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“The Measurement of Electrical Conductivity.”

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IN specifying particulars for the manufacture of submarine and other electric cables, it is the universal custom to assign a minimum value for the conductivity of the copper, on the basis of the Matthiessen standard. When the length and weight of a given wire or strand of uniform cross-sectional area are known, it is a simple matter to determine the conductivity of the material from a measurement of the resistance of the wire; but if the cross-sectional area is not uniform, the conductivity is not so readily deduced. Wire, as received from the wire-drawer, may vary in diameter to the extent of 0.5 to 1.0 per cent. in a length of a few yards; hence, for practical cable-work, greater accuracy than 0.1 per cent. in the determination of conductivity is superfluous. Length, weight, and resistance, however, are quantities easily measured with far greater precision than 0.1 per cent.; consequently, in tests of conductivity, there is a margin of precision which may be encroached upon with a view to facilitating calculation and shortening the operations. The object of the following Paper is to show that, for this degree of precision, an instrument can be designed which will indicate the conductivity *directly*, upon a scale, corrected for small differences of diameter, as percentage conductivity on the Matthiessen standard, or, if required, in C.G.S. units.

If, by any method of testing, a stranded cable-conductor reveals low conductivity, it is then too late for the manufacturer to throw out a bad component wire. In such a case, the whole length of the strand might have to be rejected. It is therefore prudent to test, by some simple method, the conductivity of each hank of wire as it arrives from the dies. For the purposes of this Paper it will be supposed that a long submarine cable is to be made, for which several hundreds of tons of copper wire of nominally uniform diameter are to be tested, hank by hank, to determine whether they fulfil certain requirements of conductivity. From the whole

of the hanks a few are selected as roughly representative; known lengths are cut from these and weighed, and the respective resistances of these lengths are measured at some known temperature, so that they can be used as standards of known conductivity. Thirty feet is a convenient length for such standards. Equal lengths of wire from every hank required for the cable are then balanced successively on a slide-bridge against one or other of the standards. These lengths need not be cut off the hanks, but merely unwound from them. It is evident that if means can be devised whereby the readings of the slider can be made to indicate at once the percentage conductivity of the test-wire, and whereby they can be compensated mechanically for small differences of mean diameter between the standard-wire and the wire of any hank under test, the operation of testing will be greatly facilitated.

In determining whether this is possible, it is necessary to consider what are the conditions of equilibrium of the slide-bridge, (1) assuming that the standard-wire and the test-wire balanced against it are equal in diameter but unequal in conductivity; and (2) assuming that the standard-wire and the test-wire are equal in conductivity but of unequal mean diameters.

Case 1.—Let R_1 and R_2 be the resistances respectively of two such wires of equal lengths and equal diameters, but of different percentage conductivities on the Matthiessen standard. Let w and x be the respective differences, positive or negative, between their conductivities and 100 per cent., so that—

$$(100 + w) R_1 = (100 + x) R_2 \quad (1)$$

and suppose that these two resistances are balanced against each other upon an ordinary straight-scale bridge of any convenient length, graduated into one hundred equal divisions numbered from the left. Let l be the reading of the slider, when there is balance. The length of bridge-wire to the right of this position is then $(100 - l)$ hundredths of the total length of the scale; hence—

$$(100 - l) R_1 = l R_2 \quad (2)$$

Dividing (1) by (2)—

$$\frac{100 + w}{100 - l} = \frac{100 + x}{l}$$

$$\therefore l = \frac{100(100 + x)}{200 + x + w} \quad (3)$$

Since copper of less conductivity than 95 per cent. would certainly be rejected, and as it is improbable that conductivities higher than 105 per cent. will be met with in practical cable-work, x and w

will be within the limits +5 and -5. As a matter of fact, the range for which precision is necessary is less than this, the usual minimum conductivity specified being 98 per cent. For all ordinary purposes, therefore, it is sufficient if conductivities between, say, 96 per cent. and 104 per cent. can be measured to within, say, 0.1 per cent. of accuracy.

In order to evaluate equation (3) it will be assumed, for the present, that $w = 0$ (which may be interpreted as meaning that the conductivity of the particular standard-wire is 100 per cent.), and that the conductivity of the test-wire is $(100 + x)$ per cent. The justification for assuming $w = 0$ will appear later. Equation (3) now takes the form

$$l = \frac{100(100 + x)}{200 + x} \quad (3a)$$

If, in this equation, x is given successive positive and negative values, the values of l corresponding to unit increments of conductivity are obtained, always supposing that the standard-wire and the test-wire are of equal diameters. These values of l determined from equation (3a) are expressed in hundredths of the length of the bridge-wire, of whatever material, length, and uniform diameter that may be. The values and their differences are given in Table I.

TABLE I.

Conductivity Per Cent.	x	l	Differences.
105	+5	51.220	
104	+4	50.980	0.240
103	+3	50.739	0.241
102	+2	50.495	0.244
101	+1	50.249	0.246
100	0	50.000	0.249
99	-1	49.749	0.251
98	-2	49.495	0.254
97	-3	49.239	0.256
96	-4	48.980	0.259
95	-5	48.718	0.262

From this Table the scale of the slide-bridge could be graduated to read directly in percentages under the conditions assumed. It is seen that the differences in l corresponding to unit increments of conductivity are not strictly equal throughout the range; if, however, it can be shown that the error introduced by assuming them to be equal is so small as to be negligible, the scale becomes a very simple one. The average of the differences in the fourth column of Table I. is 0.25. To evaluate the errors which would result from graduating the slide-wire in equal divisions having this value, another Table, assuming constant differences of 0.25 between successive values of l , can be prepared from equation (3a). Putting these successive values of l into equation (3a), corresponding values of x are obtained. Adding 100 to each of these values of x , the true values of the corresponding conductivities of the test-wires are arrived at. For comparison with these calculated conductivities, the conductivities indicated by the instrument when calibrated in equal divisions, each 0.25, are also shown in Table II.

TABLE II.

Conductivity by Instrument (observed).	$x = \frac{100(2l-100)}{(100-l)}$ From (3a).	$l = 50 \pm n \times 0.25$ (calculated).	Conductivity = $(100 + x)$ (calculated).	Error. Difference between Columns 1 and 4.
105	+5.128	51.250	105.128	0.128
104	+4.082	51.000	104.082	0.082
103	+3.046	50.750	103.046	0.046
102	+2.020	50.500	102.020	0.020
101	+1.005	50.250	101.005	0.005
100	+0.000	50.000	100.000	0.000
99	-0.995	49.750	99.005	0.005
98	-1.980	49.500	98.020	0.020
97	-2.956	49.250	97.044	0.044
96	-3.922	49.000	96.078	0.078
95	-4.878	48.750	95.122	0.122

By inspection of the fifth column of Table II. it is seen that so long as $w = 0$, the error involved by graduating the scale in equal divisions is negligible, since for test-wires between 96 per cent. and

104 per cent. conductivity the error is less than the prescribed 0.1 per cent. For special circumstances, however, it might be preferable to adopt some other value than 0.25. For example, since many modern specifications set the "minimum conductivity allowed" at 98 per cent., it might be well to choose, from Table I., a value for the length of a division which makes the differences vanish at 98 per cent. Such a length would, for that case, be 0.255 hundredths. Since, however, the value 0.25 happens to be a very convenient figure, as it represents exactly one four-hundredth part of the total bridge-wire, it will be retained for the purposes of this Paper. The scale so divided will be called the "conductivity scale," and it will be supposed that its middle point is accurately set to the electrical middle of the bridge-wire, and that this point on it is marked 100. The electrical middle of the bridge-wire can be easily found by balancing two equal coils, and by checking the position of balance by interchanging the coils. So long as the conductivity of the standard-wire is exactly 100 per cent., and the diameters of the standard-wire and test-wire are equal, the position of the slider, when balance is obtained, now indicates on the conductivity-scale the conductivity of the test-wire.

Having thus determined the graduations of the conductivity-scale, the more general case, in which w is not equal to zero, will be investigated. From equation (3)

$$x = \frac{l(200 + w) - 10000}{100 - l} \quad (4)$$

Substituting successively in this equation the values of l given in the third column of Table II., and assigning any selected value to w , a corresponding set of values for x is obtained. By assigning a limiting value to w , the greatest error that can arise in using the bridge with various standard-wires in practical cable-work will be determined. As already stated, 98 per cent. is the usual minimum conductivity specified; a standard at least as near to 100 per cent. as this should therefore always be obtainable. But in order to take the most extreme case it will be supposed that for some reason the conductivity of the standard-wire is as low as 96 per cent., so that $w = -4$. The results are shown in Table III.

If, when the standard $(100 + w)$ is only, say, 96 per cent., balance is obtained at the 100 mark on the conductivity-scale, then x is of course equal to w (i.e., $x = -4$). Thus, although the true conductivity of the test-wire is 96 per cent., the instrument indicates 100 per cent. By adding w to all the instrument-readings, however, approximately correct results are obtained, as shown in the

TABLE III.

$l = 50 + 0.25 n$ Hundredths.	$x = \frac{196 l - 10000}{100 - l}$	True Conductivity ($100 + x$).	Scale Reading ($100 \pm n$).	Scale Reading plus w ($100 \pm n$) - 4.	Differences between Columns 3 and 5.
51.250	+0.882	100.882	105	101	0.118
51.000	-0.082	99.918	104	100	0.082
50.750	-1.056	98.944	103	99	0.056
50.500	-2.041	97.959	102	98	0.041
50.250	-3.015	96.985	101	97	0.015
50.000	-4.000	96.000	100	96	0.000
49.750	-4.955	95.045	99	95	0.045
49.500	-5.901	94.099	98	94	0.099
49.250	-6.838	93.162	97	93	0.162
49.000	-7.764	92.236	96	92	0.236
48.750	-8.684	91.316	95	91	0.316

fifth column of Table III. The errors entailed by this method are also shown in Table III. It is seen that the limit, 0.1 per cent., is exceeded only towards the ends of the range. If tables similar to Table III. are prepared for values of w between +2 and -2, they indicate that if the standard is 98 per cent., wires between 98 per cent. and 105 per cent. can be tested with no greater error than 0.059 per cent., and that if the standard is 102 per cent., wires between 95 per cent. and 102 per cent. can be tested with no greater error than 0.061 per cent.

In the method represented by Table III. the slide-bridge is not direct-reading; a final correction has to be made by adding w to the result of each test. The Author has found, however, that this operation may be dispensed with, and the accuracy of the results increased, by moving the conductivity-scale relatively to the electrical middle of the bridge-wire, so that the ($100 + w$) mark on the conductivity-scale, and not the 100 mark, coincides with the middle of the bridge-wire, and clamping it there so long as that particular standard is in use. In Table IV. are given the readings obtained by this sliding-scale method, with $w = -4$, so as to make the results comparable with those of Table III. If by this method, and with $w = -4$, balance is obtained when the slider

is at 100, say, then $l = 50 + (4 \times 0.25) = 51$; and this value of l put into equation (3) gives $x = -0.082$. Hence the conductivity of the test-wire is $(100 + x) = 99.918$. Again, if the conductivity $(100 + x)$ were 96, it is clear that balance would be obtained at the electrical middle of the bridge-wire, *i.e.*, at 96 on the conductivity-scale as set, for in this case $x = w$. Table IV. is constructed on this principle.

From Table IV. it is seen that although the extreme case of the standard being of only 96 per cent. conductivity, has been taken, tests can be made by this new method between 95 per cent. and 105 per cent. conductivity, without involving any error greater than 0.082 per cent. And since, in practice, the standard will never be less than 98 per cent., the errors are well within the prescribed limit.

TABLE IV.

$l = 50 \pm 0.25 n$ Hundredths.	$x = \frac{196 l - 10,000}{100 - l}$	True Conductivity. (100 + x)	Conductivity Scale Reading (100 $\pm n$)	Differences between Columns 3 and 4.
52.25	+5.047	105.047	105	0.047
52.00	+4.000	104.000	104	0.000
51.75	+2.964	102.964	103	0.036
51.50	+1.938	101.938	102	0.062
51.25	+0.923	100.923	101	0.077
51.00	-0.082	99.918	100	0.082
50.75	-1.056	98.944	99	0.056
50.50	-2.041	97.959	98	0.041
50.25	-3.015	96.985	97	0.015
50.00	-4.000	96.000	96	0.000
49.75	-4.955	95.045	95	0.045

If some other material, such, for example, as iron wire, is required to be tested in this way, similar accuracy can be attained by proper selection of the length of a graduation of the conductivity-scale. The best length for any special work can at once be found from Table I., or from an extension of that Table.

Case (2).—So far, it has been assumed that the standard-wire and the test-wire are of equal diameters. The difficulty presented

by inequality of their diameters will now be investigated, as well as the possibility of devising a mechanical arrangement whereby to compensate for such differences as exist between the mean diameters of all wires that come under test. For this purpose it will be supposed that the conductivity of the standard-wire is equal to that of the test-wire against which it is balanced on the bridge, but that there is a slight difference between their diameters.

If the diameters be represented by d and $(d + y)$, then, in equation (1), since $(100 + x)$ is now equal to $(100 + w)$, R_1 and R_2 are proportional respectively to $\frac{1}{d^2}$ and $\frac{1}{(d + y)^2}$. Equation (2) therefore takes the form—

$$\frac{100 - l}{d^2} = \frac{l}{(d + y)^2}$$

$$\text{and } l = \frac{100 d^2 + 200 dy + 100 y^2}{2 d^2 + 2 d y + y^2} \quad (5)$$

This, as it stands, is a somewhat inconvenient expression to interpret mechanically for the position of balance. It may, however, be reduced to a progression, thus—

$$l = 50 + 50\left(\frac{y}{d}\right) - 25\left(\frac{y}{d}\right)^2 + \frac{25}{2}\left(\frac{y}{d}\right)^4, \text{ etc.} \quad (6)$$

This result is arrived at from (5) by ordinary division. Obviously the first term, 50, is the middle point of the bridge-wire, expressed in hundredths; and the sum of the remaining terms is the deviation, from the middle of the bridge-wire, due to the difference of diameter y ; for if $y = 0$, $l = 50$. A mechanical interpretation can therefore be applied to equation (6), and the deviation can be evaluated with a view to compensating for differences of diameter between the standard-wire and the wire under test.¹ In the following Table this deviation is given for values of d to cover practically the whole range of cable-work. From inspection of equation (6) it is seen that since d and y in each term have like powers, the value of l is independent of the particular unit chosen for d and y . The expression (6) is therefore perfectly general, and gives l in terms of hundredths of the length of the bridge-wire. As the mil (1 mil = 0.001 inch) is in common use as a unit of

¹ It is clear that this method of mechanical correction is of general application to many problems involving small differences of diameter. For example, it should be useful in potentiometer tests, as in the Kelvin-bridge arrangement, described by the Author in the *Proceedings of the Physical Society*, vol. xiv., pp. 243-245, 1896.

measurement in cable-work, the values of d and y in the following Table are given in mils. The values given in the fifth column of Table V. are applicable to any slide-bridge, whatever its dimensions. They represent, in hundredths of the total length of any bridge-wire, the distance from the middle of the bridge-wire, through which the sliding contact must be moved in order to balance when the standard-wire and the test-wire are of equal conductivity, but differ in diameter by 1 mil. The values are independent of the length of the standard-wire and of the test-wire, provided that these are of equal length. In practice, the differences in diameter between the standard-wire and the test-wire are less than 1 mil,

Fig. 1.

and are generally expressed in tenths of 1 mil. It is therefore suggested that when testing hanks of wire of any particular gauge, a list should be made out, of deviations for every tenth of a mil, for that gauge. This can be done at once from Table V., by taking one-tenth of the figure in the fifth column, corresponding to any particular gauge, and multiplying it successively by all whole numbers up to 10. It is also convenient to plot a curve, once for all, from the first and fifth columns of Table V., so that the deviation corresponding to any gauge can be seen at a glance, Fig. 1. There are generally two persons employed on the test,

TABLE V.

d Mils.	$50 \left(\frac{y}{d}\right)$	$25 \left(\frac{y}{d}\right)^2$	$\frac{25}{2} \left(\frac{y}{d}\right)^4$	$50 \left(\frac{y}{d}\right) - 25 \left(\frac{y}{d}\right)^2 + \frac{25}{2} \left(\frac{y}{d}\right)^4$
10	5.000	0.2500	0.00125	4.751
15	3.333	0.1111	0.00025	3.222
20	2.500	0.0625	0.00007	2.437
25	2.000	0.0400	0.00003	1.960
30	1.667	0.0278	0.00002	1.639
35	1.429	0.0204	0.00001	1.409
40	1.250	0.0156	0.00000	1.234
45	1.111	0.0123	..	1.099
50	1.000	0.0100	..	0.990
55	0.909	0.0083	..	0.901
60	0.833	0.0069	..	0.826
65	0.769	0.0059	..	0.763
70	0.714	0.0051	..	0.709
75	0.667	0.0044	..	0.663
80	0.625	0.0039	..	0.621
85	0.588	0.0035	..	0.584
90	0.556	0.0031	..	0.553
95	0.526	0.0028	..	0.523
100	0.500	0.0025	..	0.497

one gauging the wire with a micrometer, and one operating at the slide-bridge. If the gauger calls out the observed differences in tenths of mils, the bridge operator has only to refer to his list to know what is the corresponding deviation to correct for on the slider.

Mechanical Interpretation.—There are several ways in which the difficulty arising from the fact that w is not always equal to zero may be overcome. For instance, a series of conductivity tests of the ordinary kind may be made, in order to find a wire of 100 per cent. conductivity, and this may then be used as a standard. Or, from the known length and diameter of the standard-wire the

resistance corresponding to 100 per cent. conductivity may be calculated¹ for wire of the gauge under test, and this resistance may then be inserted in the bridge-arm instead of a standard-wire. Or, using a preliminary test-wire of known conductivity, setting the slider to the position on the bridge which would give balance if the standard were 100 per cent., and keeping the slider in this position, the length of the standard-wire may be adjusted until balance is obtained. Each of these methods has a certain amount of interest, and might be applied in special cases, but the method indicated by Table IV. is probably the best. This method consists in providing the slide-bridge with a small scale, C, *Figs. 2*, which can be set with respect to the electrical middle M of the bridge-wire, S, to the right or left, through a distance ($w \times 0.25$) hundredths of the length of the bridge-wire, *i.e.*, through a distance corresponding to the conductivity of the standard-wire expressed in terms of the graduations of the scale C, as already explained. This small scale, which may be called the conductivity-scale, is marked "100" in the middle, and is graduated into five divisions on each side of this mark, so that it reads from 95 to 105, the length of each division being 0.25 hundredths of the length of the bridge-wire. It is set so that the point upon it corresponding to the conductivity of the standard-wire is at the electrical middle M of the bridge-wire, and it is clamped in that position so long as that standard is in use.

To compensate for differences of diameter, another scale is provided. This scale, D, *Figs. 2*, is marked "0" in the middle, and is graduated to the right and left in divisions each equal to one-hundredth of the length of the bridge-wire. It is seen from the fifth column of Table V. that no more than five such divisions on each side of the zero mark are required. This will be referred to as the "diameter-scale." The edge of this scale slides against the edge of the conductivity-scale, parallel to it. The zero mark of the diameter-scale projects across to the conductivity-scale as shown in *Figs. 2*. The contact-slider can be clamped at

¹ Assuming that copper of 100 per cent. conductivity is such that a length of l feet, weighing w grains, has a resistance $R = \frac{l^2 \times 0.2162}{w}$ standard ohms. at 60° F., it follows that the resistance of 30 feet of copper wire of 100 per cent. conductivity, and of diameter d inches has a resistance of $\frac{0.000305795}{d^2}$ standard ohms. at 60° F. The linear temperature-correction per ohm. per 1° F. between 57° F. and 72° F. is 0.00239. Standard-resistance coils can thus be made for a range of values of d . When using these, the 100 mark of scale C is set to M, *Figs. 2*.

any position along the diameter-scale, as determined by the differences of diameter, corresponding to the fifth column of

Table V. When thus set and clamped, the contact-slider and the diameter-scale move together, and are so moved with respect to the conductivity-scale, until the galvanometer indicates balance. At

[THE INST. C.E. VOL. CLIV.]

2 A

the position of balance, the zero mark of the diameter-scale points to the required conductivity as indicated by the scale C.

The method of operation is therefore very simple and rapid. The first thing necessary is to find the electrical middle of the bridge. To do this, the middle point of the conductivity-scale, the middle point of the diameter-scale, and the contact point of the slide, are set all in line, at the position of balance, when two wires of equal resistance are in the bridge-arms, and this position is marked on the base-board. A small sliding-block M is provided for this purpose, *Figs. 2*. As a check, this operation may be repeated with the equal resistances reversed. The electrical middle of the bridge having thus been found, the graduation of the conductivity-scale, corresponding to the conductivity of the standard, is set to this

Fig. 3.

point, and the conductivity-scale is then clamped. It need not be touched again so long as the same standard is in use; the only adjustment that has to be made for successive test-wires is that entailed by differences in their diameters. This adjustment consists in moving the contact-slider, K, to the proper position with respect to the diameter-scale. If the instrument is divided and marked as in *Figs. 2*, one end of the standard-wire is connected to the left-hand terminal A, and one end of the test-wire is connected to the left-hand terminal B. The other ends of the wires are connected together, and to one terminal of the galvanometer, the other terminal of which is connected to the terminal E. The terminal E is permanently connected to the slider K, through the rod on which K slides. The connections are shown diagram-

matically in *Fig. 3*. If the test-wire is heavier than the standard-wire the right-hand half of the diameter-scale *D* is used. If the test-wire is lighter than the standard-wire, the left-hand half of the diameter-scale *D* is used. Balance is then found in the usual way, by sliding the contact *K* and the scale *D*, clamped together, to the right or left.

It is obvious that only a small portion of the bridge-wire will ever be traversed by the slider; consequently a good deal of space may be saved by coiling the ends of the bridge-wire upon bobbins; these may be placed near one another for compactness, and to equalize their temperature. If larger divisions are required, they can be secured by using a greater length of bridge-wire, and the same relations and Tables will hold throughout. Or, if desired, resistance-coils, each having a resistance equal to 4.5 times that of the whole bridge-wire, can be added, one at each end of the bridge-wire, by plugs or otherwise, so as to multiply all the indications by 10. It must not, however, be supposed that by magnifying the divisions in either of these ways the small errors are eliminated. These small errors are inherent, but fortunately, as has been proved in the foregoing, they are well within the limits prescribed by practical requirements.

If for any purpose it is necessary to express the readings of the conductometer in C.G.S. units instead of in relation to the Matthiessen standard, the same instrument can be used, with different calibration of the conductivity-scale only. There arises, however, some uncertainty as to the proper conversion-factor to adopt in passing from the Matthiessen to the C.G.S. system.¹ The best value for the specific resistance (centimetre-cube) of copper of 100 per cent. conductivity is probably 0.0000016944 standard ohm at 60° F. The reciprocal of this is 590187, which is the specific conductivity. If the standard is taken as 100 per cent., and the test-wire as $(100 + w)$ per cent. conductivity, the specific conductivity is $(100 + w) \times 590187$ at 60° F. Hence, to make the indications of the conductometer correspond with these units, the divisions of the small scale might be subdivided into, say, 59 parts. A convenient length for the bridge-wire is 2 metres; this gives divisions of $\frac{1}{2}$ centimetre for 1 per cent. on the Matthiessen basis.

In conclusion, the Author desires to express his indebtedness to Professor W. N. Stocker, of Brasenose College, Oxford, who has very kindly checked the work in this Paper, and to Mr. H. Berg, who

¹ This matter is discussed by the Author in an article entitled "The Electrical Conductivity of Copper," in *The Electrical Review*, 19 June, and 3 and 10 July, 1903.

has supervised the construction of the instrument at Silvertown. The Report of the National Physical Laboratory fully bears out the foregoing calculations as to the accuracy of this conductometer. The Report mentions that the following crucial test was made to check its indications: "Two pieces of copper-wire differing slightly in diameter, but of equal lengths, were compared by an accurate method which gave the ratio of their conductivities as 100·05 to 100; the ratio as measured by the conductometer was 100·02 to 100." If the conductivity is constant for a series of test-wires, their mean diameters can be measured by this instrument to a small fraction of one-thousandth of an inch.

The Paper is accompanied by two drawings, from which the Figures in the text have been prepared.