whence, finally,

$$\begin{aligned} & \left\{ a \left(n \frac{\partial}{\partial y} - m \frac{\partial}{\partial z} \right)^2 + \dots \right\} (vw) \\ &= (q+r-1) [rv \{bc'' + cb'' - 2ff''\} l^2 + \dots] + qw \{ (bc' + cb' - 2ff') l^2 + \dots \} \\ & \quad - qr [(A_1 l^2 + \dots + 2F_1 mn + \dots) s + (A_1 a + \dots + 2F_1 f + \dots) L \\ & \quad - 2 \{ (A_1 l + H_1 m + G_1 n) s_1 + (H_1 l + B_1 m + F_1 n) s_2 \\ & \quad + (G_1 l + F_1 m + C_1 n) s_3 \} L] \dots\dots\dots(9). \end{aligned}$$

From the foregoing investigation it is evident that (7)

$$\begin{aligned} & \left\{ a \left(n \frac{\partial}{\partial y} - m \frac{\partial}{\partial z} \right)^2 + \dots \right\} (uvw) \\ & (p+q+r-1) [p v w \{ (bc + cb - 2ff) l^2 + \dots \} + q w u \{ (bc' + \dots) l^2 + \dots \} \\ & \quad + r u v \{ (bc'' + \dots) l^2 + \dots \}] \\ & \quad - \{ q r u (A_1 l^2 + \dots) + r p v (A_1' l^2 + \dots) + p q w (A_1'' l^2 + \dots) \} s \\ & \quad - \{ q r u (A_1 a + \dots) + r p v (A_1' a + \dots) + p q w (A_1'' l^2 + \dots) \} L^2 \\ & + 2 [\{ q r u (A_1 l + H_1 m + G_1 n) + r p v (A_1' l + \dots) + p q w (A_1'' l + \dots) \} s_1 \\ & \quad + \{ q r u (H_1 l + B_1 m + F_1 n) + r p v (H_1' l + \dots) + p q w (H_1'' l + \dots) \} s_2 \\ & \quad + \{ q r u (G_1 l + F_1 m + C_1 n) + r p v (G_1' l + \dots) + p q w (G_1'' l + \dots) \} s_3] L \\ & \dots\dots\dots(10); \end{aligned}$$

and that according to the same law the result of operating on the product of four, or more, functions may be at once written down.

A Theorem in Combinations. By R. W. D. CHRISTIE.

[Read Jan. 10th, 1889.]

Let $\Sigma ({}_n S_r)^m$ represent the sum of the m^{th} powers of the sums of the letters which form the several combinations of n letters taken r at a time; then

$$\Sigma ({}_n S_1)^m - \Sigma ({}_n S_2)^m + \Sigma ({}_n S_3)^m - \Sigma ({}_n S_4)^m \dots \pm \Sigma ({}_n S_n)^m \equiv 0,$$

where $m < n$, e.g., if $n = 5$, $m = 4$,

$$\begin{aligned} & a^4 + b^4 + c^4 + d^4 + e^4 - (a+b)^4 - (a+c)^4 - (a+d)^4 - (a+e)^4 - (b+c)^4 \\ & - (b+d)^4 - (b+e)^4 - (c+d)^4 - (c+e)^4 - (d+e)^4 + (a+b+c)^4 \end{aligned}$$

$$\begin{aligned}
& + (a+b+d)^4 + (a+b+e)^4 + (b+c+d)^4 + (b+c+e)^4 + (b+d+e)^4 \\
& + (c+d+e)^4 + (a+c+d)^4 + (a+c+e)^4 + (a+d+e)^4 - (a+b+c+d)^4 \\
& - (a+b+c+e)^4 - (a+b+d+e)^4 - (a+c+d+e)^4 - (b+c+d+e)^4 \\
& + (a+b+c+d+e)^4 \equiv 0.
\end{aligned}$$

$$\begin{aligned}
\text{Here } \Sigma ({}_6S_6)^4 &= \Sigma a^4 + 4\Sigma a^3b + 6\Sigma a^2b^2 + 12\Sigma a^2bc + 24\Sigma abcd \\
\Sigma ({}_6S_4)^4 &= 4\Sigma a^4 + 12\Sigma a^3b + 18\Sigma a^2b^2 + 24\Sigma a^2bc + 24\Sigma abcd \\
\Sigma ({}_6S_3)^4 &= 6\Sigma a^4 + 12\Sigma a^3b + 18\Sigma a^2b^2 + 12\Sigma a^2bc \\
\Sigma ({}_6S_2)^4 &= 4\Sigma a^4 + 4\Sigma a^3b + 6\Sigma a^2b^2 \\
\Sigma ({}_6S_1)^4 &= \Sigma a^4.
\end{aligned}$$

Thus the number of times that

$$\begin{aligned}
\Sigma a^4 \text{ or } a^4, b^4, c^4 \dots \text{ will occur will be } {}_4C_0 - {}_4C_1 + {}_4C_2 - {}_4C_3 + {}_4C_4 &= (1-1)^4 = 0, \\
4\Sigma a^3b \text{ or } 4a^3b, 4a^2c \text{ or } 6\Sigma a^2b^2 \text{ or } 6a^2c^2 \dots -{}_3C_0 + {}_3C_1 - {}_3C_2 + {}_3C_3 &= -(1-1)^3 = 0, \\
12\Sigma a^2bc \dots \dots \dots {}_2C_0 - {}_2C_1 + {}_2C_2 &= (1-1)^2 = 0, \\
24\Sigma abcd \dots \dots \dots -{}_1C_0 + {}_1C_1 &= -(1-1)^1 = 0,
\end{aligned}$$

and there is no term containing the five letters.

In the general case, the number of times that $a^m, b^m, c^m \dots$ will occur, will be

$${}_{n-1}C_0 - {}_{n-1}C_1 + {}_{n-1}C_2 - \dots {}_{n-1}C_{n-1} = (1-1)^{n-1} = 0;$$

that $\frac{m!}{p! q! r!} a^p b^q c^r \dots$ will occur, will be

$$(-1)^{k-1} [{}_{n-k}C_0 - {}_{n-k}C_1 + \dots] = (-1)^{k-1} (1-1)^{n-k} = 0,$$

where $p+q+r \dots = m$, and the letters $a, b, c \dots$ are k in number.

The general explanation is, that the number of ways of selecting r things out of n so that each selection shall contain k given things is

$${}_{n-k}C_{r-k}.$$

The theorem will enable us to obtain an unlimited supply of equations such as the following:—

$$\text{Let } a = 1, b = 2, c = 3, d = 5, e = 7, f = 8.$$

Then, after cancelling terms like O^m with $-(a+b)^m$, &c., we get

$$0^m + 4^m + 9^m + 17^m + 22^m + 26^m \equiv 1^m + 2^m + 12^m + 14^m + 24^m + 25^m,$$

where $m = 1, 2, 3, 4$, or 5 ; and therefore for the same magnitudes

of m

$$n^m + (n+4)^m + (n+9)^m + (n+17)^m + (n+22)^m + (n+26)^m$$

$$\equiv (n+1)^m + (n+2)^m + (n+12)^m + (n+14)^m + (n+24)^m + (n+25)^m$$

for all magnitudes of n .

Thursday, February 14th, 1889.

J. J. WALKER, Esq., F.R.S., President, in the Chair.

Mr. Baker was admitted into the Society.

The following communications were made:—

On the Diophantine Relation $y^2 + y'^2 = \text{square}$: Prof. Cayley, F.R.S.

Sur la Transformation des Équations Algébriques: Signor Brioschi.

On Projective Cyclic Concomitants or Surface Differential Invariants: E. B. Elliott, M.A.

On Secondary Invariants: Prof. L. J. Rogers, M.A.

Remarks upon Algebraical Symmetry, with particular reference to the Theory of Operations, and the Theory of Distributions: Major Macmahon, R.A.

The following presents were received:—

"Proceedings of the Royal Society," Vol. xlv., Nos. 273 and 274.

"Educational Times," for February.

"Greek Geometry, from Thales to Euclid," by Dr. G. J. Allman, 8vo; Dublin and London, 1889.

"Bulletin des Sciences Mathématiques," November and December, 1888, and January, 1889.

"Beiblätter zu den Annalen der Physik und Chemie," Band xii., Stück 12; Band xiii., Stück 1.

"Bollettino delle Pubblicazioni Italiane, ricevute per Diritto di Stampa," No. 73; Index to Ditto, for 1888 (part only).

"Memorias de la Sociedad Científica—'Antonio Alzate,'" Tomo ii., No. 5; Mexico, 1888.

"Atti della Reale Accademia dei Lincei—Rendiconti," Vol. iv., Fasc. 6, 7, 8, 9.

"Annales de l'Ecole Polytechnique de Delft," Tome iv., Livraison 3^{me}; Leide, 1888.

"Annali di Matematica," Tome xvi., Fasc. 3.

"Journal für die reine und angewandte Mathematik," Band 104, Heft. ii.

"Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften," Band 14, Nos. 10, 11, 12, 13; Leipzig, 1888.