation producing of course a change in the value of Δ . With the 1-8th in. objective the posterior focal plane may be very near the back lens, and we have a long Δ : with a 1 in. objective its posterior focal plane may be further from the back lens (higher up the tube), and we have a diminished Δ . We might have with the 1-8th in. objective $\Delta = 10$ in., and a power of $(80 \times 10 =)$ 800, but with the 1 in. objective we should not have $(10 \times 10 =)$ 100, or a total power in proportion to the powers of the objectives. Δ might be 8 in. only instead of 10 in., and the total power would be only 80.

The converse case of different eye-pieces with the same objective is similarly explicable. The anterior focal planes of the eye-pieces may be at different points of the tube, and we shall have a varying Δ .

As to the general character of the variations in Δ , it may be noted that the position of the anterior focal plane of the eye-piece does not vary much in the Huyghenian form; a substantial difference is, however, found in this respect between the Ramsden and Huyghenian, the former having its anterior focal plane at some distance below the field lens, and the latter above it. With the objective, however, a very wide range is possible. Its posterior focal plane may be (1) some distance above the last surface of the objective; (2) close to this surface outside or within the objective; or (3)—though a more exceptional case—as a virtual focus below the stage or even below the table. Practically, however, with objectives of ordinary construction, the difference in position of the posterior focal plane is not great with powers higher than 1-2 in., and it is only when we come to the lower powers that the difference is a substantial one.

Greater differences in the power will also be found with short tubes than with long ones. With a 10 in. tube a difference of 2 in. reduces the 10 to 8, but with a 6 in. tube from 6 to 4, quite different percentages of variation.

The process, therefore, of multiplying together the powers of the eye-piece and the objective to obtain the total power of the

Microscope is a fallacious one, as it supposes a constant tube-length; whilst, as we have seen, the true tube-length varies with the different objectives and eye-pieces used.

To determine the power of the Microscope from the powers of the eye-piece and objective, it is necessary, in addition, to know the position of the focal planes of each of the latter. How these may be readily determined must be deferred for a subsequent occasion.