COEFFICIENT OF EFFLUX FROM AN ORIFICE FURNISHED WITH A SHORT PIPE.

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It is well known that the application of a short pipe to a simple orifice in a reservoir results in a material increase of the discharge. Most writers on hydraulics treat this phenomenon as an ultimate fact of science to be determined only by experiment. It is, of course, sufficient for practical purposes, when this case occurs, to

be able to make use of the experimental coefficients of efflux. It is greatly conducive, however, to clearness of knowledge to be able to refer this fact to the well known principles of impact and momentum. Moreover, in hydraulic computations, the same phenomenon presents itself in many forms to which no experimental coefficient is applicable.

The writers* of the splendid article "Hydro-mechanics," in the new *Encyclopædia Britannica* give their view of the rationale of this phenomenon, and express the coefficient of efflux from a short pipe thus :

$$c = \frac{1}{\sqrt{1 + \left(\frac{1}{\mu} - 1\right)^2}}$$

μ being the coefficient of efflux from a simple orifice, c that from the short pipe. If μ be put = 0.615, this gives $c = 0.85$, which is an approximation to the truth, but too large.

My own reflections on the subject lead to a somewhat different formula, which I think it worth while to put in print.

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Let v_1 be the velocity with which the water enters the pipe, *i. e.*, the velocity in the smallest section of the contracted vein at the entrance.

v_2 = the velocity of discharge from the pipe.

h = the static head acting on the orifice. This is sometimes measured to the upper edge of the orifice and sometimes to the centre. For this reason tabular coefficients of efflux differ. It must be understood in the same sense with reference to the entrance and exit of the pipe.

μ = the coefficient of efflux from a simple orifice of the form under consideration.

ϕ = coefficient of velocity from the same. So that the discharge from a simple orifice, of area A , is $A \mu \sqrt{2 g h}$, and the velocity is $\phi \sqrt{2 g h}$. It is well known that the velocity of issue from a simple orifice falls a little short of that due the static head; a fact which may ordinarily be neglected, but must be taken account of in computations relative to impact and living force.

The water enters the pipe with the velocity v_1 and issues with the velocity v_2 . During the passage through the pipe, therefore, an amount of energy, represented by

$$\frac{v_1^2 - v_2^2}{2g},$$

has disappeared. I say energy, although the idea of energy involves the weight of water discharged per second. But as this element does not vary during the computation, it is just as well to regard the energy as represented by the head.

A certain loss of energy occurs in the abrupt change of velocities from v_1 to v_2 . According to well known principles, this is represented by

$$\frac{(v_1 - v_2)^2}{2g}$$

This accounts for a part of the energy that has disappeared in passing the pipe, but not all of it. The remainder is represented by

$$\frac{v_1^2 - v_2^2 - (v_1 - v_2)^2}{2g} = \frac{2 v_2 (v_1 - v_2)}{2g}.$$

What has become of it? It has been expended in diminishing the atmospheric pressure, which opposes the flow of the water. In other words, the flow of the stream has been increased at the expense of its energy. The water enters the pipe with the velocity that would be imparted to it by the static head

$$h + \frac{2v_2(v_1 - v_2)}{2g}; \text{—say } h + H.$$

but

$$\frac{v_1}{\Phi} = \frac{v_2}{\mu} \text{ or } v_1 = \frac{\Phi}{\mu} v_2$$

whence

$$H = 2 \left(\frac{\Phi}{\mu} - 1 \right) \frac{v_2^2}{2g}.$$

The head required to impart the velocity v_1 is

$$\frac{1}{\Phi^2} \frac{v_1^2}{2g} = \frac{1}{\Phi^2} \left(\frac{\Phi}{\mu} \right)^2 \frac{v_2^2}{2g} = h + 2 \left(\frac{\Phi}{\mu} - 1 \right) \frac{v_2^2}{2g}$$

whence

$$v_2^2 = \frac{2gh}{\left(\frac{1}{\mu} \right)^2 - 2 \left(\frac{\Phi}{\mu} - 1 \right)} \quad \dots$$

$$\frac{v_2}{2gh} = \text{coefficient of efflux} = \frac{1}{\sqrt{\frac{1}{\mu^2} - 2 \left(\frac{\Phi}{\mu} - 1 \right)}} = c.$$

The value of Φ is, according to Weisbach (*Mechanics*, etc., Coxe, p. 855), 0.975. μ varies slightly with the head. Weisbach takes 0.615 as the working value of μ . Substituting these values in the formula for c , we have $c = 0.824$. Weisbach gives 0.815 as the mean experimental value of c , but the values found by different experimenters differ considerably.

ELECTRIFIED BUBBLES.—At the Royal College of Greenwich, soap bubbles were blown of a cylindrical form, and in a vertical position between two concentric platinum rings which served as electrodes. When the bubbles are left to themselves, they exhibit a series of colors in horizontal bands which gradually enlarge and descend; a black band soon appears toward the top and also extends downwards. Electric currents tend to transport the liquid to the negative pole, thus retarding the motion of the bands if the negative electrode is at the top, and accelerating it if the negative electrode is at the bottom.—*Cosmos*, Sept. 7, 1885.