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Discovery of the Cause of the Sunspots, and of their 11-year Periodicity, and of the Cause of the Cepheid, Geminid, and Cluster Variable Stars. By *T. J. J. See.*

[With 9 Plates.]

I. Introductory Remarks on the Problem of Sunspots and their 11-year Periodicity, and on the Elements, Conjunctions and Oppositions of Jupiter and Saturn.

The year 1610 is especially notable in the history of astronomy from *Galileo's* invention of the telescope and his almost immediate discovery of conspicuous spots on the sun. In the scholastic philosophy then current the sun appeared so glorious, that *Dante* had made it the abode of the saints, while under *Aristotle's* original doctrines it was held to be perfect and therefore free from all physical defects; and thus the solar spots noticed by *Galileo* soon became objects of much philosophic attention and of violent controversy. Hence it happens that our records of these spots run back 311 years, and during much of this period the observers have left us reliable and fairly continuous observations.

But although in this interval of ten generations of astronomers the combined mass of the observations accumulated is enormous, it is a somewhat embarrassing fact that science still is unable to give us an adequate physical theory of the origin of the spots or of the celebrated 11-year periodicity discovered by *Schwabe* of Dessau in 1843. For in a special article on the Sun, New International Encycl., New York, 1916, Prof. *Hale*, Director of the Mt. Wilson Solar Observatory, expressly remarks: »The cause of this periodicity is unknown.«

It always has seemed to me very unfortunate that although the observational record accumulates century after century, no substantial progress is made in the physical theory of the sun. Now there naturally will be different views of what criteria a valid physical theory should meet, and where the accumulated mass of observations is so enormous as is true of the solar records, it would not be surprising if some division of opinion should arise from outstanding details. Yet if the great body of solar phenomena clearly conforms to the theory, it will have the presumption of truth, and become the stronger the more fully it meets a geometrically rigorous test which may serve as an experimentum crucis.

Accordingly, having been somewhat occupied with the problem of the cause of sunspots since 1917, when the dynamical theory here outlined first took definite form, and having recently developed certain exact criteria which appear to be absolutely decisive, I would not be justified in longer delaying the publication of the results at which I have arrived.

It is evident that solar phenomena do not stand alone, but are closely related to other processes recognized to be going on in the solar system. Thus both Jupiter and Saturn have been found to have very sensible equatorial accelerations,

analogous to the swifter rotation of the spots observed in the gaseous photosphere near the sun's equator. Can any one doubt that these three similar phenomena, on the three largest bodies of our solar system, depend on a common cause?

And what can this common cause be? To get a long range view of this subject we may recall the capture theory of satellites developed in 1909, (AN 4308, 4341, 4343) and the resulting theory of the rotations of the principal planets (AN 4358). This development of 1909 enables us to recognize that the processes which operated in the formation of the solar system still are at work, and thus the precipitation of meteors upon these bodies leads to the equatorial accelerations noticed on the sun, Jupiter and Saturn.

And all we are required to do is to develop the dynamical theory now recognized to be valid and to have its roots firmly set in the foundations of the solar system. This will lead us to a critical study of the mutual actions of Jupiter and Saturn, in precipitating meteors upon the sun. And thus we cite first the elements of these great planets as given in *Hill's* New Theory of Jupiter and Saturn, Collected math. works of *G. W. Hill* 3.19.

Elements of Jupiter and Saturn.

Epoch 1850 Jan. 0.0 m. t. Greenw.

$L = 159^{\circ} 56' 26''.60$	$L' = 14^{\circ} 49' 34''.04$
$\pi = 11 \ 56 \ 9.33$	$\pi' = 90 \ 6 \ 46.22$
$\theta = 98 \ 56 \ 19.79$	$\theta' = 112 \ 20 \ 49.05$
$i = 1 \ 18 \ 42.10$	$i' = 2 \ 29 \ 40.19$
$e = 0.04824277$	$e' = 0.05605688$
$n = 109256''.55563$	$n' = 43996''.07844$
$m = 1/1047.870$	$m' = 1/3501.6$

Since *Laplace's* discovery of the physical cause of the great inequality, 1785, it has been generally recognized that the most important planetary perturbations of the solar system are those depending on the mutual actions of Jupiter and Saturn. The mean motions were found by Dr. *G. W. Hill* to be:

$$n = 109256''.62552 \quad n' = 43996''.21506 \quad (1)$$

And as these motions are nearly in the ratio of 5 to 2, so as to be almost commensurable, the argument $2n - 5n'$ becomes a very small quantity, namely:

$$2n - 5n' = -1467''.82426. \quad (2)$$

Accordingly, by the double integration, the term in the mean longitude,

$$\delta L = k \iint dR/d\varepsilon \cdot d\varepsilon \cdot dt \quad (3)$$

becomes quite large, owing to the smallness of the divisor $(in - i'n')^2$, thus:

$\delta L = k i P \sin \{(i n - i' n') t + n - n' + Q\} / (i n - i' n')^2$ (4)
(cf. *Laplace*, *Méc. Cél.*, lib. II, chap. VII, § 54, chap. VIII, § 65).

Laplace shows that the great inequality thus arising extends over a period of about 918 years. In the first half of this period, or 459 years, Jupiter is accelerated, (at the maximum about $21'$), while Saturn is retarded, (at the maximum about $49'$); but in the second period of 459 years, the changes in the mean motions are reversed, Jupiter being retarded, and Saturn accelerated, by corresponding amounts.

The conjunctions of Jupiter and Saturn occur at points of their orbits differing in longitude by about 120° , near the points $0^\circ, 240^\circ, 120^\circ$, as shown in the following diagram, which indicates also the shift of the line of conjunctions, in the recurrence near the same point, after about 60-year periods, — the successive subscripts to the letters J and S , showing where the conjunctions recur.

Fig. 1. Diagram of the shifting conjunctions of Jupiter and Saturn, for illustrating *Laplace's* discovery of the cause of the great inequality, 1785.

This figure and the above analysis of the relations of the mean motions will make known to us a series of periodic impulses arising from the mutual actions of our two greatest planets, and profoundly effecting the other bodies of the solar system. In particular, it is easy to see that if there be meteor swarms, with their perihelia near the sun's surface, and their aphelia near Jupiter and Saturn respectively, their paths will be subjected to considerable periodic disturbances at intervals of about 20 years, with secondary maxima in half this period, yet the whole period of meteoric downpour depending on Jupiter's sidereal revolution in 11.86 years, and thus compounding into a shorter cycle of average length 11.18 years, which is the well known sunspot cycle.

Accordingly, we propose to investigate the effect of the mutual actions of Jupiter and Saturn in precipitating meteors upon the sun, in the hope of finding this average observed period, and also of assigning the cause of the variation between the observed lower limit of about 8 and the upper limit of some 14 years.

If the average period of 11.18 years can be found from dynamical theory, and the lower and upper limits likewise assigned, in substantial accordance with the observed minimum and maximum of duration noticed during the past 311 years, — involving some 28 cycles, since *Galileo's* invention of the telescope in 1610, — the presumption will be very strong that we have found the true physical cause underlying these remarkable appearances.

No phenomena in nature have been better established than the periodic fluctuations in the frequency of sunspots, and none have proved more utterly bewildering to natural philosophers, when they have attempted to assign the cause.

For notwithstanding the abundance of observational material, we have remained hopelessly in the dark as to the cause underlying these perplexing phenomena. The sagacious investigator is therefore justified in departing from traditions which lead only to failure. It is not by pursuing these beaten paths that we shall find the hidden laws of nature.

Taking the motion in a Julian year to be as indicated in equation (1) above, we find the difference in the mean motions to be $n - n' = 65260''.41046$, which will amount to a whole circumference in a period of

$$\tau = 1296000'' / 65260''.41046 = 19.8589 \text{ Julian years.} \quad (5)$$

This is the average period between the conjunctions of Jupiter and Saturn; and half of it, or

$$\frac{1}{2}\tau = 9.92945 \text{ Julian years} \quad (6)$$

will give the average period between conjunctions and oppositions with the sun.

Now by the American Ephemeris for 1921, we find that Jupiter and Saturn will be in conjunction in heliocentric longitude $\lambda = 177^\circ 0$, at the epoch t_1 ,

$$t_1 = 1921 \text{ August } 22.5775 \text{ Gr. m. t.} \quad (7)$$

According to the mean period in equation (5) the conjunction of 1901 should have occurred on Oct. 12.5, but from the data of the *Berliner Jahrbuch* for 1901, we find that the two planets actually were in line at the epoch $t = 1901$ Sept. 29.64.

This indicates that a difference of 12 or 13 days may arise from the relative positions and forms of the orbits. If we disregard this slight change, as unimportant for our present purposes, we find that in 1881 the conjunction should have occurred on Dec. 3, and at each 20-year period earlier the date moves forward by 51.537 days. In about 140 years the conjunctions will occur at all seasons of our calendar. Thus the conjunction of 1941 will occur on July 1.0, and in 1961 on May 10.5.

2. Periodic Impulses for the Precipitation of Maximum Multitudes of Meteors upon the Sun under the combined actions of Jupiter and Saturn.

It is well known from the researches on *Jacobi's* integral for the restricted problem of three bodies (cf. my *Researches on the Evolution of the Stellar Systems*, vol. II, 1910, p. 170), that we have an hour-glass shaped space about the sun and a planet such as Jupiter. In his celebrated memoir on Periodic Orbits, *Acta Math.* 1898, Sir *George Darwin* calculated these surfaces for a planetary mass Jove having one-tenth of the solar mass. This planetary mass is relatively about 100 times that of our actual Jupiter, and the closed surface about that planet therefore is about ten times too large in linear dimensions. Yet such enlargement of the scale enables us to see the nature of the closed surface about the planet more distinctly, and as the other relations are unchanged, we shall make use of *Darwin's* figure, as follows.

It is shown that a meteoric body may pass from the region controlled by the sun, through the neck of the hour-glass, to that under the control of the planet, and, under some conditions, collisions occur.

Fig. 3. Section of the limiting *Hill* surfaces, of hour-glass form, connecting the closed spaces about the planets with that about the sun. The scale of the space about Jupiter is magnified 10 times, that about Saturn 18.3 times; and the two sets of surfaces superposed as at the conjunctions of the two planets.

by absorption and of course by far the greater chance is that it will find its Nirvana by absorption in the sun.

The modern researches of *Poincaré*, *Darwin*, *Strömberg* and others have now established this collision theory on the firmest basis; and thus we assume that the reader is familiar with the chief conclusion of modern mathematicians.

Now if so much is established by the researches of geometers, in respect to the precipitation of the Jovian meteor particles into the sun, it follows that similar reasoning will apply to the independent system of particles passing near the sun and Saturn. For just as Jupiter has an immense mass, rapid rotation, and captured satellites, as shown by the retrograde revolutions, of the outer members of the system; so also has Saturn large mass, rapid rotation and also captured satellites — showing that this great outer planet has played almost as great a part in the development of our system as has Jupiter himself. We may therefore assume meteoric swarms to touch all points of Saturn's orbit, and often pass also near the orbit of Jupiter in their circuits about the sun.

The reader should be referred especially to a paper by the author in AN 4307, on the Physical Cause which has produced the small obliquity of Jupiter (cf. Researches . . . , vol. II, 1910, p. 393-397). It is there shown that if Saturn's

mass could be trebled, so as to become equal to Jupiter's, the increase of the Saturnian mass would be accompanied by a destruction of the obliquity, — so that instead of 27° as at present, it would become less than three degrees. This throws important light on Jupiter's small obliquity — it has been destroyed in building up the planet by the capture and absorption of millions of meteors moving near the plane of the planetary orbit.

But of course in building up Jupiter and destroying his original obliquity, the sun has been built up also. In fact the precipitation of meteors upon the larger bodies of the solar system has been going on from the earliest ages. And it is this process of sweeping up cosmical dust which has built up the planets. It is definitely established from valid dynamical laws that they never were parts of the sun, as supposed by *Laplace*, but have been formed in the distance by the accretion of smaller masses, and gradually drawn to the centres about which they now revolve, while the original eccentricities of their orbits have been gradually destroyed by the secular action of the nebular resisting medium formerly pervading our solar system.

The conjunction of Jupiter and Saturn previously described occur at average intervals of 19.8589 years. Midway between these conjunctions there occur oppositions, in average period of 9.92945 years; which is shorter than the average sunspot period.

Thus whilst Jupiter's own period of 11.86 years is somewhat in excess of the observed value, the conjunction or opposition with Saturn is shorter than the sunspot period, which usually is given as 11.18 years.

Is it dynamically possible that the observed period of 11.18 years may depend on the superposition and composition of orbital motions in these periods of 9.92945 and 11.86 years?

If so, the mystery of the sunspot periodicity would be solved. A question of such importance deserves the most careful examination in the light of the modern theories of dynamics. But before forming a judgement on this critical problem, we may advantageously examine certain oscillatory phenomena of the earth's surface, more especially the tides of our oceans, which in practice combine a free and forced oscillation of different periods.

It is pointed out by *Darwin* and other authorities on tides, that the equilibrium figure of our actual ocean is subjected to forces which tend to change its shape in a lunar day, $24^h 51^m$. But the oceans are too shallow to respond to such rapid oscillation, and the tide-wave therefore travels westward more slowly than the forces by which the disturbance is generated. Hence the tide-wave is retarded in its progress around the earth by the friction incident to the small depth of the sea.

Accordingly the disturbance of our actual sea is repeated at the average interval of a lunar day, and the wave thus generated travels around the world in about two days. The tide-wave forcibly generated by the moon's attraction has its period of free oscillation nearly doubled by the shallowness of the sea; where as if the sea were about 13 miles deep it would keep pace with the moon in its westward movement.

It follows that the resultant oscillation of the ocean must be the summation of a series of partial waves, generated at successive intervals of time, yet constantly falling behind the moon, and the integration of the partial waves, which yields the aggregate wave, being the same at each instant, (in an ocean or canal of uniform depth), this aggregate wave must travel westward at the rate of a thousand miles an hour (cf. *Darwin*, Tides, Encyc. Brit., 9th ed., p. 354). It follows therefore that the period of an oscillating system may be altered by composition, as under resonance influence, when there are such periodic impulses at work to change or modify the oscillation time. And in any given case the final result will depend on the composition of the superposed periodic forces to which the whole system is subjected.

3. The Effect of Two Forced Oscillations, in Different Periods, when the Number of Free Bodies is Infinite, is an effective Composition according to the Weights of the two Impulses, or the Spheres of Activity of the two Disturbing Planets.

The principle here stated is analogous to the celebrated theorem adopted by *Laplace* in his theory of the tidal oscillations (*Méc. Cél.*, vol. II), that the state of any system subject to periodic impulses must be periodic like the forces to which it is subjected. Accordingly, we need only apply the principle to the problem now under consideration.

Jupiter is the largest planet of our solar system, with mass $3500/1047.35 = 3.34177$ times that of Saturn; and thus if forced oscillations of meteor swarms arise depending on the actions of these two great planets, it is natural to expect that the impress of Jupiter's action depending on his sidereal revolution will be considerably the more powerful of these periodic phenomena.

Now meteors are moving about the sun in elongated orbits, with aphelia near the orbits of Jupiter and Saturn, and under the mutual actions of these two great planets their paths are so sensibly perturbed that a considerable mass of them are brought into collision with the sun. If the meteors traveled in a straight line ellipse about the sun's centre, the period for the Saturnian meteors would be 10.416 years, and for the Jovian meteors 4.1958 years. In practice, however, both of these periods would be somewhat lengthened, the exact amount depending on the distance of the perihelion, and whether the aphelion is beyond or within the orbit of the great planet in question. All these various cases will occur in nature.

We have next to consider the relative extent of the spheres of influence carried by the two planets Jupiter and Saturn in their motions about the sun. The gravitational actions are given by the expressions:

$$\begin{aligned} f &= k^2 m / r^2 &&= \text{Jupiter's action} \\ f' &= k^2 m' / r'^2 &&= \text{Saturn's action} \end{aligned} \quad (8)$$

where k^2 is the Gaussian constant.

To compare these actions at all distances it suffices to note that Jupiter's mass exceeds that of Saturn in the ratio 3.34177 to 1. Hence the actions will be equal at distances in the ratio of the square roots of these numbers, or 1.82805 to 1; for obviously whatever be r and r' , we may always write $r = 1.82805 r'$, and have the valid equation:

$$3.34177 / (1.82805 r)^2 = 1 / r'^2 \quad (9)$$

Accordingly, in order to make the Jovian action always equal to that of Saturn, we have to take Jupiter's action at a distance $r = 1.82805 r'$. These spheres of influence as drawn to scale are as indicated in the following figure 4; but the volumes are as the cubes, $(1.828)^3$ to 1, so that Jupiter's sphere exceeds Saturn's 6.108 to 1.

Fig. 4. Illustration of the radii of the relative spheres of influence of Jupiter and Saturn, which give the relative weight of these planets in fixing the resulting periodicity of the meteoric downpour.

It thus appears that the volume of Jupiter's sphere of activity is 6.108 times that of Saturn's. But in order to get the dynamical effect on meteors traversing these spheres of influence, we have to consider the velocities of the meteors when near Jupiter and Saturn respectively. This will enable us to estimate the relative lengths of time in which they are subjected to these disturbing influences, and the precipitative effects in throwing them upon the sun.

Now at Jupiter and Saturn respectively the sun's gravity is in the ratio of $(9.54)^2 : (5.20)^2 = 3.36$. And hence the moving meteors, under this stronger central action at Jupiter's orbit, will be in his larger sphere of influence, or under greater force, a longer duration of time, proportional to $6.108 / 3.36 = 1.82$ nearly.

By comparing the parabolic velocities at the orbits of Jupiter and Saturn respectively (cf. AN 3992, p. 136), we find them to be 8.0941 km and 4.4148 km respectively. This yields the ratio of 1.83. Accordingly, by the above reasoning we find that, Jupiter's relative perturbative efficiency over Saturn's, in their mutual actions on meteors, whether moving with moderate or very great (parabolic) velocity, and thus not subject to appreciable change, is in the ratio of 1.828 to 1.

Dynamically this means that whatever distance be taken as the unit of distance, and of action, Jupiter's sphere of influence and his efficiency, always is 1.82805 times greater than Saturn's. In other words, if there be two periods based on meteor swarm precipitation, — namely 11.86172 years, which is Jupiter's sidereal period, and 9.92945 years, which

is the average period in which Saturn passes Jupiter's radius vector in the reverse direction at conjunction or at opposition — we must get their mean period by combining them with different weights as follows:

$$\begin{array}{rcl} 11.86172 & \text{wt. } 1.82805 & \\ 9.92945 & 1 & \\ \hline \end{array}$$

Mean period $T_c = 11.17846$ years. (10)

Let us now examine for a moment the observational period of the sunspot cycle as found by the best authorities. These periods are as follows:

1. *R. Wolf*, 1852-1877, by the various observations since 1610, AN 35.369, and Mem. R. A. S., vol. 43, 1877, p. 202. $T = 11.111 \pm 2.030 \pm 0.307$.

In this result 2.030 years is a periodic oscillation, and 0.307 the uncertainty in the determination of the period.

2. *Faye*, 1878, from a careful discussion of *Schwabe's* observations under *Faye's* criterion, CR 86.911, July 30, 1877. $T = 11.120$.

3. *Spoerer*, 1881, by a discussion of the accumulated observations since 1732, AN 97.102. $T = 11.131$.

4. *Newcomb*, 1900, from a discussion of the accumulated data since the time of *Galileo*, ApJ 13, Jan. 1901. $T = 11.113$.

Each of these determinations has high and special merit, and if we take the simple mean, which gives them equal weight, we get

$$T_0 = 11.1187 \pm 0.062; \quad (11)$$

and hence the difference from the above calculated period is only

$$T_0 - T_c = 0.009 \quad (12)$$

or about 3.3 days. This difference is so extraordinarily small as to be remarkable — only one seventh of the probable error in the above value of T_0 .

In his memoir of 1877, *Wolf* remarks that *Schwabe* himself, the discoverer of the sunspot cycle, had later adopted the period of 11.11 years (AN 1521). And the same is true of later investigators, — they all agree that the period is slightly over 11 years.

Accordingly, it appears that we can only view the above results as establishing the true cause of the sunspot cycle; and naturally the theoretical mean period is preferable to that derived from observations. Thus the true mean period of the sunspot cycle is 11.178 years.

The accompanying diagrams, plate 3, fig. a, of the sunspot cycles observed since 1831 are from *Wolf's* memoir of 1875 in the memoirs of the Royal Astronomical Society for 1877. It will be noticed that the curves are not of equal heights, nor the periods of equal lengths; but that the different cycles show notable variations both in height of curve and period of duration.

This problem will be examined in more detail hereafter, and we shall endeavor to assign the cause for the oscillation of the period in length, and for the variation of the amplitude or vertical height. At present we merely point out two salient features:

1. An indication of double periodicity, in about 89 years, 4 high maxima followed by 4 low maxima, as from 1834 to 1923, or from 1745 to 1834.

2. The persistence of the tendency to a secondary maximum, which is especially well shown in *Wolf's* curve from 1831 to 1875.

Explanation of the Sunspot Curves here adopted.

1. It is evident that the modern observations of the sun kept up during the last half century at Greenwich, Potsdam, Zürich, and other observatories, on a uniform basis, and thus giving comparable data decade after decade, are vastly preferable to the older observations. Yet we find fairly satisfactory data back to the beginning of *Schwabe's* records in 1826; and we may even go back half a century further, to 1776, without passing out of the era of modern observers.

2. It is stated in Miss *Clerke's* History of Astronomy during the 19th Century, 4th ed., 1902, p. 53, that "for 164 years, then, after *Galileo* first levelled his telescope at the setting sun, next to nothing was learned as its nature"; and that beyond the time of rotation of the sun on its axis, which was immediately deduced by *Galileo* and *Fabritius*, there was no development or increase of precision for five generations of astronomers.

3. The turning point for renewed solar observations seems to have been reached when Prof. *Alexander Wilson* of the University of Glasgow, in November, 1769, noticed a great sunspot and followed it so systematically as to deduce the celebrated Wilsonian theory that the solar spots are depressions in the photosphere. In 1774 *Wilson* proved his theory by geometrical evidence then considered satisfactory, (Phil. Trans. 1774, part 1, p. 7-11). In the same epoch other notable solar observers appeared:

(1) *J. E. Bode*, Gedanken über die Natur der Sonne und die Entstehung ihrer Flecken, Berlin, 1776.

(2) *J. H. Schroeter*, Beobachtungen über die Sonnenfackeln und Sonnenflecken, Erfurt, 1789.

(3) Sir *Wm. Herschel*, On the Nature and Constitution of the Sun and Fixed Stars, (Phil. Trans. 1795, p. 46); and Observations tending to investigate the Nature of the Sun, in Order to find the Causes of its Variable Emission of Light and Heat, (Phil. Trans. 1801, p. 265-334):

(4) *Lalande* and his associates at Paris also were active in all lines of observations, and from the Mém. de l'Acad. d. Sc., 1776, (pub. 1779), we find that the solar spots had enough attention for *Lalande* to develop the eruptive theory first outlined by *Derham* in 1711.

4. If we contrast this considerable development of systematic solar observations during the last third of the 18th century with the almost total lack of good observations in the century and a half preceding 1770, we shall find reason to reject the assumption sometimes made that the sunspot curves of frequency prior to 1770 can be depended upon. This no doubt is the tacit reason why *Rudolf Wolf* and other investigators of sunspot curves often terminate these curves at 1770. In his memoir of 1875, *Wolf* indeed extends his curves back to 1745; and in a recent extensive examination of this subject by Prof. *A. Wolfer* of the Zürich Observatory the curves are carried back to 1749. (cf. Die Häufigkeit der Sonnenflecken in den Jahren 1749-1901, *Wolfer*, Astron. Mitteilungen no. 93, and Monthly Weather

Review, Washington, April 1902; and a later summary to 1920, Monthly Weather Review, Aug. 1920).

5. In the paper of August, 1920, *Wolfer* finds from his profound studies on the 11-year periodicity of sunspots, 1610 to 1920, that a revision of the mean period now changes the value from 11.12 to 11.2 years. *Wolfer's* new value thus agrees perfectly with my theoretical value 11.178, found above from the motion of Jupiter and Saturn, and the observed value resulting from the work of *Wolf*, *Spoerer*, *Faye* and *Newcomb*, 11.187, which I had deduced before I received the papers so kindly furnished by Prof. *Wolfer*.

6. We now turn to *Wolf's* sunspot frequency curves, 1745-1798, and notice that in his latest work, *Wolfer* has reduced *Wolf's* maximum of 1788 to about 86 percent of the height given it by *Wolf* in his memoir of 1875. After some study of this question I am led to think this curve ought to come down to 0.7 of the height given it by *Wolf*. This would make the maximum of 1788 a low maximum. On the other hand, I would raise the *Wolf* maxima of 1760 by about 40 percent, and that of 1750 by about 50 percent, as shown by the pointed lines in the diagrams of *Wolf's* curves revised, Plate 3 (b).

7. My reasons for going further than Prof. *Wolfer* does in this concluded revision of *Wolf's* curves are the following:

(a) The observers from 1745 to 1770 were few in number and desultory in attention to the observation of the spots which actually existed on the sun. The chances are therefore that a great percentage of the spots escaped notice.

(b) On the other hand in the period of the French Revolution, 1784 to 1798, — when all Europe and especially France was under an exaltation of intellectual activity, — it is probable that many relatively minute sunspots were noticed, and the number thus unduly increased. Hence to make the record from 1745 to 1798 of any scientific value we must depress the maximum of 1788 and elevate the maxima of 1750 and 1760, as here indicated.

(c) This revision of *Wolf's* curves would make the 4 maxima 1745-1785 all high, and of comparatively short period, 10 years each, which accords with the tendency noticed in the later and more reliable records of our own time.

(d) After careful reflection I am of the opinion that no known fact stands in the way of this revision, while on the other hand it brings the 18th century sunspot record into harmony with that since 1834, which is the best and most accurate of all. Incidentally it confirms the great sunspot saros in 88.9 years, which is founded upon the numerical relations of cycles recognized to exist, the writer's theoretical period of 11.178 years being independently found by Prof. *Wolfer* as 11.2 years in his latest work.

8. In conclusion, the author acknowledges his great indebtedness to Prof. *A. Wolfer*, for invaluable data, including his very latest results; and to Sir *Frank W. Dyson*, Astronomer Royal, who kindly furnished the latest Greenwich sunspot and magnetic records. It is to the continuous observations maintained at Greenwich and the systematic classification and reduction kept up at Zürich that we mainly owe our present knowledge of sunspots and their 11-year periodicity. Such records will always be of the highest usefulness to the investigator.

4. The Calculation from the Motion of Jupiter of the Theoretical Sunspot Saros in 88.9 Years confirmed by the Observed Periodicity in about 90 years.

From a study of the sunspot cycle, in relation to Jupiter's orbital motion, made on Aug. 12, 1921, I was able to derive the Great Theoretical Saros, or Restitution Period, in which the sunspots should recur, and to confirm the theory by a careful comparison with *Wolf's* researches on the observed periodicity since 1745. This theoretical periodicity was found to correspond to 88.9 years, and was derived by the following process of calculation.

1. We have seen that the theoretical period of the sunspot cycle is 11.178 years, while the mean observed period found by *Wolf*, *Faye*, *Spoerer* and *Newcomb* was 11.187 years, which differs only 0.009 from the above theoretical period. The difference between the theoretical and observed period is insensible, so that we naturally prefer the theoretical period of 11.178 years.

2. Now Jupiter's sidereal revolution occurs in 11.86172 years; and thus the sunspot cycle is 0.68372 year shorter than the planet's revolution. The sunspot cycle is therefore displaced 22°0201 in respect to Jupiter's orbit, in a sidereal revolution of the planet, as shown by the following equation:

$$n:n_1 = 1/11.86172 : 1/11.178 = 360^\circ : x \quad (13)$$

$$x = 382^\circ 0201$$

$$x - 360^\circ = 22^\circ 0201; \quad (14)$$

or dividing 22°0201 by 11.86172, we have 1°8564 per annum.

3. But each sidereal revolution of Jupiter finds the conjunction-opposition line displaced according to the proportion:

$$n:n_2 = 1/11.86172 : 1/9.92945 = 360^\circ : x \quad (15)$$

$$x = 430^\circ 0559$$

$$x - 360^\circ = 70^\circ 0559; \quad (16)$$

whence, dividing 70°0559 by 11.86172, we have for the motion per annum 5°90605.

4. And now the difference in these motions, 5°90605 — 1°8564 = 4°04965, is the amount by which the conjunction-opposition line gains upon the sunspot cycle in a year. The periodicity resulting from this displacement is therefore

$$\tau = 360^\circ / 4^\circ 04965 = 88.89657 \text{ years} \quad (17)$$

which is equal to 7.952816 sunspot cycles.

Accordingly, in slightly less than 8 sunspot cycles, the observed cycles of the phenomena should be repeated as a kind of sunspot saros. If the sunspot phenomena were made up of 4 high maxima, and 4 low maxima, which is very nearly the case, the whole of the phenomena should become periodic after about 88.9 years. With the cycle fixed at 11.178, we find the eight periods to amount to 89.424 years, which corresponds very closely to the above saros period.

5. Now let us examine *Wolf's* curves of the sunspot cycles from 1745 to 1921, — a period of 176 years, very nearly two of the above theoretical cycles, $2 \times 88.9 = 177.8$ years, — for which the observations are most trustworthy. It is to be understood that since *Wolf's* death in 1893, his curves have been continued by his successors, so that the

data of the last 28 years are dependable. The periods of the frequency curves of the accompanying chart since 1745 probably are free from serious defect of any kind.

6. First, we notice that in the whole interval of 176 years, since 1745, there appears to be two great divisions in the spot phenomena:

(a) A marked period of three low maxima, 1798–1834 = 36 years; and a similar marked period of four low maxima, 1877–1921 = 44 years.

(b) Each of these groups of low maxima were preceded by series of high maxima. That of 1745–1798 = 53 years, apparently contained five maxima, with average period of 10.6 years; and that from 1834 to 1877 = 43 years contained four well defined high maxima of average period 10.75 years each. As the old records are less satisfactory than modern ones, we cannot be too certain of minor details; for example one high maximum, most likely that of 1785–1798, might really be a low maximum, which would balance the distribution more perfectly.

7. As the beginning and end of these periods is somewhat indistinct, we must not expect too perfect an agreement in the minor details; yet in a general way there is very perfect agreement in the observations showing two groups of high series, and two groups of low series of the sunspot cycles. These are the larger and more outstanding features of the observed phenomena; and thus the records since 1745 certainly point to two great periods of solar activity, of about 89 years each. Can such a record be the result of chance, or does it depend on the theoretical Jovian cycle above discussed?

8. To judge intelligently of this problem, we notice that each of the lows in the groups of the cycles comes along in proper succession: that is, they are all together, each of the lows of the low groups being placed with the other low groups; and each of the highs of the high groups being placed with the other high groups of its series. Thus there is no mixing of the highs and lows, but each class of these phenomena is well separated, and stands out conspicuously by itself.

9. Now if this arrangement or order of development depended on chance, the probability of this orderly recurrence would be excessively small. As 16 spot cycles are involved, we should have to consider the probability of a chance event depending on either of two possibilities, each equally probably. Hence we should have:

$$P = (1/2)^{16} = 1:65536. \quad (18)$$

The improbability of this orderly arrangement being due purely to chance is so obvious that further argument probably is not required in this connection, — though additional considerations may be adduced as follows, which render the above divisor very much larger yet.

10. We may consider the chance of each low being in elevation of the same order as the other lows. This would be $P' = (1/2)^4 = 1/64$; and the chance of each high being of the same order as the other highs, which is $P'' = (1/2)^4 = 1/64$. The chance of the compound probability is therefore $P_c = PP'P'' = 1/65536 \cdot 1/64 \cdot 1/64 = 1/268435456$ (19)

11. Moreover, we should consider the chance of the lows occurring when the moveable epoch of the sunspot saros is on the side of Jupiter's orbit corresponding to minimum meteorites; and the highs occurring when this epoch of the sunspot saros is on the side of Jupiter's orbit corresponding to maximum comet orbits, or maximum meteors. This again leads to a compound probability almost infinitely small:

$$P_c = PP'P''P''' = 1/65536 \cdot 1/268435456 = 1/17592186044416 \quad (20)$$

12. It is needless to extend these calculations any further, since a compound probability of 17 trillions to 1, that the arrangement is not due to chance may be regarded as a certainty.

Accordingly, we conclude that the great saros or theoretical periodicity in 88.9 years — found from the relations of the cycles 11.178 and 9.92945 years to Jupiter's sidereal revolution in 11.86172 years — is strikingly confirmed by the sunspot records of the past 176 years. The observations point to the theory in such a way that not the least doubt can remain as to reality of the connection.

It is true, therefore, that there is a great periodicity or saros in the spot cycles, extending over about 89 years, yet heretofore not recognized, because we did not see the connection with Jupiter's motion, and the process by which the meteors were precipitated upon the sun, and the larger features of the spot phenomena recurring after the lapse of so great a period.

Accordingly, it only remains to write the formula for the length of the sunspot cycle:

$$II_i = II_0 - AII_0 (\sigma_0/\sigma_\mu) \sin[\beta - \alpha(t - t_0)] \quad (21)$$

$$II_i = 11.178 - 2.05(\sigma_0/\sigma_\mu) \sin[177^\circ - 4^\circ 04965(t - t_0)], \quad (22)$$

$$t_0 = 1921.64224$$

Here the variable term 2.05 years is the »oscillation« found by *Wolf* (Mem. R. A. S. 1877, p. 202) from the observations between 1610 and 1870, and σ_0 is the mean density and σ_μ represents the average variable density of the meteors in the different parts of Jupiter's orbit successively described in the motion of that great planet about the sun.

It will be found that this formula is capable of representing the lengths of the sunspot cycles with remarkable accuracy, and that the highs and lows are indicated by the factor (σ_0/σ_μ) , in the variable term having the period of 88.9 years, with four highs and about 4 lows, as above explained. The amplitudes A_i of the successive cycles depend on the density of the meteors, and thus on the reciprocal expression:

$$A_i = \sigma_\mu/\sigma_0 \quad (23)$$

For it is an observed fact that the highs are somewhat shorter in period than the lows, the former having length say 10.7, and the latter extending to about 12.0 years. The highs therefore are deficient in length by 0.50, and the lows exceed the average length by about 0.83.

It thus appears that to some extent there is a process of partial compensation in the sunspot phenomena. When the amplitudes are greater the periods are somewhat shorter than the average; and when the amplitudes are smaller the

periods are considerably longer than the average period. Perfect compensation probably is not to be expected here, yet this tendency towards partial compensation is clearly indicated by the observed phenomena and may not be rejected without doing violence to the observations. Such processes of partial compensations are frequently met with, and may be said to be a general tendency in nature.

Finally it is to be noticed that even if perfect compensation on the two sides of Jupiter's orbit be attainable, and a good agreement in the individual cycles could be predicted from our formula, under legitimate variation of the arbitrary constants, yet there is little assurance or probability of this concordance being permanent, owing to the changing perturbative influences affecting the meteors, and the unknown distribution or density of these swarms in the different parts of their orbits. Accordingly, if there is a general accordance between theory and observation it is all we can expect; and that we now have certainly attained, in much greater perfection than any one heretofore has dared to hope for.

5. A Method for making the Spot Frequency Curve an Experimentum Crucis for the Discovery of the Cause of Sunspots and of Cepheid and Cluster Variable Stars.

1. If we have a series of recurring phenomena, but do not know the underlying cause to which they are due, yet can trace the curve of these phenomena in an accurate and dependable way, we may proceed to infer the hidden cause involved from other periodic phenomena showing similar frequency curves, if it is allowable to suppose that the two causes may be of like nature, as indicated by the frequency curves of the phenomena.

2. Now in the case of the sunspots, the observations during the 311 years since *Galileo's* invention of the telescope and discovery of these spots, do indeed give us reliable frequency curves; yet the underlying cause is left utterly obscure, because no one appears to have applied successfully the above principle. Perhaps the difficulty was increased, because we did not anticipate that it would be possible to find curves in other periodic phenomena so accurately similar to the sunspot curve as to constitute an experimentum crucis; yet on Aug. 19, 1921, such an identity of curves was discovered by the writer and shown to be so rigorously applicable as a crucial test, that the chances are infinity to one that it reveals the true cause of sunspots. Heretofore the nature of these phenomena has been so completely hidden from our view that the cause involved has proved utterly bewildering to natural philosophers.

3. The accompanying curves (Plates 4 and 5) for the form of the tide wave in a river or shallow canal of uniform width and depth are from *Airy's* celebrated article on Tides and Waves, *Encyc. Metr.*, 1845. To explain them it suffices to say that the upper curve Nr. 9, shows the theoretical form of the surface of the water in a shallow river, as the tide wave advances from the sea, to the 1st station; then up the river, to the 2nd, 3rd and 4th stations. It will be noticed that the wave front becomes steeper, owing to resistance; thus the form of the wave is slowly modified, and it finally breaks up by the development of a secondary wave in the rear of the chief wave. The formula¹⁾ for the elevation of the water is:

$$H = -bk \sin(mvt - mx') + \frac{3}{4} b^2 kmx' \sin(2mvt - 2mx'). \quad (24)$$

The first term of this expression is a sine curve and holds for the elevation when the displacement of the particle

¹⁾ In the propagation of a simple unresisted wave along a canal of uniform width and depth, we have for the uniform motion of the disturbance, under the accelerating forces X and Y :

$$X = \Phi(x, t) = \chi(vt - x) = A(e^{my} + e^{-my}) \cos(nt - mx - B) \quad (a)$$

$$Y = - \int_{y=0}^{y=y} dX/dx \cdot dx = -A(e^{my} - e^{-my}) \sin(nt - mx - B) \quad (b)$$

$$\text{which satisfy the differential equations: } dX/dx + dY/dy = 0 \quad d^2X/dx^2 + d^2Y/dy^2 = 0 \quad X = \Phi(y + x\sqrt{-1}) + \Psi(y - x\sqrt{-1}) \quad (c)$$

But under resistance the equations of wave motion in a canal are more complicated, just as the theory of disturbed planetary motion is more complex than the simple theory of undisturbed motion. In general our equations have the form:

$$d^2X/dt^2 = F + d/dx \left[-gK - \int_{y=\eta}^{y=k} d^2Y/dt^2 \cdot dt \right], \text{ equation of equal fluid pressure} \quad (d)$$

$$Y = \Xi d\eta/dx - \int_{y=\eta}^{y=y} dX/dx \cdot dx, \text{ equation of continuity} \quad (e)$$

where Ξ = the value of X at the bottom of the canal, and $d\eta/dx$ = tangent of the inclination of this sloping bottom to the horizon. These two equations contain the whole theory of the motion of fluids in canals of uniform width, but of uniform or variable depth, when the motion is taken to be entirely longitudinal and vertical.

If in this oscillatory motion of the fluid about a mean position, which is called a wave, we put:

$$X = a \cos(mvt - mx) \quad (f)$$

we get

$$dX/dx = -ma \sin(mvt - mx) \quad (g)$$

and

$$d^2X/dx^2 = -m^2 a \cos(mvt - mx).$$

These expressions satisfy the differential equation of wave-motion,

$$d^2X/dt^2 - v^2 d^2X/dx^2 = 0 \quad (h)$$

and, by double integration, accurate to a second approximation, lead to the solution:

$$X = \Phi(vt - x) + \Psi(vt + x) - \frac{3}{16} a^2 m^2 (vt + x) \cos(2mvt - 2mx) \quad (i)$$

which is easily transformed to give the elevation of the water above mean level, as cited above (cf. *Airy*, Tides and Waves, p. 286-300).

is small in comparison with the depth of the water; but the second term is of a different type, since the multiplier α' is outside of the periodic function, as in the integrations for the secular variations of the elements of the planetary orbits (cf. *Méc. Cél.*, Liv. II, Chap. V, §§ 40-42). This latter expression for the elevation of the water may be conceived to be a secondary wave, infinitely small at first, but whose magnitude increases continually as α' increases, or the disturbance travels along the canal, with the increase of the time t .

4. When the tide wave leaves the sea, at the first station, the second term is insensible, and the motion harmonic, in the curve of sines; but as the disturbance travels up the canal, the secondary wave grows, as shown in *Airy's* curves, which of course exaggerate the effects, to render them visible to the eye of the reader. In our actual seas the vertical motion is of course very small compared to the horizontal motion, and this exaggeration of the vertical component is necessary in the diagram.

Now if at any station along the river we record the passing elevation of water, as by a tide-gauge, we get the next lower series of figures, nos. 10, 11, 12, 13. We can easily see that this is a reversal of the above form of the surface; for a tide-gauge is made by fixing a marking pencil on a sheet of paper, moving uniformly in the contrary direction to the advance of the wave up the river, as shown by the arrow on the right, upper part of Plate 4. Thus as the pencil is held fast, and the paper on a cylinder revolves beneath it, to the left, the tidal curve due to the change in the level of the water, with the periods shown, is recorded, the record being the exact reverse of the above form of the tide-wave.

5. It is the tidal curve of resisted wave motion which especially interests us in the theory of the sunspots; for it gives an exact and very wonderful reproduction of the curve of frequency, or form of the sunspot cycle, as found by *Schwabe*, *R. Wolf*, *Faye*, *Spoerer*, *Newcomb* and other investigators. Let the following typical sketches serve to remind us of the steep slope on the left, the high maximum, with more gradual slope and distinct indication of secondary maximum on the right.

Fig. 5. Typical forms of the curve of frequency of sunspots.

These curves are typical of the records found from the laborious researches of *Wolf*, which covered the whole period from 1610 to 1875, and were extended till *Wolf's* death in 1893, since which time *Wolf* has kept up these solar researches with great and commendable continuity. Thus our curves run from 1610 to 1921, but different portions of the record have been made by different investigators.

6. From the above simple argument it follows that the sunspot cycle obeys the mathematical law of a wave movement advancing against resistance, as shown by the actual records of oceanic tides in shallow rivers. These

forms of tides and the reversed tidal curves have been treated by *Airy* in the ablest manner, and illustrated by many figures which now become of extreme interest in connection with the sunspot cycle. As the forms of the sunspot frequency curve and the tidal curve are absolutely identical in every detail, we are forced to admit that the sunspot frequency depends on a movement in the sun analogous to a tide, but resisted by other parts of the sun's body in which such motion does not exist.

7. The next question naturally is: What kind of solar tide can this be? Obviously it is nothing but the periodic forward rush of the equatorial acceleration, produced by the sudden downpour of meteors against the equatorial regions of the photosphere, under the combined precipitative action of Jupiter and Saturn, yet not occurring in the polar regions, and therefore this equatorial tide in the solar photosphere is resisted by the slower rotation in higher latitudes — just as the tidal waves entering the river Severn are resisted by its banks, as so well and fully illustrated by *Airy* in his great work on tides and waves. Accordingly this resistance of the equatorial rush or photospheric tide at the solar equator is the true cause of the form of the curve of the sunspot frequency. On this point there is not the slightest doubt to those who know the geometrical properties of curves shown to be of identical type.

8. It would be possible to build up an argument from the theory of probability, showing that the chances are infinity to one that this is the correct and only admissible interpretation of the identity of form between the tidal curve of resisted wave motion and the curve of spot frequency. The incontestable nature of such an argument is obvious. By no possibility could the two curves have identically the same form throughout their courses — involving the analogous ordering of an infinite number of points or neighboring elements of curvature — without both curves representing resisted tidal wave motions.

9. It only remains to add that when *Schwabe* began to record sunspots in 1826, he acted unconsciously as a tide-gauge. The spots are the solar analogue of the height of the water in the tidal river; and *Schwabe* by noting the number of spots with the uniform flight of time made a useful gauge of the disturbances in the equatorial regions of the sun. Thus *Schwabe's* method was very simple, yet extremely effective, since it gave an accurate register of the ebb and flow of spots, and therefore of the resistance to the current movements at the sun's equator due to the downpour of meteors under the combined actions of Jupiter and Saturn.

10. As soon as I had perceived the cause thus underlying the curve of sunspot frequency, Aug. 19, 1921, I had the good fortune to be able to submit the argument to the critical judgement of commander *L. M. Cox*, U. S. N., the eminent civil engineer at Mare Island, who concurred at once in the view that the proof is overwhelming. It was not without the concurrence of several other trusted and sagacious thinkers — Capt. *E. L. Beach*, U. S. N., Commandant, Mr. *A. E. Axland*, Mr. *L. Tiernan*, Mr. *W. S. Trankle*, and especially Mrs. *See* — that I was induced to announce

the discovery by the cablegrams of Aug. 20-22, to the *Astronomische Nachrichten*, and the *Astronomische Gesellschaft* at the Potsdam meeting.

For upon reflection it was evident that whatever cause regulates the sunspot curve of frequency — the whole flow of spots — must necessarily regulate the origin of the individual spots. Therefore there could be no doubt whatever of the discovery of the cause of the sunspots. They could be due to nothing but the cause here assigned, and all other causes are wholly and forever excluded from consideration.

11. We have therefore no hesitation in introducing into astronomy the tidal curve of resisted wave motion as of great importance in the science of the heavens. The doubly periodic function

$$H = -bk \sin(mvt - mx') + \frac{3}{4}b^2kmx' \sin(2mvt - 2mx') \quad (25)$$

as here interpreted, gives us the true clue to the long standing mystery of the sunspot frequency curve.

And not only will this development enable us to understand great masses of phenomena in the solar system, but also others equally or more bewildering in the sidereal universe. It is well known that the two great classes of variable stars heretofore utterly bewildering to investigators are the Cepheids including Geminids and the Cluster variables, the latter numbering thousands, and discovered chiefly by Professor *Solon I. Bailey*.

As Dr. *Harlow Shapley* has used the Cepheid and Cluster variables to find an indirect correlation method for measuring the distance of the globular clusters, — ranging in distance from 7000 to 240000 light-years — it is evident that we must explain the forms of the Cepheid light curves, and also the light curves of the cluster variables, in order to be sure that they come under the causes assigned by the theory of resisted wave motion.

12. The accompanying figures, Plates 6, 7, will make this theory sufficiently obvious. The resemblance of the light curves of the Cepheids to the sunspot frequency curve has been emphasized by several writers, but especially by Miss *Clerke's* *The System of the Stars*, 1905, from whose excellent work the typical illustrations here used are taken.

The propriety of applying the theory of the tidal curves of resisted wave motion to these variables is at once obvious, if we turn to *Airy's* figures Nr. 53 and 59, Plate 5.

(a) These tidal curves of the resisted waves at Newnham and Weymouth respectively are so exactly similar to the light curves of certain typical Cluster variables found by *Bailey*, that one could not wish for a more perfect geometrical correspondence. The rapid rise to high maximum, more gradual decline, and long dead level track constituting the minimum, are very notable features, and will strongly commend the theory of a resisted tide-wave to the investigator of variable star phenomena throughout the sidereal universe.

(b) It might be possible to ascribe the light curves of cluster variables of this type largely to mere surface conflagration incident to the downpour of meteors, since the rapid rise in brilliancy and slower decline, under cooling, could thus be accounted for. The objection to the purely

conflagration theory is that it would separate the cluster variables from other variables, with secondary maxima. Nature will not allow such discontinuity.

(c) Thus whilst the conflagration of meteors occurs, there are also currents and inequalities of level set up in the photospheres of the stars; and the resulting thermal and gravitational oscillations are closely associated and obey the same wave laws, — though usually in slightly different periods, as we see by the results cited below from Lord *Kelvin*, eq. (32), *Ritter*, eq. (49) and (50), and *Moulton*, *ApJ* 29 — and therefore the changes, with the whole of the phenomena of light variation, obey the curves of resisted tide waves. This is the substance of the new theory, and it is applicable alike to the sunspots and to the periodic fluctuations of starlight shown by the light curves of Cepheid, Geminid and Cluster variables.

(d) It only remains to point out that in the case of the Cluster variables the secondary maximum on the descending slope of the light curve, is either missing or nearly insensible. It is important to know how to interpret this deficiency, and a clear light is shed on the mystery by extending the researches on the theoretical form of the tide-wave in a shallow river, to second approximation given in *Airy's* figure Nr. 9. If this curve be continued to the 6th or 8th station, it is evident that the secondary wave will break away more distinctly from the chief wave — the secondary wave becoming steeper in front, and allowing the chief wave to be followed by a long dead level track, just such as we find in the light curves of Cluster variables.

(e) Accordingly we conclude that if enough time were to elapse before the next meteoric downpour, under the orbital revolution of the satellite, the secondary wave would duly appear. But the forced oscillation under orbital motion in say half a day gives a pulsation so rapid that the thermal and gravitational disturbance, due to meteoric downpour at their equators, has no time to show an effect of sensible resistance by disrupting the pulsation into two oscillations. There is not time for sensible resistance effects to follow as a separate wave; hence the secondary maximum in the light curves are largely or entirely lacking, because too soon overtaken and swallowed up in the succeeding forced oscillation depending on orbital motion, which is of much greater magnitude. The fact that the resisted wave theory overcomes this great mystery in Cluster variables shows how secure is the physical foundation underlying the theory. Thus all the chief phenomena of Cepheid and Cluster variables are accounted for in a manner much simpler than any investigator heretofore has dared to anticipate.

6. Brief Notice of the Cepheids, Geminids and Cluster Variables: Explanation of why the Sunspot Frequency and Stellar Light Curves follow the Law of the Tidal Curve of a Resisted Wave.

1. The following table from Miss *Clerke's* *Problems of Astrophysics*, London, 1903, p. 324, will convey to the reader a very good idea of the periods, and extent of the light changes in a dozen typical Cepheid variables.

Name	Limits of change	Period
R Muscae	6.6 to 7.4 mag.	0 ^d 21 ^h 10 ^m
R Triang. Austr.	6.6 » 8.0 »	3 9 20
ST Cygni	6.6 » 7.4 »	3 20 10
T Vulpeculae	5.5 » 6.5 »	4 10 28
Y Sagittarii	5.8 » 6.6 »	5 18 33
U Sagittarii	7.0 » 8.3 »	6 17 46
X Sagittarii	4.0 » 6.0 »	7 2 50
W Sagittarii	4.8 » 5.8 »	7 14 16
S Sagittae	5.6 » 6.4 »	8 9 7
X Cygni	6.4 » 7.7 »	16 9 15
W Virginis	8.7 » 10.4 »	17 6 30
T Monocerotis	5.8 » 8.2 »	27 0 18

The system of δ Cephei, from which this class of variables is named, fluctuates in brightness between 3.7 and 4.9 magnitude, in a period of 5^d 8^h 47^m 39^s, of which 1^d 14^h 36^m yield the increase of light, while the remaining interval, 3^d 18^h 11^m, is devoted to the decline in brightness, with secondary maximum at 3 days from the minimum. The secondary maximum is therefore slightly less than 0.6 of the whole oscillation from the minimum, and the whole series of changes is well shown by the accompanying light curve (s. Plate 6a).

2. The accompanying table of eight Geminid variables, also from Miss *Clerke's* Problems of Astrophysics, p. 331, will sufficiently illustrate the light changes in these systems.

Name	Limits of change	Period
R ² Centauri	7.4 to 7.8 mag.	0 ^d 7 ^h 16 ^m
S Antliae	6.7 » 7.3 »	0 7 47
U Pegasi	9.3 » 9.9 »	0 9 0
24 ω Centauri	13.4 » 14.0 »	0 11 5
V Puppis	4.1 » 4.9 »	0 17 27
U Vulpeculae	6.9 » 7.6 »	8 0 4
δ Serpentis	5.0 » 5.7 »	8 17 17
β Lyrae	3.4 » 4.5 »	12 21 47

This class of variables is named after ζ Geminorum, discovered by *Schmidt* in 1847 (AN 624). It was soon afterwards found by *Belopolsky* and *Campbell* that the light variations are associated with orbital motion in 10^d 4^h, as if the system were so tilted that its orbital plane lie nearly but not quite in our visual ray. One component is invisible, yet the observed minima are not due to its intervention, — since their epochs do not correspond to spectroscopically determined conjunctions.

In the case of the Geminids the periods are fairly short and the light curves rise and fall almost as symmetrically as a sine curve: thus the resisted wave effect does not develop into any prominence, since the tendency to secondary maxima are nearly insensible. In the case of U Vulpeculae and V Puppis, however, the tendency to secondary maxima is more pronounced, as shown by the »humped« decline (s. Plate 6b).

3. While making photographic comparisons of certain Globular clusters in 1895, *Bailey* discovered the most numerous of all the types of variable stars — the so-called Cluster variables. Over 500 have been found in certain clusters, and more than a thousand in the Magellanic clouds (Harv. Circ. 82, 96, 100).

Their periods generally are remarkably short — decidedly less than one day. The following brief table will give an idea of the nature of these cluster variables (*Clerke's* System of the Stars, 1905, p. 379):

Name	RA. 1900	Decl. 1900	Range in mag.	Period in days	Rem.
No. 8 ω Centauri	13 ^h 20 ^m 8	-46° 57'	12 ^m 8-14 ^m 3	0.52	1
No. 7 Messier 5	15 13.5	+ 2 27	13.5-14.9	0.50	2
S Arae	17 51.5	-49 25	9.5-10.8	0.45	3
Y Lyrae	18 34.2	+43 52	11.3-12.3	0.51	4
XX Cygni	20 1.3	+58 40	10.7-11.6	0.13	5
UY Cygni	20 52.3	+30 3	9.6-10.4	0.5	6

Remarks: 1. One of 128 variable components of the great southern cluster. Nearly constant minimum of 6^h duration. — 2. Situated in the globular cluster in Libra. Minimum lasts 0.4 of total period. — 3. Rises to maximum in 1^h 10^m. — 4. Increase of light effected in 1^h 30^m. — 5. Discovered by Mrs. *Ceraski* in 1904. — 6. Rise completed in 1^h 53^m.

The Cluster variables are characterized by a swift rise to maximum, and a prolonged halt of change at minimum. Hence the minima are more or less dead level tracks, and this low light phase is about half of the whole period. The brightness then blazes up rapidly and after passing the maximum, declines much more slowly. The light curves of these stars show distinctly the backward slope of the tidal curve of a resisted wave. Yet the time of the whole change is so short that secondary maxima do not become sensible (s. Plate 7).

After thus briefly showing the applicability of the theory of the tidal curve of a resisted wave to the Cepheid, Geminid, and Cluster variables, it may perhaps be appropriate to give a short notice of how this advance in astronomical geometry became possible.

4. Having been occupied with the problem of the sunspot periodicity since early in July, 1921, I finally resolved on Aug. 18, to search diligently in all available mathematical and physical treatises for the geometrical form of the typical curve of sunspot frequency, which, if it could be found, would enable one to predict the normal development of spot phenomena, with confidence and rigorous accuracy. This daring adventure might not succeed, but at least the experiment was worth trying, in the hope of finding the mathematical and physical law underlying the spot development.

(a) On Aug. 18, I turned to the elaborate tables of Spherical Harmonic Functions, with curves for illustration, given in *Thomson* and *Tait's* celebrated Treatise on Natural philosophy, ed. 1883, sections 782-784; and after some examination, settled upon the functions $\Theta_6^{(5)}$, $\Theta_7^{(6)}$ as being of all the graphs there given the one most like the form of the sunspot cycle. The resemblance was not perfect and after some disappointment, I put that treatise aside, to look into certain wave forms resulting from *Fourier's* analysis of wave-motion, and illustrated by curves in *Riemann's* Partielle Differentialgleichungen, *Hattendorff's* edition. 1882, p. 59, 159, 161. The two latter wave figures, based on the transcendental equation

$$\varphi \cos \varphi + p \sin \varphi = 0 \quad (26)$$

which in an oscillating periodic function of great interest, seemed likely to be most applicable, but after a brief examination I had to drop the work until the following day.

(b) On Aug. 19, I made a careful sketch of the Harmonic Functions $\Theta_6^{(5)}$ and $\Theta_7^{(6)}$, here reproduced, s. Plate 8 Fig. 6, but found the resemblance to the sunspot curve not sufficiently close to justify the adoption of these curves. Thus I put aside the harmonic curves altogether and turned to the *Fourier* wave curves treated by *Riemann*, p. 159, 161.

On tracing the curves reduced in the vertical ordinate by the factor $1/4$ I found that although these reversed wave curves rose more rapidly than they fell, and in general form were like the sunspot frequency curves, yet they did not have a secondary maximum on the downward slope, as indicated so frequently in the solar records. Hence whilst I did not despair of progress I deemed it prudent to extend the search for other curves, in the hope of finding a more exact type for the sunspot frequency curve.

(c) Having been so long engaged upon the wave-theory of physical forces, (1914-1921), in completing the New Theory of the Aether, 1921, nothing was more natural than to turn to *Airy's* great work on Tides and Waves, 1845. Remembering his analysis of the theory of long waves as resisted in shallow canals (Art. 201) with the figure reproduced in the Second Paper on the New Theory of the Aether, AN 5048, p. 141-2, I turned to that illustration, and immediately recognized the exact form required for the sunspot curve. In a few minutes the problem was perfectly solved and the further the examination of the tidal curves was extended the more incontestable the proof of the identity became. The rest of the discussion is given above, and any one may now form his own judgement both as to the logical character of the search for curves which would be useful in the geometry of the heavens, and as to results of that search in finding curves of the deepest interest to the investigator of the physical universe.

5. It only remains to add that in view of the evidence for the sun's curve of spot frequency cited above and confirmed by the light curves of Cepheid, Geminid and Cluster variables, we have no alternative but to give up any idea that the effect of meteoric downpour upon the sun or a star would follow the simple law of the well known curve of sines, of which the equation is:

$$y = a \sin(2\pi x/\lambda + \alpha) = a \sin(2\pi/\lambda)(Vt - x) = a \sin(2\pi/\lambda)(mVt - mx). \quad (27)$$

In practice the disturbance would be a disintegrating oscillation or broken wave; and hence we must frankly admit that all the physical oscillations of matter in nature are accompanied by resistance. The tendency of this friction is a gradual modification of the chief wave, and, if kept up long enough, a disruption into another of shorter length but greater amplitude; and the formation of a secondary wave, as shown by the second term of the expression for a tidal curve of a resisted wave:

$$H = -bk \sin(mvt - mx') + \frac{3}{4}b^2k mx' \sin(2mvt - 2mx'). \quad (28)$$

This curve with two maxima represents a modification which in time develops into double periodicity, or a move-

ment developing into two unequal oscillations. Therefore it is more complicated than the above simple curve of sines, with single periodicity — just as motion in the ellipse, investigated by the Greek geometer *Apollonius* of Perga in his great work on conic sections (250 B. C.), and introduced by *Kepler* in 1609 for the motions of the planets, is more complicated than simple motion in a circle.

Up to *Kepler's* time circular motion had been used by astronomers, since the days of *Hipparchus* and *Ptolemy*; and it required a heroic effort on *Kepler's* part to get rid of this usage of eighteen centuries. For it must be remembered that the Greek geometers and natural philosophers regarded the circle as a perfect figure, and hence it was held that the celestial motions necessarily are circular.

6. If α, β be the coordinates of the centre of the circle, the equation of the path of the eccentric referred to the origin, distant $A = \sqrt{\alpha^2 + \beta^2}$, becomes:

$$\begin{aligned} (x - \alpha)^2 + (y - \beta)^2 &= r^2 \\ x &= r \cos \omega + \alpha \\ y &= r \sin \omega + \beta. \end{aligned} \quad (29)$$

The radius vector ρ and polar angle θ , in reference to any point as origin may be obtained, by a roundabout process; and this usage was kept up till *Kepler* introduced the simpler theory of motion in the ellipse, of which the polar equation is:

$$r = a(1 - e^2)/(1 + e \cos v). \quad (30)$$

Here the radius vector is given, and the true anomaly v is easily found by the solution of *Kepler's* equation.

7. Increased geometrical rigor, as well as a better basis for physical action, followed *Kepler's* innovation of 1609. For it was upon *Kepler's* laws of planetary motion, — which placed the sun in the focus of the ellipse, with the radius vector describing equal areas in equal times, and made the squares of the periodic times to vary as the cubes of the mean distances, — that *Newton* established the law of universal gravitation, 1686:

$$f = mm'/r^2. \quad (31)$$

With these explanatory remarks on the progress of past ages, it must be borne in mind that in our present problem of sunspots and variable stars, we introduce the theory of the tidal curve of a resisted wave with the following understanding:

(a) Under the law of gravitation and the doctrine of the conservation of energy, we hold that the downpour of meteorites generates the energy of light and heat radiated away by the disturbed heavenly body. As the meteors fall, their gravitational energy generates a disturbance in the photosphere, which upsets both the hydraulic and radiative equilibrium, whether on the sun or stars; and the first result is a photospheric outburst, with rush of flaming fluid — the magnitude of the disturbance depending on the amount of meteors precipitated, the intensity of gravity, the star's velocity of rotation, surface temperature, etc.

(b) The hydraulic and radiative equilibrium being thus upset, there will follow with time, a gradual reaction or subsidence, to be followed by a secondary wave of disturbance like the first, but of feebler intensity.

T. J. J. See. Cause of the Sunspots and their Periodicity.

Diagram illustrating the relative motion of Jupiter and Saturn, and the theoretical method for calculating the sunspot period of 11.178 years.

8. (α) The problem of the gravitational oscillation of spheres has been treated by Lord *Kelvin* in the Phil. Trans. for 1863; and the period found to be very short, except in case of masses of small density, which are thus subjected to feeble forces, — the time of oscillation being inversely as the square root of the density. *Kelvin's* time for the gravitation oscillation of the sun is found to be:

$$T = 3^h 8^m. \quad (32)$$

(β) In his *Anwendungen der Mechanischen Wärme-Theorie auf kosmologische Probleme*, Leipzig, 1882, *Ritter* finds that when the sun's thermal equilibrium is disturbed the period of the oscillation is $T = 2.422$ days. No change of *Ritter's* constants will alter this result materially; so that we know the time of the thermal oscillation is only a few days.

(γ) In the *Astrophysical Journal* for May, 1909, *F. R. Moulton* reached a period of thermal oscillation for the sun of only a few days, — and thus substantially in accord with *Ritter's* work of 1882. These results hold for expansions and contractions of the globe represented by a harmonic of the second order, which in view of the high effective rigidity of the sun (AN 4104) is an excellent approximation to the truth.

9. Now we dismiss all such gravitational and thermal oscillations as are here described in paragraphs α , β , γ , as inapplicable to the sun and variable stars; and consider only what will happen as the secondary wave or tidal current of the equatorial disturbance develops on the sun or a star, under the effect of frictional resistance to the original rush of the flaming fluid. It is for this readjustment of the equatorial flow under friction that we introduce the curve of the resisted tide wave — the double or secondary oscillation being due to the effects of friction and reaction, as in the observed progress of the tide waves noticed upon the earth.

10. When the current is set up at the equator of a sun or star by a meteoric downpour, there is generated ¹⁾ besides the outburst of light and heat, a forward rush of the photospheric fluid. Now a river, as we know in practical hydraulics, runs swifter and at greater elevation in the centre of the channel, where the fluid is deepest. Owing to the tendency to maintain the hydrodynamical condition implied in the equation of continuity, (cf. *Airy*, *Tides and Waves*, Art. 72, or *Darwin*, Art. *Tides*, *Encyc. Brit.*, 9th ed., § 11. eq. 481):

$$du \gamma \sin \theta / d\theta + \gamma \sin \theta \cdot dv / d\varpi + w \sin \theta = 0 \quad (33)$$

the rushing column of liquid in the centre tends to drag along the fluid near the banks of the river, and thus by suction the level of the fluid is elevated in the rapidly moving centre of the channel.

11. Now in the same way a rapidly moving current set up in the photosphere, at the equator of the sun or of a star, by the downpour of meteors, tends to carry the fluid

in the higher latitudes along with it, by the suction of friction, — the hydrodynamical conditions specified in the equation of continuity still holding approximately true. Moreover, by the increased centrifugal force given to the equatorial current by the meteoric downpour, there is additional suction towards the swifter current at the equator. Hence the level of the spherical or spheroidal photosphere is slightly raised at the equator; and the level of the fluid in higher latitude undergoes a slight adjustment as the surface fluid moves slightly towards the equatorial region.

Hence in time the shallow surface current is checked by friction at the sides, and from the layers beneath, which revolve less rapidly: the result is a revulsion and readjustment of level in all latitudes, as when the resisted tide wave in our rivers breaks up. This subsiding oscillation calls forth a secondary movement or rush of fluid like the first, but much feebler, and thus arises the secondary maximum in the curves of the sun and stars. Such readjustment in the surface layers of our sun are also implied in *Spoerer's* law of sunspots, which we have shown confirm the theory in every respect.

The views here expressed on the sun's equatorial photospheric currents and their reactions, apply of course to the stars throughout the sidereal heavens, — though the time of reaction will vary from star to star, according to physical conditions. In view of the extreme fluidity of the stars, at the high temperature of their photospheres, it is evident that the period of modification for the formation of a secondary wave, as in the resisted wave noticed upon the earth, will not be very short, except where the frictional effects are very great.

12. In the case of the sunspots, for example, the period to the secondary maximum is some six years, — half of the periodic time of the chief disturbing planet. In the case of a variable like η Aquilae the secondary maximum follows in about four days, about two-thirds of the period of the companion shown by the spectrograph to revolve in some $6\frac{1}{2}$ days. If the period be very short — say less than half a day — it seems certain that secondary maxima cannot develop, for lack of time of decay; and this inference is confirmed by the general absence of secondary maxima in Cluster variables.

As Commander *Leonard M. Cox* was kind enough to confirm the early results at which I arrived, on Aug. 19–22, I may be pardoned for quoting some passages from a letter in which he gives the conclusions of a practical worker in the hydraulics of rivers and channels.

»My dear Dr. See:

»I am so profoundly impressed with your remarkable discovery of the cause and periodicity of sunspots and of the Cepheid and Cluster variable stars, that I am impelled to express, in my feeble way, my appreciation of the achievement.« . . .

¹⁾ In the present paper we have scarcely alluded to the elliptic tide which in fact is a periodic change of oblateness of figure depending on the revolution of the companion in its orbit: yet we assume that the reader is familiar with this variation of tidal forces, and the resulting equatorial and polar rush of the flaming fluid. This change of oblateness under the tidal forces producing the elliptic tide would give rise to some change of brightness, but not the great blazing up of light actually noticed in the case of many variables. Hence the chief effects are ascribed to the meteoric downpour, yet the subordinate effects of the elliptic tide are not overlooked in our reasoning, but recognized to exist without special discussion.

»I am particularly impressed with the use of the geometrical theory of terrestrial wave motion, as in the case of the tides — which are of so much interest to engineers. Apparently you have rejected the complicated and used the simple and obvious — though not obvious until you pointed the way. In this, you remind me of *Kepler's* innovation (in 1609, I think) in boldly rejecting the clumsy theory of eccentrics as handed down by *Ptolemy*, for the simple and perfect theory of elliptical motion with the sun in the focus of the ellipse.«

»As I recall the development and application of geometry to the heavens, not many species of curves are used even now — chiefly the conic sections. Your introduction of the tidal curve of a resisted wave, and your development of a general equation describing it, makes a new doubly periodic function applicable to the most intricate and varied of celestial phenomena. Undoubtedly we shall have occasion to observe indications of this resisted wave motion throughout the solar system as well as in the globular clusters and other portions of the sidereal universe.«

»Since the cluster variables are used for calculating the distance of the clusters, recently found to be removed from the earth hundreds of thousands of light years, this application of the tidal curve of a resisted wave seems destined to become as useful in astronomy as the theory of elliptical motion introduced by *Kepler* the year before *Galileo* invented the telescope and discovered the sunspots from which your new discovery arises.«

»It will always be a source of pride to me that, despite my inability always to follow you on the lofty plane of your work, you have done me the honor of including me among the few to whom you first communicated your discovery. I shall not attempt to express my gratitude, for the reason that personal considerations seem out of place in connection with such a momentous addition to the knowledge of man.«

7. Explanation of the Binary Character of the Formula for the Length of the Sunspot Cycle, with the resulting Secondary Periodic Oscillation affecting the Duration, and itself Variable with the Amplitude.

It will be noticed that our formula for the length of the sunspot cycle, given in section 5 above, has the binary form, and thus the secondary term may become positive or negative, through the changes of sign in the expression

$$\sin[\beta - \alpha(t - t_0)]. \quad (21)$$

Hence a word of explanation is desirable in justification of the form adopted.

Already in 1875 *Wolf* found the period, — during the whole interval from 1610 to 1870.6 — about 261 years — to be so clearly modified by a secondary term which might become either positive or negative, that this eminent investigator adopted the designation »oscillation« for this secondary term, which was to be added to the mean term of period 11.11 years (Mem. R. A. S., 1877).

Such a secondary oscillation, for modifying the mean sunspot period, is simply a fact of observation shown in the records from the age of *Galileo*; and therefore we are

authorized to use it, yet at the same time we shall endeavor to explain how it arises from the eccentric distribution of the orbits of comets and meteors about Jupiter's orbit.

1. The distribution of comet orbits in *Sivaslian's* diagram of 1893 (s. plate 1) resembles the positive and negative character in the sine function, — nearly all the orbits being between longitudes 90° and 270° , while the lack of orbits between 270° through 360° to 90° corresponds to the negative value of the sine between 180° and 360° .

2. Thus by taking the average density for the mean value we may introduce the sine function to represent the density of the orbits; and with the axes arranged as here described, we find that by taking the average density of the orbits for the zero of our function, the variable density of the orbits from 0° through 360° will correspond very accurately to the sine function.

3. So much for the law of distribution of the orbits of comets about the orbit of Jupiter. The question remains what the density of the meteors will be in the different parts of the planetary orbit. Here again we may take σ_0 for the average density of the meteors throughout the planetary orbit, and σ_μ for the average density of the meteors in the part of the orbit being described; the ratio σ_0/σ_μ will vary somewhat as the distance function r/a in the theory of elliptic motion, and thus will not become negative in any part of the orbit. The factor σ_0/σ_μ will give therefore the variable amplitude required.

4. In his memoir of 1877 (Mem. R. A. S., 1877), *Wolf* noticed that the variation in length of the sunspot cycle could properly be regarded as an »oscillation«, and his mean values from all his series of observations 1610–1870, turns out to be:

	Oscillation	
1610.8 } 1734.0 }	Minima, $\Delta\pi = \pm 2^y 11$	(34)
1615.5 } 1837.7 }	Maxima, $\Delta\pi = \pm 2^y 06$	
1745.0 } 1867.2 }	Minima, $\Delta\pi = \pm 1.54$	
1750.3 } 1870.6 }	Maxima, $\Delta\pi = \pm 2.52$	
Mean value $\Delta\pi = \pm 2.0575$		(35)

5. Hence we may take the oscillation or variable part of the period as 2.05 years, and by introducing the law of the sine above described, our formula would become

$$\pi_i = 11^y 178 - 2^y 05 (\sigma_0/\sigma_\mu) \sin[\beta - \alpha(t - t_0)] \quad (36)$$

$$= 11^y 178 - 2^y 05 (\sigma_0/\sigma_\mu) \sin[177^\circ - 4^\circ 04965(t - t_0)]$$

$$t_0 = 1921 \text{ Aug. } 22.5775 \quad (37)$$

where $\beta = 177^\circ$ = heliocentric longitude of Jupiter, Aug. 22, 1921, and the oscillation is adjusted to give four high maxima during the next 44 years.

6. Now it is found by the sunspot records that when the amplitudes of the cycle are large, or the maxima are high, the period is materially shorter than the mean; and when the amplitudes are low, or the maxima are low, the periods are considerably longer than the mean. Thus there is a

tendency to compensation in length for lack of height, as in many other phenomena of nature.

This is an additional reason for the form of the above equations, where the amplitude

$$A_i = \sigma_0/\sigma_\mu. \quad (38)$$

7. The amplitudes A_i are therefore large on the side of the orbit $\lambda = 90^\circ$ to $\lambda = 270^\circ$, as shown in the figure. On this side of Jupiter's orbit the meteors are dense, and the effect of their downpour is therefore intense.

It appears that the combined actions of Jupiter and Saturn, when the phase line representing the appulse with Saturn enters the region of dense orbits, are nearly twenty years preparing for the precipitation of meteors upon the sun, probably in rearranging the meteor paths relatively to the planetary orbits. The effect is thus delayed, as if to let orbital motions develop. The eccentricities of the meteor orbits no doubt are readjusted¹⁾, so that increasing numbers of meteors pass very near the sun.

8. But as the effect, when once developed, runs on for a like period before it is exhausted, we see that the solar phenomena are prolonged after the phase line of Jupiter and Saturn has passed on, and entered into the region of rare meteors. Thus our formula for the oscillation has the form written above, and is justified by experience.

This theory enables us to predict that the sunspot cycle beginning next year will have high maxima, and after 44 years the low maxima will again return. Such is the plain indication of the past 176 years, and we cannot depart from it without doing violence to the observations.

9. In practical mechanics it is well known that if a wheel roll upon an eccentric, such as an elliptical cylinder, with axis through the focus of the ellipse, the centre of gravity of the wheel will oscillate relatively to the axis, in the period of the movement around this centre, as in the theory of planetary motion.

The oscillation of the radius vector is given by the formula for elliptic motion:

$$(r/a) = (1 - e^2)/(1 + e \cos v). \quad (39)$$

And the component of this central displacement, depending on the true anomaly v (which in our problem might increase or decrease the precipitation of meteors upon the sun, owing to the asymmetry of the orbital arrangement) is easily seen to be:

$$(r/a) \sin v = [(1 - e^2)/(1 + e \cos v)] \sin v. \quad (40)$$

The magnitude of these eccentricity effects depends upon the eccentricity of the meteor orbits, (usually very great), and upon the asymmetry of the ensemble of cometary orbits, which appears to be very pronounced. Hence it is not remarkable, but perfectly inevitable that in the sunspot cycle there should exist a double oscillation, — that is, an oscillation variable both in period and amplitude, as in the above formulae (39) and (40), where v and (r/a) oscillate, and the resolved component may reach large proportions, in view of the eccentricities and asymmetry involved.

Fig. 8. Illustration of the effect of elliptic motion, which gives the secondary term of the sunspot period depending on $\sin[\beta - \alpha(t - t_0)]$ also a variable amplitude.

Accordingly, if the reasoning here outlined be admissible, it will follow that the sunspot cycle does not depend on two separate periods of $4^y62 + 6^y51 = 11.13$ years, as *Newcomb* concluded in his paper »On the Period of the Solar Spots«, *Ap. J.* 13.1, January, 1901. In view of the impossibility of such long periods arising from internal causes, it is difficult to recognize any physical or geometrical ground for the view that our sunspot cycle depends on two shorter periods each several years in length. The only possible physical basis for *Newcomb's* theory would be the downpour of meteors from Jupiter's system, yet having perihelia near the sun; but here the periods would lie between 4.2 and 5.3 years, and thus do not correspond to *Newcomb's* hypothetical component periods.

It remains to point out clearly a series of false premises heretofore very widely current, and which have added to the disorder of our thinking on sunspot phenomena.

1. It has been very usual to ascribe certain meteorological phenomena to the influence of sunspots; but any attempt to prove the truth of this assumption has met with insurmountable difficulty, and added to the confusion of the subject.

2. Below in section 10 we set forth the conclusions which seem to follow from accurate calculation, yet even here it is necessary to be on our guard against misinterpretation of the present theory.

3. For example, the sunspots are now held to be an indirect effect of the repulsive forces forming the corona and of the circulation and readjustment within the solar atmosphere, due to the periodic downpour of meteors precipitated by the actions of Jupiter and Saturn; and thus the spots are comparatively, if not entirely, harmless in their effects upon climatology and other terrestrial weather conditions, — unless the »Magnetic Storms« associated with the spots have some influence not yet recognized. This latter possibility still exists, but we do not elaborate it in the present paper.

4. It is a very different question when we come to consider the seasonal elevation of terrestrial temperature by the downpour of meteors, which appears to occur at intervals of 9.92945 years, — thus giving us with the equatorial expansion of the corona, terrible decennial droughts. If our

¹⁾ Compare *Herschel's* tabulation of the effects of the disturbing force, given below near the end of section 12, for details of these adjustments.

theory be admissible, these droughts do not follow either the sunspot cycle, or Jupiter's orbital motion.

5. In Section 4 above we deal with the shift of the epochs of the several cycles in respect to one another.

(a) The droughts appear to occur about every ten years, yet there is a shifting of their epoch amounting to 51.5 days in 19.8589 years, or about 25.8 days every ten years, so that relatively to our calendar, the seasons of the decennial droughts will circulate around the 12 months of the year in about 140 years.

(b) The drought cycle depending on meteoric downpour and the action of Jupiter and Saturn shifts in respect to the average sunspot cycle in 11.178 years, according to the formula:

$$n_1 : n_2 = 1/11.178 : 1/9.92945 = 360^\circ : x \quad (41)$$

$$x = 405.267$$

$$x - 360^\circ = 45.267 \quad (42)$$

(c) Accordingly in about 8 drought cycles, of 9.92945 years each, (about 79 years), the epoch will be displaced through a whole circumference, relatively to the mean sunspot cycle, and the phases may begin over again.

6. It is therefore vain for us to search for a fixed relationship between the drought cycle and the sunspot cycle. No such relationship exists, and the search merely adds to our bewilderment. In the same way we show, in section 4 above, how the epoch of the mean sunspot cycle circulates in respect to Jupiter's sidereal motion in his orbit. Thus it happens that in time the sunspot maxima and minima occur with Jupiter in every conceivable relative position: the shift is 22.0201 each Jovian revolution, or 1.8564 per annum, thus completing the circulation through 360° in slightly less than 200 years.

7. In view of the considerations here brought to light, it is important on the one hand to avoid hasty deductions which can have no basis in cyclic or dynamical theory, and on the other not to abandon hope of real progress in a subject heretofore unduly confused. If a few firm foundations of fact can be established, and harmonized in our theory of sunspot phenomena, the clearing up of the rest will present less difficulty. Accordingly, in this paper we aim to develop a theory based on carefully ascertained facts, and which may unfold to us the true laws of nature.

8. If the dynamical theory at first sight seems strange, it is only because it is unfamiliar, — not having been thought out by any previous investigator. When its obviousness is once pointed out many will wonder that it had not occurred to some of the eminent astronomers heretofore occupied with the deeper problems of solar physics. The road to new light is not along the familiar beaten paths; and thus I have not hesitated to follow where the light leads.

8. Calculation of the Amount of Meteoric Matter falling into the Sun.

In the Researches on the Evolution of the Stellar Systems, vol. 2, 1910, p. 320, the writer has shown how to calculate the amount of the meteoric matter falling into the sun, from the observed secular acceleration of the earth's motion, which may be found from researches on total eclipses of the sun noticed by the Greeks.

The formula for the secular acceleration of the earth depending on the increase of the sun's mass is found to be:

$$\Delta L_1^0 = \frac{328714}{328715} \frac{3H}{R_0} \left(1 + \frac{q^2}{R_0^2}\right) \frac{k}{a^{3/2}} t^2 \quad (A) \quad (43)$$

where q is the mean radius of the earth, $q = 6370000000$ mm; R_0 is the radius of the sun, $R_0 = 696098000000$ mm; $k/a^{3/2}$ = the mean motion of the earth in a Julian century = $129600000''$, the combined mass of the sun and earth being unity; and H is the thickness of the layer of cosmical dust of the mean density of the earth falling upon the sun from regions of celestial space beyond the terrestrial orbit, and thus effecting secular changes in the earth's motion.

Now since

$$1 + q^2/R_0^2 = 1.0001$$

which differs but little from unity, we get by the above equation (A)

$$\Delta L_1^0 = 388800000/698098000000 \cdot H t^2 = 0.0005584 H t^2. \quad (B) \quad (44)$$

If we take $H = 1000$ mm, and use the factor 1.0001, we obtain

$$\Delta L_1^0 = 0.5590 t^2. \quad (C) \quad (45)$$

This is the amount of the secular acceleration¹⁾ of the earth's motion, arising from the fall of a layer of matter of the mean density of our globe, 5.5, upon the sun's surface in a Julian century. This falling of matter on the sun produces the principal secular change in the earth's motion.

But there are small changes due to tangential resistance in the curved cylindrical space swept over by the earth in its motion about the sun, which is found to be given by the equation

$$\Delta L_2^0 = 9/2 \cdot (k/a^{3/2}) (h^2/q) = 0.0916 t^2. \quad (D) \quad (46)$$

And, lastly, there is a retardation of the earth's rotation by the downfall of miscellaneously directed cosmical dust upon our globe, and by tidal friction. In the same Researches, vol. 2, 1910, pp. 317–321, we have shown that these two secular effects upon the earth's rotation, and affecting the apparent secular motion of the sun, may be reduced to the expression

$$\Delta L_3^0 = 0.0650 t^2. \quad (E) \quad (47)$$

All these three terms are plus, and they lead to an apparent secular acceleration of the sun in a century amounting to:

¹⁾ Since preparing the above discussion, largely as given in my Researches, vol. 2, 1910, I have been surprised to find that *Newton*, *Halley* and *Flamsteed*, in 1694, were earnestly occupied with the problem of the acceleration of orbital motion due to increase of the central mass. In *Brewster's Life of Newton*, 1855, vol. 2, p. 170, we have the following letter from *Flamsteed* to *Newton*:

»Yesterday at London, I had a great deal of talk with Mr. *Halley* about the moon's motion. He affirmed the moon's motion to have been swifter in the time of *Albatignius* than at present, and that the cause of it was by reason that the bulk of the planets continually increased. I gave him the hearing, and at last told him that his notion was yours, he answered, in truth you helpt him with that«.

This passage is very remarkable, and so far as I know it is the earliest discussion of the subject except what is given in *Newton's Principia*. *Flamsteed* says he told *Halley* that the argument was *Newton's*, and *Halley* admitted that *Newton* had helped him with it.

$$\sum_{i=1}^3 \Delta L_i^0 = \begin{Bmatrix} +0.5590 t^2 \\ +0.0916 t^2 \\ +0.0650 t^2 \end{Bmatrix} = +0.7156 t^2 \quad (F) \quad (48)$$

This reasoning is founded on a meteoric layer of dust at the earth's surface of thickness $h = 1$ mm in a century, density = 5.5.

The researches of Dr. *P. H. Cowell*, on ancient eclipses, made some 15 years ago, and Dr. *Fotheringham's* recent extension of these researches, as reported in the Monthly Notices, indicate that the probable amount of the earth's secular acceleration may be as great as 1".43 per century, which would correspond to a downfall of meteoric dust upon the sun of thickness 2000 mm, and upon the earth of thickness $h = 2$ mm each of density 5.5, which is not excluded by any known observational phenomenon.

In a letter written to the Royal Society, in 1749, *Euler* expressed the conviction that the motions of the planets are accelerated, and that the earth once was beyond the present orbit of Saturn. This was an early indication of the view developed in recent times that the planets are steadily nearing the sun, chiefly from the secular increase of the sun's mass under the downfall of meteorites upon its surface.

The above calculation is accurate and dependable, and as the researches of *Cowell*, *Fotheringham* and others on ancient eclipses of the sun, show that a secular acceleration of the earth really exists, it is natural to attribute this secular change in the major axis of the earth's orbit mainly to an increase of the sun's mass by the downpour of meteorites upon the surface of the sun.

Quite recently there was a report of a paper to the Roy. Astron. Soc. in the Observatory (May, 1920, p. 178) stating that no cause was known adequate to account for the earth's reported secular acceleration. In reply to this comment (Observatory, Aug., 1920, pp. 287-8) I called attention to calculations like the above, given in my Researches vol. 2, 1910, pp. 319-321; where upon Mr. *Harold Jeffreys* acknowledged the adequacy of the assigned cause for explaining the real secular acceleration of our planet indicated by ancient solar eclipses, particularly that of 128 B.C., total at the Hellespont, and supposed to have been observed by *Hipparchus*.

Accordingly, the cause assigned seems adequate, and founded upon the downpour of meteors upon the sun, which is known to be at work; and thus we are justified in adopting the above explanation as true and sufficient to explain all the known phenomena. It seems therefore justifiable to hold that in this case we have brought to light the true cause of the earth's secular acceleration¹⁾.

This implies an increase in the sun's mass by nearly twice that of the moon is a century, yet as the moon is

only 1:27000000th of the solar mass, such a secular augmentation of the central mass is by no means improbable. The only question remaining therefore is whether the increase of mass goes on uniformly, or by gusts, varying in intensity like the observed showers of rain. It would seem that the downpour cannot possibly be uniform.

It is quite remarkable, as pointed out by Sir *John Herschel* (Outlines, 10th ed., 1869, § 325), that as far back as 1852 (Compt. Rend., Aug. 20, 1852, and AN 806, 833), the celebrated Father *Secchi* made careful measurements of the intensity of the thermal radiation of different parts of the solar surface, finding the heat emitted at the centre of the disc nearly twice that at the borders, and moreover that the equatorial region of the sun is somewhat hotter than the polar regions.

This early result of *Secchi* coincides with the requirements of the meteoric theory, and, as *Secchi's* work always is accurate, it may even be considered as an observational confirmation of the postulated meteoric downpour. What other cause except falling meteorites could heat the equatorial regions to a temperature beyond that of the polar regions? Obviously no other cause is known. And as the sun has a recognized very noticeable equatorial acceleration of the photospheric surface at the equator, over that in higher latitude, where the spot zones appear, we infer that the equatorial acceleration is due to the whirl of cosmical dust continually colliding with the sun's surface, and thus imparting to it an increase of momentum, owing to the meteorites having a direct motion of revolution like those of the planets.

The tangential velocity at the sun's surface would be 617 kms. (cf. AN 3992, p. 136), and thus a small mass, by this great velocity, would contribute considerably to the augmentation of the sun's equatorial momentum. Such secular downpour of meteorites would thus maintain the sun's equatorial acceleration against decay by friction; and also maintain a higher temperature in the region of greatest impact, since probably nine-tenths or more probably 95 percent of the meteors would have direct motion, like the planets, by whose disturbing action they are precipitated upon the sun's disc.

By this reasoning, it will be noticed, we give a direct and simple explanation of the sun's equatorial acceleration, which is accounted for by principles operating also to give the equatorial accelerations observed in Jupiter and Saturn, and confirmed by the direct motion of all their inner satellites, and of Saturn's rings. The vortex of dust here assumed to be precipitating upon the solar surface is therefore definitely known to exist, and to have a direct motion like the acceleration observed in the sun's equatorial regions.

The greater temperature observed at the sun's equator in 1852 by *Secchi*, has therefore the highest probability. But it does not follow that such inequality in temperature would

¹⁾ Several recent British writers persist in the antiquated habit of regarding the apparent secular acceleration of the sun and moon indicated by observation as due chiefly to a retardation of the earth's rotation, holding that only a small part of it is real. This was the view current half a century ago, as in the speculations of *Delaunay* and *Adams*, and in *Thomson* and *Tait's* Treatise on Natural Philosophy, 1873, which underlies *Darwin's* researches on the origin of the moon, 1880.

Such views were justifiable enough before the development of the Capture Theory of Satellites (AN 4341, 4343); but since that development — with definite proof that the sun and planets have acquired their rotations by the collisions of meteors against their equators (AN 4307), and that tidal friction is everywhere vastly overshadowed by the acceleration of rotation due to the downpour of meteors against the surfaces — the persistence of this antiquated mode of reasoning is entirely out of place, as inconsistent with recent progress in cosmogony (cf. Researches on the Evolution of the Stellar Systems, vol. 2, 1910, p. 293-326).

lead to spot development in higher latitudes till the circulatory system somewhat adjusted itself, which probably would be a process of four or five years.

9. All other Explanations of the Earth's Secular Acceleration, except the Downpour of Meteorites upon the Sun, excluded. No Cause within the Sun can give Rise to the Sunspot Cycle, nor produce a Sensible Change in Solar Radiation. Sir *John Herschel's* Cyclonic Theory of Sunspots, 1847, still more Appropriate under the Periodic Conflagration of Meteorites.

It is important to emphasize the indisputable fact that the theory of a considerable downpour of meteorites upon the sun offers the only known explanation of the earth's observed secular acceleration. Thus we have the definite fact of the secular acceleration of the earth's motion, — which is confirmed by the researches of *Cowell*, *Fotheringham*, and other investigators of ancient solar eclipses, and only one cause adequate to explain this acceleration.

Our solution of the problem of the earth's acceleration is therefore unique: there is one solution and only one, just as in the case of a linear equation with one unknown quantity. Such a problem, with unique solution, is most welcome to the investigator; for not the least uncertainty attaches to the result, and thus the solution illuminates a great secret of nature.

And not only is the unique solution of the secular acceleration of the earth found to be satisfactory mathematically, but also physically. It is especially welcome, as having no inherent improbability; for we know that billions of meteors daily fall upon the earth (cf. observations by the writer in AN 3618, and *Researches*, vol. 2, 1910, p. 300), and hence we hold that these same bodies must therefore also fall upon the sun, in vastly greater quantities. The sun naturally draws in great quantities of small bodies and thus builds up its mass.

If we come to examine the causes within the sun which could give rise to a periodic disturbance or oscillation of that globe depending upon the equilibrium of gravitational or thermal forces we shall find that any such supposed cause cannot be admitted, owing to the absence of known physical agency adequate to produce and especially to prolong the oscillation. Internal causes at most would explain an oscillation having a period of a few days — but not a sunspot cycle of 11.18 years, which is at least four thousand times the duration of any admissible internal disturbance.

In his well known *Anwendungen der Mechanischen Wärme-Theorie auf kosmologische Probleme*, Leipzig, 1882, *Ritter* investigates the duration of the pulsation of the sun. Taking the ratio of the specific heat of the gas under constant pressure to that under constant volume to have the value $k = 1.41$, as in ordinary biatomic gases, *Ritter* finds (p. 69), that the duration of the double pulsation would be

$$T = 20928 \text{ seconds} = 5.812 \text{ hours} = 0.2422 \text{ day. (49)}$$

If the sphere of gas were expanded to fill Neptune's orbit and the density equally distributed, the time would be 340 years. Thus the only way to get a long period for the thermal oscillation is to have a very expanded mass

subjected to forces which become very feeble, in the reactions of the parts of the mass upon itself. Now, such a very expanded state does not correspond to the sun's actual condition, and is therefore excluded.

If $k = 1.66$, as in monatomic gases, *Ritter's* formula for the sun's double pulsation period would reduce to:

$$T = 2\pi r^{1/2} / [(3k-4)Ng]^{1/2} = 10046^s = 3 \text{ hours, nearly (50)}$$

where $r = 688\,000\,000 \text{ m}$; $k = 1.66$, $N = 27.435$ and $g = 9.81 \text{ m}$.

Accordingly, whatever be k , whether the gases be biatomic or monatomic, the duration of the double oscillation at most will be only a few hours. It is not possible to prolong the pulsation into days without adopting assumptions unwarranted by experience.

In 1909 *Moulton* also made a thorough investigation of the pulsation period of the sun (ApJ 29, May, 1909): but although his details were different from those reached by *Ritter*, he concurred in the above conclusion that the sun's pulsation period is very short. It could not be made to exceed a day without doing violence to physical experience, and departing from every indication of probability.

Accordingly, we reach the incontestable conclusion that no oscillation depending on thermal or gravitational causes within the sun could arise of longer duration than a day. The average sunspot cycle is at least 4000 times longer than this theoretical oscillation period; and thus by no physical possibility can the sunspot fluctuation depend on an internal oscillation of the sun's mass.

It follows, therefore, that the long duration of the sunspot cycle compels us to look beyond the sun's interior for the causes which underly this periodic fluctuation in the sun's condition. No thermal or gravitational cause within the sun can possibly generate a cycle of 11.18 years duration, because the forces at work are very powerful, and the period of their reaction on the sun's mass very short, a day or less, and thus less than $1/4000^{\text{th}}$ of the sunspot cycle.

In turning away from the sun's body to the surrounding heavens, for the cause of the sunspot fluctuation, nothing appeals so directly to our common sense as the mutual reactions of Jupiter and Saturn, which come into conjunction-opposition in 9.92945 years, while Jupiter himself revolves in 11.86172 years, and has an unsymmetrical or lopsided distribution of cometary orbits about his sidereal path.

Ever since the researches of *Adams*, *Leverrier*, *Schiaparelli*, *Newton* and *Alexander Herschel* on the November meteors, and other meteoric swarms, and the comets with which they are connected, which researches were carried out in the middle sixties of last century, — we have referred meteors to cometary origin. And thus we must hold that not only are the cometary paths distributed unsymmetrical about Jupiter's orbit, but also the corresponding meteor swarms. Thus with the meteor swarms arranged in a lopsided, or unsymmetrical form, the cycle of the downpour of meteors upon the sun will depend upon Jupiter's sidereal revolution as well as on the conjunction-opposition with Saturn. Accordingly we reach the theory above developed, and every other explanation is excluded.

Sir *John Herschel's* Cyclonic Theory of Sunspots, 1847.

1. Those who read Sir *John Herschel's* lucid account of sunspot distribution in the two zones north and south of the solar equator (Results of Observations at the Cape of Good Hope, 1847, p. 433) cannot fail to be impressed with the argument there adduced for the cyclonic theory of sunspots, afterwards confirmed observationally by *Secchi* and *Faye*, and in recent times often illustrated by our photographs of the sunspots.

»Now, whatever be the physical cause of the spots,« says *Herschel*, »one thing is certain, that they have an intimate connection with the rotation of the sun upon its axis. The absence of spots in the polar regions of the sun, and their confinement to two zones extending to about 35° latitude on either side, with an intermediate equatorial belt much more rarely visited by spots, is a fact notorious in their history, and which at once refers their cause to fluid circulations, modified, if not produced, by that rotation, by reasoning of the very same kind whereby we connect our own system of trade and anti-trade winds with the earth's rotation. Having given any exciting cause for the circulation of the atmospheric fluids from the poles to the equator, and back again, or vice versa, the effect of rotation will necessarily be to modify these currents as our trade winds and monsoons are modified, and to dispose all their meteorological phenomena on a great scale which accompany them as their visible manifestations, in zones parallel to the equator, with a calm equatorial zone interposed. It only remains, therefore,

Fig. 9. Illustration of the circulatory system of the sun's photosphere and corona. The meteoric downpour yields the acceleration of spots observed at the equator, while the resulting expulsion of dust, falling in higher heliographic latitude, produces the zones of spots on either side, with *Spoerer's* law of drift in the observed sunspot period.

to inquire whether any such cause of circulation can be found in the economy of the sun, so far as we know and can understand it?»

2. In section 424 *Herschel* continues:

»Recurring now to the question whether any probable or possible cause can be assigned, from what we know of the sun's economy, capable of giving rise to circulatory movements to and from its poles, in the fluids which cover its surface and having at the same time a dependency on its rotation; it may be observed that if any physical difference in the constitution or circumstances of its polar and equatorial regions tend to repress the escape of heat in the one and to favor it in the other of these regions, the effect will be the same as if those regions were unequally heated from without, and all the phenomena of trade winds, mutatis mutandis, must arise.«

3. Here it may be observed that the downfall of meteors upon the sun's equator not only represses the escape of the sun's internal heat, but also adds greatly to the temperature of the surface layer of the photosphere. *Herschel* then proceeds to reason that the sun is surrounded by an atmosphere of considerable thickness, as an aliquot part of the sun's radius, and with the ellipticity of the layers increasing outwardly, in accordance with the known laws of dynamics, (cf. *Laplace*, *Méc. Cél.*, Lib. III, Chap. IV, § 30). The thickness of the atmospheric layers, density for density, must differ between the equator and the poles. He concludes that this elliptical layer about the sun, with increased obstruction to the equatorial radiation, should maintain the equatorial and polar regions at different temperatures, and by the interchange thus produce the spots. If this difference of temperature would be maintained by mere difference of radiation, due to greater thickness in the elliptical layer at the solar equator, how much more certainly would such an effective blanketing of the heat at the equator arise from the downpour of meteors? The blanketing of the equatorial zone is shown to be a fact by the observed acceleration of the earth's motion, made known by the researches on ancient eclipses.

The exchanges of solar atmospheric circulation between equator and poles should be accompanied by vorticose motions, in very full agreement with observation, not only in the case of terrestrial tornadoes, but also of sunspots, which appear to be filled in by the collapse of their sides, the penumbra generally closing in upon the spot and disappearing after it.

4. Such is *Herschel's* cyclonic theory of sunspots. It rests on a very secure dynamical basis, as respects the effects of rotation, but the premise postulating an atmosphere thick enough to constitute a considerable part of the sun's radius, and decreasing in depth towards the poles, is the weak link in the chain of reasoning. For the sun's atmosphere is known to constitute a very thin layer of gases, with the density falling off rapidly as we ascend, (cf. AN 4053), and all internal disturbances dying out rapidly owing to the intensity of the gravitative action as explained in AN 4053.

Zonal obstruction of the sun's radiation therefore would not give rise to long period disturbances in the

atmospheric circulation, unless this obstruction was superposed periodically, by an external cause, such as the downpour of meteors, acting at fixed intervals corresponding to the tardy return of the appulse with Jupiter and Saturn, and giving sunspots in cycles of 11 years' duration.

5. *Herschel's* Results at the Cape appeared in 1847, but his theory of sunspots doubtless was prepared some years earlier, and probably without knowledge of *Schwabe's* discovery of a sunspot periodicity in 1843. Thus *Herschel's* cyclonic theory took no account of the problem of periodicity, which is much more difficult to solve than might be supposed at first sight. For several investigators long ago considered the possibility of the sun being an oscillating gaseous globe, but all of them have found the theoretical periods much too short to be compared with the immense 11-year cycle noticed for sunspots (cf. section 6, paragraph 8, α , β , γ above).

10. Somewhat Violent Variations of the Sun's Radiations indicated by World Famines and by other Large Features of Terrestrial Meteorology.

Up to the present time our studies in solar radiation have extended over short periods, in recent times, or else are long range studies based upon the records of snow and ice in Europe during the winters of various years in the period since the Middle Ages. Thus in the Smithsonian Report for 1857, pp. 339-345, there is a very philosophical summary of the evidence for secular changes of the sun's intensity of radiation. Though *Wolf* and other high authorities are cited, no decision is reached, and it is doubtful if the record is continuous enough to enable a modern investigator to pursue the subject to a definite result.

Lord *Kelvin* and some other authorities, including *Fourier*, in his celebrated *Théorie Analytique de la Chaleur*, have considered the distribution of plants and animals in the classic period as affording comparatively good evidence that no considerable change in the sun's radiation has taken place since the Greek and Roman writers *Aristotle*, *Theophrastus* and *Pliny* described the flora and fauna best known 2000 years ago.

It would seem that the records of antiquity exclude the possibility of very great secular change, since the distribution in modern times confirms that recorded by the Greek and Roman naturalists. But up to the present time there are only general indications of the secular constancy of the solar radiation, whilst decennial variations have not been investigated at all, and we can affirm nothing, except that in his recent measurements *Abbot* found the so-called »solar constant« to be 1.93 (small) calories per second, and variable by as much as 5 or even 10 percent within a single week.

That very sensible and comparatively sudden changes do occur from time to time in the sun's radiation is indicated by our common sense, and by the consensus of opinion of mankind. Yet this result heretofore has not found definite scientific expression, chiefly because it was not suspected nor sought for in the larger aspects of terrestrial meteorology.

So long as we look only at the changing multitude of details of climate, in particular countries, and not at the condition of the globe as a whole, we naturally do not suspect violent variations in the sun's radiation.

But the moment we venture to depart from traditional teaching, and begin to think for ourselves, we discover abundant evidence of changes in the solar radiation which are sensible throughout the terrestrial globe. They may even prove fatal, through the effects of unusual heat and drought, to a considerable portion of mankind¹).

We shall therefore examine this question on its merits. No great problem could be more worthy of the meditation of philosophers who aim at the discovery of the laws of the universe.

From Plate Ia, *Sivashian's* Diagram of the Comet Family of Jupiter, 1893, we notice that the next opposition in 1931 along the line $\mathcal{F}_{1/2} S_3$, in Plate Ib, will be in a somewhat sparser region of comet orbits; whilst the conjunction of 1941 will be in heliocentric longitude $\lambda = 57^\circ$, a region much sparser in meteors, according to the indications of the cometary orbits. Hence we should expect the season of 1941 to be considerably less violently hot than 1921, which is a record breaker for violence of temperature, and disastrous droughts in nearly all the leading countries of the world.

The conjunction of 1961 will again become violent, from the downfall of meteors and the development of excessive heat; but in 1981 the effects will be even worse, — being a repetition of the record breaking heat of 1921. Yet the heat will occur in our winter, corresponding to summer in the southern hemisphere, as in 1861, when in fact the conjunction fell on Feb. 20, 1862. And as the southern terrestrial hemisphere is so largely covered with water, the effect of the heat upon the earth may be very much less important than in 1921, when the conjunction occurred in August, with the sun as directly as possible over the land hemisphere of the globe.

In 1802, July 26, there were conditions for great heat development similar to that of 1921. Sir *William Herschel* (Phil. Trans. 1801, pp. 265-318) was then studying solar effects upon the earth, and considered the general aspects of terrestrial meteorology to depend largely upon the sun's condition. »The influence of this eminent body« says *Herschel*, p. 265, »on the globe we inhabit, is so great, and so widely diffused, that it becomes almost a duty for us to study the operations which are carried on upon the solar surface«. »A constant observation of the sun with this view (i. e. determining the radiation) and a proper information respecting the general mildness or severity of the seasons, in all parts of the world, may bring this theory to perfection or refute it, if it be not well founded.«

Even today, after the lapse of 120 years, our data are very incomplete, yet may afford some indications of the truth.

¹) In the Literary Digest of Aug. 6, and Aug. 13, 1921, it is stated that no rain has fallen in the wheat belt of Russia, along the Wolga, from Samara to Perm, since March. After a drought of six months vegetation is parched, the ground cracked open to great depths, forest trees dying and millions of people fleeing west in the greatest distress ever known in Russian history.

The summer heat and drought of 1901 was so terrible that in large regions of the Mississippi valley a considerable fraction of the forest trees perished. The season of 1881 was similar, but perhaps less severe all over the globe. In 1901 the normal conjunction was due on Nov. 5, and in 1881 on Dec. 31. These hot waves therefore were nearer the winter season than in the season of 1921, where the conjunction occurred on August 22.

Russia is a very large and uniform country much like the Mississippi valley, and the Russian annals of 1891 make the famine from drought in that year very notable. The drought and heat in America was less pronounced, yet widespread, and severe. The year 1891, it will be noticed, corresponds to the opposition of Jupiter and Saturn $J_{\frac{1}{2}} S_1$, Plate 1b, when a very considerable meteoric downfall may have occurred.

The only observational record in partial conflict with this inference, is some observations by *Frost* at Potsdam in 1892 (cf. AN 3105-6, and *Astronomy & Astrophysics*, 11. 720). Yet it may be observed that *Frost's* work was directed chiefly to the determination of the absorption of the solar atmosphere, at the sun's limb, while the relative temperature of different parts of the disc was a secondary problem, and given only casual attention.

Accordingly, whilst the observations were held to indicate the practical uniformity of the heat radiation from all parts of the solar disc, we must accept such a conclusion with great reserve, both because the observations were insufficient, and because contrary to *Secchi's* careful work of 1852. Moreover, *Frost's* work was done nearly a year after the great Russian drought and famine of 1891, and thus somewhat late for detecting the effect of meteoric downpour a year or more previous to the observations.

A complete search of past records is much beyond the scope of the present paper. We can only say the indications point strongly to the theory here developed, and no contradiction of its conclusions is known, nor do we believe any can be established by the most painstaking and impartial research.

Thus we turn to the season of 1921, and ask ourselves for a summary of the evidence that the heat has been unusual.

1. There has been excessive heat throughout the interior and nearly all parts of the United States, and Canada, as well as Mexico. Recognition of this unusual torridity has found expression in a thousand ways; and it is remarked not only that the heat is great and unabated, but accompanied by conditions which preclude rain — so that great droughts have occurred. Thus the crops in Mexico are officially reported to be greatly injured by the excessive drought. What occurs in the tropical and semitropical land of Mexico, is true of the temperate zone of the interior United States and Canada. The great heat is everywhere associated with excessive and long-continued drought, — so that not only are corn and other crops greatly injured, — the cotton crop being the smallest since 1888 — but even large numbers of forest trees have been killed by the violence of the heat.

2. Up to August 1, England has had a drought of over 100 days, and the most terrible heat ever known. Attempts to produce rain about London by throwing great quantities of explosives high into the air, about the middle of July, failed to produce a drop of moisture. The *Manchester Guardian* of July 1, reports that the iron locks of Monument Bridge in the ship canals at Hull had become bound from the excessive heat, and that pouring water on them had failed to cool them enough to release the locks, so that the ships continue tied up and unable to move.

3. France has had this summer the most terrible heat recorded within half a century, with almost no rain for four or five months; so that all the French rivers are low and stagnant, and the ship canals unable to operate. At Paris the summer has been almost unbearable, so that the forests at Fontainebleau were dried out and have partly perished by accidental fires, while at Bordeaux the heat and drought have been equally bad. No appreciable rain has fallen for five months, and the vineyards, gardens and fields of the country are parched, so that the forests are everywhere endangered.

4. In Spain and Italy the conditions of extreme heat and dryness correspond to those in France. At Milan, Venice, Florence, the heat was extreme, while at Rome the summer heat has been so excessive that for the first time within the memory of the oldest inhabitant, relief against the torrid climate has had to be sought in discarding the coat at all dances given by the Roman aristocracy. The significance of this record is sufficient to show the extreme heat all over Italy.

5. In Germany the summer heat and dryness has been excessive. Reports show that whilst the early grain crops escaped without great injury, the late crops, such as potatoes, will be largely burnt up, and will result in a serious food shortage. The forest trees in the Tiergarten, the largest park in Berlin, are losing their leaves so rapidly under the drought, at the end of July, that already they have to be cleaned up and carried away; whereas such fall of the leaves usually does not occur till late in September or October. Thus even the forests of Germany, Switzerland, Bohemia, Austria, and Hungary have suffered badly before mid-summer. In the burning of a large hotel at Villars, Switzerland, Aug. 1, even the trees took fire and acted as torches for spreading the conflagration, so that they had to be felled by the troops. Such a record of drought throughout Europe is quite unprecedented since exact observations began to be made 300 years ago.

6. When we come to Russia, the story is even more distressing. The immense and fertile valley of the Wolga, from Samara to Perm, is burnt up by the heat, with long-continued and excessive drought; and in the province of Samara, millions of people, without reserves of food, or prospects for this year, have abandoned their homes and are fleeing in the hope of escaping starvation from lack of food and water. So great an exodus of whole peoples has not been known since the Middle Ages. Great alarm is felt in Germany, Poland and Rumania at the Russian migration. A wave of relief was felt in Central Europe when the United

States government took steps to relieve the distress of the fleeing Russian refugees ¹⁾.

7. In the case of China, Manchuria, and parts of Siberia, the story is similar. Millions of the Chinese and Northern Mongolians are fleeing from the excessive heat and drought which have laid waste their provinces. No such desperate migration has been observed in China since the Middle Ages; and it is impossible to foresee the extent of the affliction in vast areas, which usually are fertile and prosperous regions, supporting in comfort immense populations of industrious people. In fact reports from Shanghai show that the drought has greatly afflicted not only China, but the whole of Asia.

8. In India the intense heat and drought have done immense damage, and by raising the prices of wheat added to the unrest and agitation of the population which is of such grave concern to the authorities of the British Empire. What is true of India is equally true of Syria and Mesopotamia, and especially Siberia. In Japan, the grain, fruit and the rice fields have suffered greatly from the unusual torrid heat, so that the whole Orient has suffered from the terrible summer heat of 1921. The Literary Digest of Aug. 13 summarizes the Russian famine situation by the alarming heading: »The Third Horseman rides in Russia«.

Accordingly, it appears from this brief survey that whether we consider conditions in California — where prunes and other stone fruit are rotting from the torrid heat reflected from the pit in the core, or any other part of the world — the story is everywhere the same: an unprecedented state of heat and drought, which is not to be explained by local terrestrial conditions, but must be assigned to an elevation in the temperature of the equatorial regions of the sun itself.

There is no other possible explanation of an effect so worldwide ²⁾ and so unprecedented in the annals of the human race. And not a single objection to such an explanation can be offered. Science up to the present has no records to contradict an explanation sanctioned by our common sense, and by the consensus of the climatic evidence of the globe.

A sensible secular acceleration of the earth has been recognized as probable for about 15 years; and in 1909 I showed that the fall of meteors upon the sun is the only possible explanation of the secular acceleration of the earth indicated by the researches of astronomers on ancient eclipses of the sun. The amount of matter thus precipitated upon the sun may be double that of our lunar mass in a

century. The chances are infinity to one that it does not fall steadily, but comes down in gusts, — at the time of the conjunctions and oppositions of Jupiter and Saturn, as we have explained above. Accordingly, the theory here developed accords with all known phenomena, and not a single legitimate objection to it can be raised.

The Physical Cause of the Persistence of Excessive Droughts, as in 1921.

The unprecedented drought of 1921 is of world-wide character, profoundly affecting all the principal nations of the northern terrestrial hemisphere; and thus it is well to inquire into the physical cause of this condition, and its long persistence over the larger part of the terrestrial globe. The cause can not be local, but is evidently general throughout the length and breadth of the earth; yet it depends not on ordinary influences, — but on a hidden physical law of such a kind as not to be suspected by meteorologists, and thus apparently it has escaped detection by investigators.

1. We have seen that there is every reason to believe that the heat of the sun has been suddenly increased by the meteors falling upon that globe, under the combined precipitative actions of Jupiter and Saturn. Let us assume as probable such an increase in the solar radiation; and then consider what will follow. Is there any physical law of the molecular motions of the gases of the atmosphere not heretofore suspected to exist?

2. When the weather is very hot, a very large percentage of the water vapor in the atmosphere, say $1/n^{\text{th}}$ part — where n may equal any number as 3, 4, 5, 6, 7, and so on, — is separated out from the lower layers of the atmosphere and driven to regions of the air higher than usual, by virtue of the great heat and the smaller molecular weight of the water vapor, and its 1.27 times greater molecular velocity.

3. For the air has molecular weight as follows:

		parts
Nitrogen	14	3
Oxygen	16	1

Mean molecular wt. = 29.

In the case of water vapor we have for the vapor of H₂O:

		parts
Hydrogen	1	2
Oxygen	16	1

Molecular wt. = 18.

¹⁾ A press report from London by Sir *Hall Caine*, Sept. 13, 1921, shows that the above description of the disaster in Russia is not overdrawn. Sir *Hall* quotes the British Prime Minister's statement, made in August, in the House of Commons: »In the Russian famine we are witnessing the most terrible devastation which has afflicted the world for centuries«; and himself goes further, declaring, from the evidence furnished by distinguished Russians of all parties in various countries of Europe, that the calamity is the worst which has afflicted the human family since the Flood.

Only a part of the calamity, however, is due to the drought — the rest being due to the war, revolution and subsequent demoralization.

Sir *Hall Caine*, who personally witnessed the famine and cholera in Russia in 1891, exclaims: »Yet what was the famine of 1891 compared with the present catastrophe? My Russian friends tell me the population affected is not fewer than 40000000, and that from 15000000 to 20000000 of these are homeless, friendless and foodless, and are on the roads«. (Note added Sept. 13, 1921).

²⁾ The description of the drought in Russia, Literary Digest of Aug. 6, says that the drought is word-wide, and thus not confined to Russia, though there attaining the most disastrous proportions. A press dispatch from Spitzbergen, Aug. 5, 1921, says: »The heat wave at last has struck Lapland and the Polar regions. With the thermometer at 86 degrees (Fahrenheit), the Eskimos have thrown away their fur garments and are organizing bathing parties.«

»For most of the Eskimos, this bath will be the event of a lifetime, perhaps never to be repeated. The game and reindeers are suffering terribly.«

T. J. J. See. Cause of the Sunspots and their Periodicity.

WOLF'S Sunspot Frequency Curves Extended and Theoretically Modified for Great Sunspot Saros in 88.5 Years. T.J.J.-
R. Wolf's original frequency curves of sunspots and sunspot frequency curves, extended and theoretically modified for the Great Sunspot Saros in 88.9 years by T. J. J. See.

4. It thus appears that water vapor is 11 units less heavy than air (29), which is a difference of 38 percent. And as the mean molecular velocity is governed by the molecular weight, according to the formula of Maxwell:

$$\frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_2 v_2^2 \quad (51)$$

or

$$9v_1^2 = (14.5)v_2^2 \quad (52)$$

we find

$$v_1 = \sqrt{(14.5/9)} \cdot v_2 = 1.27v_2 \quad (53)$$

5. Accordingly, as the molecular velocity of water vapor is 1.27 times that of the air, the water vapor, under intense and continued heat, tends to rise above the air of corresponding temperature and pressure. Hence the water vapor forms a relatively greater part of the upper part of the higher atmosphere, while in the lower atmosphere there is a deficiency of moisture, due to the heat driving out the swifter molecules of water vapor.

6. Now the lower air contains most of the dust particles, which by acting as nuclei of condensation, may develop drops, and give rise to rain. And when the weather is very hot, with the water vapor tending in greater percentage upward, a difficulty arises in the saturation of the atmosphere. Thus the formation of rain is increasingly difficult.

7. Under the great and persistent heat of a drought, upon a large inland area, the water vapor passes more and more to a level relatively free from dust; and even if a lowering of temperature occurred at this height, the droplets, on descending to lower level, would again vaporize, and again ascend to greater height. Hence the cirrus clouds so prevalent in dry seasons!

8. This explanation appears to afford a solution of the increased difficulty in developing rain during prolonged droughts. It has long been a saying that the dryer it gets, the more difficult it is to rain; so that dryness adds to the prolongation of the drought, and the spell is not broken until there is change of temperature and shift of air currents with the season, in the larger world movements of the atmosphere.

9. Accordingly, the present (Aug., 1921) world-wide drought may last till late in the summer or autumn. And all the phenomena would indicate that the abnormal solar radiation had thrown the terrestrial atmospheric elements

out of equilibrium early in the season of 1921. And there seems no possibility of general relief till the whole atmosphere is stirred up by the larger current movements incident to the sun's rapid change in declination, about the autumnal equinox.

10. We have examined many treatises on meteorology¹⁾, and consulted several eminent meteorologists, without finding any record of the above separation process as applied to the lighter vapor of water. It depends on the physical laws of molecular diffusion, and apparently has not been studied in dealing with terrestrial meteorology. The cause at work is definite, and well defined, but would produce a notable effect only under persistent high temperature; and thus the effects would become notable chiefly in the summer with the sun at maximum elevation.

11. If in addition to summer altitude, the radiative power of the sun be increased, as by meteors falling into it and developing an immense equatorial extension of the corona, under the actions of Jupiter and Saturn, then the disturbance of the rainmaking power of the world would be enormous. This cause appears to be adequate to account for the unprecedented drought of 1921; and so far as we know no other cause can be assigned, which is at all adequate to account for so great a disturbance in the normal equilibrium of the terrestrial meteorological elements.

12. It is true that the cause here assigned depends on unequal molecular diffusion; but this is a definite cause operating on an immense scale²⁾, in the gigantic laboratory of the earth's atmosphere, under direct solar radiation. To deny an accumulative effect of such a process is to deny well known physical laws of molecular diffusion. And to ignore the drought as due to unusual causes is to deny the plainest evidence of our senses, and the most authentic reports of the recent extreme desiccation of the larger part of the terrestrial globe.

II. The Observed Periods, Distribution, Nature and Cause of Sunspots.

(i) The sunspot periods found by *Wolf* and later investigators.

In the *Memoirs R. A. S.* 43, 1875, *Wolf* gives the following useful table, to which the later periods have been added from the best available authorities.

¹⁾ Since the above discussion was written I have found that in his *Meteorology*, 1861, (p. 51-52), Sir *John Herschel* takes account of the diffusive power of water vapor which he says is lighter than air in the ratio of 0.6235 to 1. "It is the lightest of all known vapours", and, with exception of hydrogen and ammonia, the lightest of gases. In consequence, as soon as generated, it tends to rise in the air by its buoyancy, and in so doing, carries up with it much of the air with which it is intermixed, disengaging itself no doubt from it, in its upward progress, to become entangled, however, with fresh particles, which again it carries upward, to abandon them for others. In this way, not only is its upward diffusion far more rapid than its horizontal, but in its struggle upwards it tends to produce an ascensional movement in the air itself, and thus to act as a powerful agent in the production of winds.

²⁾ In his *Elements of Meteorology*, 1902, p. 143, Prof. *W. M. Davis* gives a table of pressures, weights of vapor and saturated air:

Temp.	Vapor Pressure mm	Vapor weight cu. met. grams	Sat. Air weight cu. met. kilogr.
-30° C.	0.38	0.44	1.45
0	4.57	4.87	1.30
+30	31.51	30.08	1.15
+40	54.87	50.67	1.11

Davis adds: "It is important to notice that the increase of capacity is much faster at high temperatures than at low temperatures", — which would make the higher air able to hold but a small amount of vapor, owing to its low temperature. Yet as the amount of the upper air is indefinitely great, I do not doubt the escape of water vapor into it, as shown also by the cirrus clouds so prevalent in droughts, yet yielding no rain.

Première Série		Seconde Série			
Minima	Maxima	Minima	Maxima	Minima	Maxima
1610.8		1615.5	10.5	1745.0	11.0
1619.0	8.2	1626.0		1755.2	10.2
1634.0	15.0	1639.5	13.5	1766.5	11.3
1645.0	11.0	1649.0	9.5	1775.5	9.0
1655.0	10.0	1660.0	11.0	1784.7	9.2
1666.0	11.0	1675.0	15.0	1798.3	13.6
1679.5	13.5	1685.0	10.0	1810.6	12.3
1689.5	10.0	1693.0	8.0	1823.3	12.7
1698.0	8.5	1705.5	12.5	1833.9	10.6
1712.0	14.0	1718.2	12.7	1843.5	9.6
1723.5	11.5	1727.5	9.3	1856.0	12.5
1734.0	10.5	1738.7	11.2	1867.2	11.2
11.20 ± 2.11		11.20 ± 2.06		11.11 ± 1.54	
± 0.64		± 0.63		10.94 ± 2.52	
				± 0.76	

Minima	Diff.	Maxima	Diff.
1878.0	10.8	1870.6	
1889.5	11.5	1883.8	13.2
1901.5	12.0	1893.0	9.2
1912.8	11.3	1905.0	12.0
(1923.0)	(10.2)	1917.6	12.6

It will be seen that the periods vary from 8.0 to 15 or 16 years, and that frequently there are associated together two periods of about 8 years each, 9 years each, or 10 years each, while if we turn to periods longer than the 11.18 year cycle, we frequently find two periods of 15-13, 14-12, 13-11 years, which must be considered very favorable to the Jupiter and Saturn meteoric theory above outlined. Owing to the variation of the meteoric density σ_{μ} in our formulae, we could obtain any desired length by admissible changes in this constant, but we have preferred not to attempt any minute agreement in the present state of this subject.

(ii) The distribution of the sunspots in latitude shows that they do not depend directly on the downpour of meteors. *Spoerer's* law of decreasing latitude explained.

In section 9 above we have cited the cyclonic theory of sunspots developed by Sir *John Herschel*, 1847, and we again invite attention to *Herschel's* reasoning.

The distribution of sunspots, in two chief zones on either side of the equator, as indicated in the following figure, shows that the spots do not depend directly upon the downfall of meteors. The spot development must be an indirect effect; for if they were a direct effect they ought to be more prevalent at the solar equator than anywhere else, which is contrary to observation.

This diagram shows the distribution of 1386 spots, distributed in heliographic latitude as shown on the left. The part of the figure on the right relates to the protuberances, which are seen to be more frequent near the equatorial regions.

It is a leading doctrine of the meteoric theory as here developed that with the downfall of the meteors chiefly at the equator, the increase of heat is developed there.

Fig. 11. Carrington's illustration of the distribution of sunspots in heliographic latitude.

1. Hence it follows that the equator would be under the greatest radiation pressure, and this should give great prominence to the expulsion of dust for building up the corona in the equatorial regions, about the time of sunspot minimum.

2. It is well known that the corona is most widely extended in the equatorial parts of the sun's disc, usually with a prong on either side of the equator, as if there were some differential pressure towards the poles, yet the outer parts of the corona are arranged as if rendered luminous by the sun's magnetic field with wave-rotations along the lines of force from the poles.

3. If the heated equatorial region be not very wide, this differential effect would arise as the fine dust is carried upward under the increased radiation pressure of the heat and light at the equator.

4. Since the dust driven away from the solar surface thus would have an under drift towards the poles, as it falls back to the sun, there would be two consequences:

(a) As the cooling matter descended on the solar surface in higher latitude, the internal hydrostatic equilibrium of the solar globe would develop a slight surface flow from the higher to the lower latitudes. For if the most matter is expelled under the radiation pressure in the equatorial regions the deficiency naturally would be made good by a surface drift towards the equatorial regions, as in the celebrated law discovered by *Spoerer*.

(b) The corona and protuberances would be relatively prominent in the equatorial regions of the sun, owing to the superior temperature and radiation pressure operative in that region, near sunspot minima, when most of the meteors are falling, and for some time afterwards.

5. *Spoerer's* law is illustrated by the following double figure from *Young's* work on the sun, 1902, p. 157, in which the heavy lines represent *Spoerer's* law of spot drift from higher to lower latitude, while the dotted line represents *Wolf's* sunspot curves, 1855-1880.

Fig. 12. Illustration of *Spoerer's* law of drift of spots from high to low heliographic latitude with the progress of the sunspot cycle.

6. It will be seen that the drift of the spots in latitude found by *Spoerer* corresponds exactly with the theory above outlined. The theory would thus appear to have vastly increased probability; for *Young* says that *Spoerer's* law is of »great theoretical importance«, yet so far as we are aware no previous explanation of it has been given, while the explanation based on the meteoric downpour appears to be perfect.

(iii) The tendency of the spots to form vortices recognized by *Secchi*, 1857, and *Faye*, 1877; and now explained by the meteoric theory.

As far back as the fifties and sixties of last century *Secchi* was a diligent student of the sun, and at this early date he noticed the tendencies of the spots to have a vortical or whirling motion. *Secchi's* drawings of vortical sunspots found their way into many handbooks of astronomy during the latter half of the 19th century.

At a somewhat later period *Faye* became a great student of the sun, and he too recognized the vortical tendencies in the spots on either side of the solar equator. In a special article on the sun, *Newcomb's* Popular Astronomy, 1878, p. 281, *Faye* says:

»These whirlpools, which tend to equalize the difference of velocity just spoken of (between the equator and higher latitudes), follow the currents of the photosphere in the same way that whirlpools and whirlwinds, tornadoes, and cyclones of our atmosphere follow the upper currents in which they originate. Like these they are descending, as I have proved (against the meteorologists) by a special study of these terrestrial phenomena. They carry down into the depths of the solar mass the cooler materials of the upper layers, formed principally of hydrogen, and thus produce in their centre a decided extinction of light and heat as long as the gyratory movement continues. Finally, the hydrogen set free at the base of the whirlpool becomes reheated at this great depth, and rises up tumultuously around the whirlpool, forming irregular jets which appear above the chromosphere. These jets constitute the protuberances.«

»The whirlpools of the sun, like those on the earth, are of all dimensions, from the scarcely visible pores to the enormous spots which we see from time to time. They have, like those of the earth, a marked tendency first to increase, and then to break up, and thus form a row of spots extending along the same parallel.«

Not very much more than *Faye* has here indicated is known about the vortical character of the spots today, though

the use of photography gives us accurate records which may be preserved for permanent study, and therefore are valuable.

The accompanying Plate 9, 3a gives *Secchi's* drawing of the great rotating spot of April and May, 1858; and by comparison with 3b, a photograph of a solar vortex recently taken at the Mt. Wilson Solar Observatory, under the direction of Prof. *Hale*, we see that the progress during the past 60 years is not so great as many persons have supposed.

By this short sketch it appears that although the detailed study of solar spots has enormously increased of late years, the students of the sun have not been able to enrich science by a general improvement of theory, by which we may understand clearly the scheme of the sun's activity.

Already in 1878 a similar remark was made by *Newcomb*, Popular Astronomy, p. 257, where he says:

»It is remarkable that modern science has shown us more mysteries in the sun than it has explained: so that we find ourselves farther than before from a satisfactory explanation of solar phenomena.«

Such a conclusion still applies, after 43 years of modern photographic effort. For although a vast mass of details have been accumulated, no hope heretofore has been held out of our being able to interpret them into a connected whole. It is for these reasons that we outline the simple consequences of the meteoric theory.

Nearly all the phenomena of the sun are shown to be related, and to follow from a few simple principles. A new theory of the sun's activity is an urgent desideratum of science, but in the present paper we merely sketch briefly the foundations of such a theory.

(iv) Calculation of the heat due to colliding meteors at the solar equator.

It is easy to calculate the heat due to collision of meteors against the solar surface. If m be the mass of the meteor, and v its velocity, which we may take to be the parabolic velocity, the energy of the collision will be:

$$E = \frac{1}{2}mv^2 \quad (54)$$

But to get the exact effect we have to deduct the effect of the sun's rotation, with equatorial velocity $v_0 = 2$ km per sec. Now if we take the parabolic velocity of the colliding meteor to be $v = 617$ km per second (cf. AN 3992, p. 136), the energy imparted to the surface of the sun's equator becomes,

$$E = \frac{1}{2}m(v^2 - v_0^2) = \frac{1}{2}m(380689 - 4). \quad (55)$$

And thus we see at once that a very small mass may impart relatively great energy to the sun's equator, because of the enormous velocity of the impinging meteors. Nearly all the orbits of the meteors have direct motion, like the planets by which they are precipitated, and thus the collisions are predominantly tangential.

We have seen that the sun's mass is about 27 000 000 times that of our moon, yet twice this amount of matter may fall into the sun in a century, or 9 sunspot cycles. Hence in a single sunspot cycle the mass of the meteors may exceed a fifth of the moon's mass, or 1 : 112 500 000th of the sun's mass.

As this is a very small fraction of the sun's mass, we must not expect it to appreciably accelerate the sun's rotation. But it may easily prevent the extinction of the observed acceleration of the outer layer of the equatorial region with surface drift like that known to exist.

As the equator is given an increase of momentum proportional to the meteoric downfall:

$$dH = m/M \cdot (v - v_0) = m/M \cdot (617 - 2) \quad (56)$$

we see that the equatorial layer tends to be accelerated by over 600 times its present velocity. Now as the falling mass m is small, compared to the total solar mass M , the only effect is to accelerate the surface layer.

From our researches on the motion of rivers, we know that the current motion is deepest in the centre, and dies out towards the banks, owing to friction. In the same way we may be sure that the acceleration or current in the solar surface is deepest and swiftest at the equator, and dies down gradually on either side. But in any case the accelerating current at the sun's equator is not of any considerable depth, owing to the tremendous increase of pressure as we descend into the sun (cf. AN 4053, 4104, 4152, and *Researches on the Evolution of the Stellar Systems*, vol. II, 1910, pp. 451, 541).

Accordingly, as there is elevation of temperature at the solar equator, and greater relative motion, we see that an effort at adjustment always is in progress. The vortical motion of the spots, with clockwise rotation in the southern hemisphere, and counter-clockwise rotation in the northern hemisphere therefore arises, and the explanation of it presents no difficulty of any kind.

The whirling of the spots originates from this differential motion, and the spots themselves arise from the descent of cooler matter, originally driven out from the equatorial heat zone, and tending to fall back in higher latitude. Hence the spots obey *Spoerer's* law of development, as before remarked.

But as the elevation of the sun's temperature at the equator under meteoric downpour is gradual, and the downpour decreases only gradually, the adjustment between the equatorial and polar regions is also gradual and may extend over several years. The heating up is comparatively rapid, but the cooling down more leisurely, while the drift under *Spoerer's* law is slow, owing to the feebleness of the forces under which adjustment is gradually accomplished.

12. The Periodicity of the Coronal Extension naturally conforms to the Sunspot Cycle.

(i) The variability of the coronal extension.

Within the last 40 years it has been established by the observation of total eclipses of the sun, that the corona is not always of the same form and extent, but that the appearance of the corona is variable with the sunspot cycle. It would thus not be difficult to trace the details of the growth of this doctrine since 1870, but we may refer the reader to the account in Miss *Clerke's* *History of Astronomy*, 1902, pp. 174-176.

The theory of a variable form for the corona has come into use gradually, and has at last been concurred

in by nearly all investigators, though our photographic data in proof of the fact is based upon the scant records of the few moments of total solar eclipses.

The fact appears to be fairly well established that the extent of the corona is variable, with the sunspot cycle. It is found that at the minimum the corona has the longest equatorial extensions, and when the spots are numerous the corona appears to be most fully developed above the spot zones, offering to the eye a rudely quadrilateral contour. On each side of the equator there are great luminous sheaves of the coronal light, curving together, and away from the pole on the outside, as if arranged somewhat along the lines of the sun's magnetic field. No doubt the magnetic field and the electrical forces operating about the sun have a good deal to do in arranging the visibility of the lines of the corona; for the polar streamers are distinct and follow the lines of force of a spherical magnet, as I have proved by careful comparison.

As far back as the Colorado eclipse of 1878, Prof. *Cleveland Abbe* reached the conclusion that the coronal light was due to streams of meteors rushing towards or from perihelion, and many others concur in the view that the solar neighborhood is crowded by swarms of such small bodies.

But since *Arrhenius'* study of the solar corona, *Lick Observatory Bulletin*, Nr. 58, 1904, it has been held that under light pressure the fine matter of the corona is actually expelled from the sun itself. Some of the fine dust driven away by radiation pressure falls back, while other parts of it float quietly in the coronal field, and still other parts of the dust are so fine as to recede to great distance or escape from the sun's control entirely.

Accordingly, we concur in the general opinion that the coronal matter is supplied from the solar surface by the varying pressure of the sun's radiation; and as the temperature of the equatorial regions exceeds that of the polar regions during the maximum of the meteoric downpour, which corresponds to the minimum of spots, there is a variable state of the corona, depending on the sunspot or meteoric cycle. In the early period, when the meteors are falling thickest, and later as the spots are being generated, the equatorial extension of the corona is notable. In the later periods of spot decline, just before the rapid growth of meteoric downfall, the outline figure of the corona appears to be nearly circular. The details of this theory have yet to be studied, but it conforms generally to the existing state of our knowledge, and is believed to rest on a substantial foundation.

(ii) 'Magnetic storms' or the variation of terrestrial magnetism, with the spot development, due to the magnetic areas about the spots being then most effective.

Ever since *Lamont* (*Annalen der Phys.* 84.580) discovered the change in the range of magnetic variation in $10\frac{1}{8}$ years, 1851, it has been recognized that the variation in the amplitude of the magnetic declination follows the curve of *Wolf's* sunspot cycle so closely as to show that the two phenomena — sunspots and magnetic storms — are immediately and directly related.

The question is: How and why are the magnetic storms related to spot development? We may answer this as follows:

1. In the Bulletin de la Soc. Astr. de France, Nov., 1918, I have explained the magnetic effect by supposing the surface openings of the spots to allow the underlying magnetic waves to escape more easily than usual. Thus when the surface is agitated by spot development, we have abundance of 'magnetic storms', and vice versa.

2. But if the spots be really due to the descent of cooler matter upon the solar surface as is implied in the theory now set forth — the zones on either side of the equator being fixed by the higher temperature at the equator, with the corresponding expulsion of dust and drift towards the polar regions, with the beginning of spot growth in higher latitude, as indicated by *Spoerer's* law — then it will follow that, as the cooler matter settles upon the photospheric surface, it looks dark by contrast, gives out less heat and light, and thus takes on more fully magnetic properties appropriate to the sun as a whole, because the iron and other vapors in the spots are cooled enough to become magnetic. Under this view the spot areas are simply better radiators of the magnetic waves. At present either view seems admissible, but time must decide which is preferable.

Accordingly, there is no difficulty in accounting for the growth of 'magnetic storms' with the progress of the spot development; yet certain discriminations are needed before we can fix on the theory of the magnetism of spots which is most admissible. This problem is difficult, yet soluble by separating the secondary effects of spot growth from the causes out of which it grows.

In 1877 *Faye* called attention to the frequently observed outbursts of hydrogen protuberances about spots as due to this volatile element being carried down into the nucleus, where the temperature is high, — whence a rapid expansion, with outburst of hydrogen, calcium and other protuberances occur. The details of spot operations thus admit of satisfactory explanation, and it is believed that no serious difficulty remains outstanding. This is remarkable in view of the bewilderment heretofore prevalent, and increasing with the accumulation of the vast masses of solar data now available.

(iii) Remarks on the incompleteness of *Sivaslian's* diagram for the dependence of the sunspot cycle, with coronal fluctuation, on the meteors connected with Jupiter and Saturn.

It is important to inquire why the sunspot cycle depends on a composition between the sidereal revolution of Jupiter in 11.86172 years and the retrograde revolution of the conjunction-opposition line with Saturn in 9.92945 years. The best answer to this inquiry is the preceding diagram of Jupiter's family of comets, made by *A. G. Sivaslian* in 1893.

This diagram is not complete, as was supposed by *Sivaslian*, in 1893, but today has to be supplemented by the orbits of the following comets, and several others.

	Period.
1896 V (Giacobini)	9.00 years
1896 VII (Perrine)	6.67
1905 II (Borrelly)	7.30
1906 IV (Kopff)	6.67
1906 VI (Metcalf)	6.89

Accordingly, it follows that *Sivaslian's* diagram of 1893 is incomplete, yet the relation of Jupiter to his family of comets is sufficiently developed to show that the comets are quite unsymmetrically distributed about the Jovian orbit. Most of the orbits are gathered together and symmetrically arranged about the Jovian Aphelion, whereas about the perihelion, in longitude 12° , almost no orbits will be found. This remarkable asymmetry of distribution is not yet explained, and it may be a long time before we know the cause on which it depends. But the fact is certain, and moreover, adding a few more orbits to the diagram will not change the distribution of the cometary paths, to alter the dependence on Jupiter's position in his orbit, or the sidereal revolution.

The above figure, Plate 1, will sufficiently illustrate the theory of a meteoric swarm under the action of Jupiter and Saturn. We must suppose Jupiter's orbit everywhere crossed by meteor paths, just as in the well known case of periodic comets. The swarms on the average revolve direct, in a period of from 4.5 to 4.8 years. Saturn's meteoric swarms may have a period of 11.125 years, as imagined by *Schuster*. The reaction of Jupiter upon his own meteors, and upon those connected with Saturn's orbit will give relative impulses in periods of 9.92945 years, and in 11.86172 years respectively, which may be compounded as before described.

All kinds of detailed transformations of the meteor paths will result. We are concerned here only with average effects, and dynamically these evidently will attain maxima in periods depending on combinations of 9.92945 years and 11.86172 years, which latter is the period of Jupiter's sidereal revolution. As Saturn's motion is retrograde relatively to Jupiter, the longer period will be shortened to an intermediate period, between 9.92945 and 11.86172 years, yielding an adjustment in accordance with the principle of least action.

But although the maximum dynamical impulse occurs at conjunctions and oppositions of Jupiter and Saturn, and the outburst of solar heat should then take place, — as in 1881, 1901, 1921, which were years of terrific heat — yet the sunspots, being an indirect effect of the equatorial acceleration and the heat then given to the sun, may not attain maximum prominence till 4 or 5 years have elapsed in the cooling and adjustment of the sun's surface layers in different latitudes. We shall not go into this problem in more detail at present.

We now observe from Plate 1 that if the conjunction of 1921, Aug. 22.57756 G. M. T., the heliocentric longitude $\lambda = 177^\circ$, be taken as the initial position $S_1 \mathcal{F}_1$, in our figures, it will be evident we are now in the densest cluster of comet orbits and meteor swarms. Hence we should expect a tremendous increase recently in the precipitation of meteors upon the sun. Is this the source of the terrible heat developed during the summer of 1921 in all the leading countries of the world? The recent weather reports accord with this view, and thus emphatically support the theory of meteoric downfall; but we have already examined the weather reports somewhat fully in section 10 above, and need not here extend the discussion.

1. Hence it follows that just as the arrangement of the cometary orbits depends on the sidereal revolution of Jupiter, so also the meteor swarms shown to be produced by the disintegration of comets must also depend on the sidereal revolution of this great planet.

2. We do not know the reason for this dependence on the sidereal position, but from the observed positions of the cometary orbits, the fact appears to be certain. Nothing could be more clearly demonstrated by the above diagrams.

3. It is to be noticed also that as Jupiter has very few comet orbits near his perihelion, in longitude 12° , but very many near his aphelion, a presumption exists that a similar law of distribution will hold for the paths of the meteor swarms, which are invisible.

4. Accordingly, when Jupiter is in conjunction with Saturn, near aphelion, Aug. 22, 1921, there ought to be great derangement of the meteor swarm paths by the mutual actions of these two great planets. If this throws down swarms of meteors upon the sun, there ought to be maximum development of heat, as during the terribly hot summer of 1921.

5. The details of the mutual actions of these great planets upon the meteor swarms is sufficiently illustrated by the Plate 1. Jupiter will so shorten the radius vector of the meteors belonging to the Saturnian system as to bring billions of them into collision with the sun; and Saturn will react correspondingly on the Jovian swarms. Hence a maximum downpour of meteors may be expected.

6. The conjunction of 1901-1961 in longitude 297° is less favorably situated for the downpour of meteors, as we see by comparing the two diagrams. The conjunction of 1881-1941 in longitude 57° , will be still less favorably situated; and we should not expect such terrific precipitation of meteors upon the sun at these conjunctions.

These diagrams facilitate the study of the dependence of the meteor swarms upon Jupiter's sidereal revolution. And as the conjunction-opposition line for Jupiter and Saturn revolves retrograde, in a period of 9.92945 years, while Jupiter revolves direct in a period of 11.86172 years, we see that the sunspot cycle, if dependent on the interactions of these great planets, should recur in less than 11.86 years, but have a period greater than 9.93 years.

It is shown in section 4 above that the combination of these periods leads to the average value of 11.18 years, in exact agreement with the observed duration of the sunspot cycle, 11.18 years. Moreover, the periods will be irregular, varying from about 8.0 years to 14.0 years, in the way we have already explained.

It is difficult to imagine a more satisfactory proof than that here adduced of the cause of the sunspot cycle, and its mysterious variation. The exact agreement as to periods here given, and their direct deduction from the retrograding motion of the conjunction-opposition line in respect to Jupiter's sidereal revolution leaves nothing to be desired.

(iv) The previous researches on meteor swarms by Sir *John Herschel*, *Lockyer*, *Schuster* and *Turner* did not lead to the observed period of the sunspot cycle, 11.18 years.

1. It is well known that Sir *John Herschel* was the first to suggest that the sunspot periodicity depends on the downfall of meteors upon the solar surface. (Outlines of Astronomy, 10th ed., 1869, Arts. 900-905a). It does not appear that *Herschel* worked out the theory in any considerable detail, but rather adopted Sir *W. Thompson's* theory of meteoric matter falling into the sun to keep up its heat.

When this theory was revived by Sir *Norman Lockyer*, the objection dating from *Herschel's* time, seemed more formidable than at first seemed probable. Meteors falling on the solar surface, for example, would not be distributed with especial density in northern and southern heliographic latitude where the maxima of the spot zones are located. Accordingly, certain fundamental difficulties of the meteoric theory were not overcome by *Herschel* or by *Lockyer*; and the difficulty still persists, unless overcome by the meteoric theory of the present writer, which we believe will be found to be true.

2. In later times the problems of sunspot periodicity has been extensively studied by *Schuster* and *Turner*¹⁾; but in spite of their extensive labors it can hardly be said that any satisfactory conclusion was arrived at.

Newcomb's conclusion that there are two periods of $4.62 + 6.51$ years = 11.13 years is also far from convincing. He holds that these are the uniform cycles underlying the periodic variation of sunspot activity; but is so bewildered by the cause involved that he adds: "Whether the cause of this cycle is to be sought in something external to the sun; or within it . . . , we have at present no way of deciding."

Now to ascertain what would happen under the mutual action of the two planets, we consider as before first the *Hill-surfaces* about the planet and the sun, and the rotating orbits resulting therefrom. It will not be difficult to infer the shifting of orbits which will take place when the orbits of the particles are not periodic and do not rotate with Jupiter and Saturn, but extend from either planet to perihelia near the sun.

(a) When the orbits are rotating, like the *Hill-surfaces*, which either planet carries about itself and the sun, it is evident that just as the surfaces may be superposed in conjunction or opposition, so also may the orbits be superposed. The outcome of this superposition is indicated by the figures.

(b) We conclude that in conjunction Saturn operates to decrease the perihelion distance of many particles near the sun, mainly within the Jovian control; and likewise, Jupiter operates to increase the perihelion distance of many particles mainly under the control of Saturn. In opposition the perturbative tendencies at work are exactly the reverse, but other particles in paths normal to the planetary line are precipitated upon the sun. If the two planetary orbits were exactly circular the two tendencies would be equal, for equal planetary actions; and the sun will have about equal masses of meteorites precipitated upon its equatorial regions, in periods less than 11.86 years, but greater than 9.93 years.

¹⁾ MN 64.543 (1904).

(c) *Herschel's* table of the effects of the tangential disturbing force.

Direction of Motion in Orbit	Situation in Orbit	Action of Tangential Force	Eccentricity
Indifferent	About Aphelion	Accelerating P	Decreases
»	»	Retarding P	Increases
»	»	Accelerating P	Increases
»	»	Retarding P	Decreases

Effects of the normal disturbing force.

Direction of Motion in Orbit	Situation in Orbit	Action of Normal Force	Eccentricity
Approaching S	Anywhere	Inwards	Increases
»	»	Outwards	Decreases
Receding from S	»	Inwards	Decreases
»	»	Outwards	Increases

3. Passing therefore to the non-periodic orbits traversed by swarms of meteorites, such as may be assumed to have accompanied *Lexell's* comet of 1770, in a period of $5\frac{1}{2}$ years, we conclude:

(a) The Jovian meteorites might revolve in periods varying from 4.2 years to 5.3 years, the average period for such masses in orbits smaller than that of *Lexell's* comet being about 4.76 years, like the periodicity found by *Schuster* for the sunspots, Phil. Trans. 1906.

(b) The meteoric swarms depending on Saturn would revolve in periods varying from about 10.4 years to 11.85 years, — the latter period corresponding to a limit beyond Saturn's orbit. This would give an average period of 11.125 years, as concluded by *Schuster* from observed sunspots by periodogram methods, Phil. Trans. 1906.

(c) *Schuster* also inferred a periodicity of 8.344 years, which corresponds very nearly to the mid-region between Jupiter and Saturn, $a = 8.231$, where many meteor swarms may be collected by an action of Saturn, analogous to that of Jupiter in gathering the asteroids within his orbit.

Thus Saturn's mean distance is 9.54, and Jupiter's 5.20, giving a central zone at 7.37; but the above zone for *Schuster's* periodicity 8.344 years, $2a = 8.231$, is a little further out, 1.31 units within the orbit of Saturn. Now it is known that Thule, one of the remotest asteroids, has a distance 4.30, and thus lies 0.90 unit within Jupiter's orbit. The situation of a zone of cosmical dust 1.3 within the orbit of Saturn may therefore be safely assumed from the established arrangement of the asteroids within Jupiter's orbit. The periodicities found by *Schuster*, namely:

$\frac{1}{3} \cdot 33.375 = 11.125$ years, $2a = 9.9672$ units, Saturn = 9.54
 $\frac{1}{4} \cdot 33.375 = 8.344$ » $2a = 8.231$ units
 $\frac{1}{7} \cdot 33.375 = 4.768$ » $2a = 5.664$ units, Jupiter = 5.20
 appear to correspond to the three zones here considered, namely the path of Saturn, the stable zone within Saturn's orbit, and to the orbit of Jupiter.

Now the orbits of Jupiter and Saturn are zones of perturbations, owing to the actions of these planets, while the stable zone between is a catch-basin, for the dust of our system in process of transformation. The particles are hurled into and away from the orbits, and also into and

away from the zone between them. These three zones are therefore the ones which should precipitate meteoric showers onto the sun's equatorial region, to produce the secular acceleration, and disturb the atmosphere so as to form clouds or spots in the readjustment between the equatorial and polar regions.

Summary and Conclusion.

We have seen how difficult, if not impossible, it has been to make progress in solar and stellar physics under the old theories handed down for the last half century. Thus already in 1878 *Newcomb* could say that the sun had developed a great many more problems than the solar physicists had been able to solve. This criticism of solar physics still holds today, in spite of the vast masses of data recently accumulated by photographic and spectroscopic research. The investigators literally are lost in the multitudinous and confusing character of their data: they are in the unfortunate position of the explorer who could not see the forest on account of the trees — because all general and collected view of nature is lost sight of.

The vast importance of the unsolved problems of solar and stellar physics may be inferred from the number of observatories recently devoted to the study of the sun. This number amounts to about one half of the larger observatories of the world. And in 1904 there was founded near Pasadena, California, the Mt. Wilson Solar Observatory of the Carnegie Institution of Washington, on which now at least two million dollars have been spent, and of which the maintainance involves an annual expenditure of \$186 000 — three times that of any other large observatory on the globe.

Such expensive equipment, with vast annual outlay of funds for the maintainance of solar and stellar research, is ample testimony to the importance of the problems treated of in this paper. It may be noted that the directors of these observatories have dwelt upon the importance of cooperation, in the solution of leading problems of astrophysics. Accordingly, in view of the simple connected view now obtained of several great problems, which have defied successful treatment for nearly a century, we may anticipate a zealous interest on the part of our solar and astrophysical observatories in extending and improving on the promising researches here outlined.

If the meteoric downpour upon the sun now going on is shown to depend on the perturbative actions of Jupiter and Saturn, the knowledge of sound theory thus gained, when confirmed by time and increased experience, may prove to be a source of safety for vast populations against worldwide drought and famine, such as are now witnessed in Russia and China, and threaten other parts of the continents of Asia, Central and Western Europe, and North America.

About 140 years from now, 2061 A. D., a very similar situation of maximum danger will recur to the inhabitants of the northern terrestrial hemisphere, from the conjunctions of the great planets Jupiter and Saturn in the middle of the summer. But lesser dangers will recur in 1941 and 1961, as explained above, — after which the danger to the northern hemisphere will decrease somewhat, owing to the conjunctions corresponding to the winter season, and thus threatening chiefly the inhabitants of the southern terrestrial hemisphere, which is largely covered by the ocean, and less likely to be adversely affected.

By the extension of these researches it may be possible for future astronomers to foretell these dangers in time to avert world drought disasters to large populations, such as we have witnessed in 1921. If this could be done with confidence, so as to give opportunity in years of plenty to provide for years of famine, incident to drought, millions of people would be saved from suffering and destruction by this humanitarian service of science. Thus in time the researches here outlined may add not a little to the safety and stability of the nations of the earth.

It always is very difficult to be sure that we have correctly connected the details of a great multitude of phenomena in the simplest and most direct way. But as simplicity is a powerful argument for truth, when we come to search for the fundamental laws of nature, the indications are that the meteoric theory here advanced alone meets the modern requirements of solar and cosmical physics. Certainly no simpler theory could be proposed, and it is difficult to imagine any valid objection to explanations which appeal so strongly to our common sense, and to the direct evidence of our senses.

When we consider the sun and the Cepheid, Geminid and Cluster variable stars in their larger aspects, we naturally ask whether the theory advanced will account for the chief phenomena of their observed changes. If so, the more of these phenomena we can bring under the theory, the greater the probability that it is correct.

For in the final confirmation of any new development, it always is a great element of satisfaction to find that the adopted theory is supported by arguments drawn from several independent sources, so that the indications all are mutually accordant.

In the investigation now concluded it appears by the records at hand that as far back as 1917 I became so fully convinced that the sunspot cycle depends upon meteors thrown upon the sun by the combined actions of Jupiter and Saturn, that I then drew up a considerable outline of the dynamical theory of this precipitation of meteors, and

hung certain sketches of it over my desk, to remind me that the subject should be resumed as soon as the new theory of the aether was concluded (June 15, 1921).

Accordingly, early in July, 1921, the work was energetically resumed, and as the summer already was one of unprecedented heat, I became convinced, as stated in a press announcement issued July 8, that an unusual number of meteors had recently fallen upon the sun. This explained the extraordinary summer heat in accordance with a cause known to be at work, from the observed secular acceleration of the earth's motion, as confirmed by the researches of *Cowell* and *Fotheringham* on ancient eclipses.

The ability to calculate the exact length of the mean sunspot cycle from the sidereal revolution of Jupiter (11.86172 years) compounded with retrograde appulse with Saturn (in 9.92945 years) was another very striking proof of the dynamical theory depending on the meteoric downpour under the combined actions of these two great planets, as was also the great sunspot saros in 88.9 years.

But final and absolutely overwhelming proof became possible only on Aug. 19, when I discovered that the form of the sunspot frequency curve was that of the tidal curve of a resisted wave. Such a gravitational and thermal tide at the sun's equator would result from the meteoric downpour, and hence the argument was seen to be absolutely complete; and as each part of the argument supported the other, and thus it hung together consistently as a whole, the result was announced by the cablegram of Aug. 20, to be followed by the cablegram of Aug. 22, when the results had been fully applied to the Cepheid, Geminid and Cluster variables.

Nothing could be simpler or more general than the present theory of the generation of surface tides rushing forward about the equators of the stars, and by the neighboring resistance to this movement, yielding curves of light, corresponding to the tidal curves of resisted waves, as observed in the canals and rivers connected with the oceans covering the terrestrial globe. Thus the tidal theory, — begun by *Newton*, 1686, extended and improved by *Laplace's* use of more rigorous dynamical principles, and made eminently practical by the researches of *Airy* on tides and waves, 1845, — unfolds a penetrating vision of the most stupendous operations of the sidereal universe.

We are enabled to understand the cause of sunspots, by the form of the curve of frequency, where the period is somewhat long, and the movement shows a secondary maximum under the sensible resistance of the equatorial rush by other portions of the photosphere in higher latitude and at greater depth.

To understand the break in the tidal curve of a resisted wave, as applied to the sun and stars, we notice the well known fact that a river flows fastest, with the water at greatest elevation, in the central deeper parts of the channel. Now this central elevation of level corresponds to the equators of the sun and stars; and hence we perceive that as the equatorial current is resisted, the equatorial level of the photosphere readjusts itself, by a revulsion of

the flaming fluid towards the poles, and a secondary oscillation follows, by reaction of the globe upon itself, yielding a flow like the first equatorial accelerated current, but feebler. These simple considerations, verified by recognized facts and experiments in terrestrial hydraulics, show that the tidal curve of a resisted wave gives an exact representation of the currents at the equators of the sun and stars as set in motion by the meteoric downpour.

And we may apply this discovery at once to the body of the Cepheid and Geminid variable stars, in which the light curve has the same form as the curve of sunspot frequency.

In the case of the sunspots we do not observe the variation of the sun's light — though indeed it does actually vary, — but the frequency of the spots, a secondary phenomenon proportional to the variations of the sun's light. Up to the present time our imperfect experimental measurements do not disclose to us the sun's light curve; yet we know from the variable stars that the type of this variation in radiation is similar to that of the Cepheid variables.

Accordingly, in the absence of direct records of the sun's light curve, we may use the curve of spot frequency, which will have the correct form, but a considerable retardation of phase, owing to the fact that the spots are an indirect effect of the downpour of the meteors upon the sun's equatorial belt.

Again, in the case of the Cluster-variables discovered by *Bailey*, we find the periods to be of the order of a half day or less, which is too short an interval to bring about any visible secondary maximum, due to break of the tidal rush under the effect of resistance; and thus *Bailey's* Cluster-variables are practically devoid of sensible secondary maxima in their light curves.

Without the present theory we could not have foreseen the cause of this omission of the secondary maximum; but in the light of it, we see how inevitable the omission is, and how very useful the theory is in dealing with the thousands of stars undergoing regular pulsations of light, though situated in globular clusters so remote that the radiation takes from 7000 to 240000 light-years to reach the earth.

Obviously the regularity of the pulsation is due to the orbital motion of a satellite about the Cluster-variable; yet the downpour of meteors thereby precipitated, chiefly owing to the considerable eccentricity of the satellite orbit, is the direct cause of the rapid increase of the star's light, as shown by the steep ascending slope of the light curve.

These remarks remind one that there is a new »Astronomy of the Invisible«, more general than that made known eighty years ago through *Bessel's* celebrated researches on the proper motions of Sirius and Procyon. The difference between the two developments consists largely in the fact that the tidal movements in the stars will be forever invisible, while the companions of Sirius and Procyon have been optically discovered. Yet as the light curves admit of direct comparison with the sunspot frequency curve and

the tidal curves recorded in our rivers, we see the cause at work as plainly as if it were observed with the telescope.

It is fortunate that we have very perfect frequency curves for the sunspots, and very similar light curves in such stars as η Aquilae. In view of the perfect geometric conformity of the curves with each other and with the tidal curve of resisted wave-motion, it is absolutely impossible to doubt that the cause at work in the two cases is identical. In the case of the sun the orbital motion of Jupiter is slow, and the meteoric downpour therefore somewhat gradual and prolonged; in the case of η Aquilae the period of the meteoric downpour is much shorter, yet long enough for the gravitational and thermal tidal wave at the equator of the star to break, partially subside, and, by reaction, form a distinct secondary maximum.

All these considerations show the exalted nature of the tidal theory as a means of future discovery. The applications of the theory in the terrestrial phenomena of the oceanic tides heretofore made by *Newton*, *Laplace* and *Airy* are small compared to those which will be made in the immensity of space. By the study of the associated gravitational and thermal pulsations, — visible through the fluctuations of starlight in the globular clusters and other remote objects of the sidereal universe, — we have a means of exploration of transcendent interest to the geometer and natural philosopher.

Here are sidereal systems of the highest order, composed of many thousands of giant stars, yet removed from us by perhaps a quarter of a million light-years, and thus by perspective condensed into a very small angular space on the back ground of the heavens, while from the mere effect of this distance these giant stars are so faint as to appear on our photographic plates as extremely delicate points of light. Nevertheless their starlight is found to undergo regular periodic fluctuations which admit of accurate measurement.

From the light curves thus defined, the geometer perceives that the gravitational and thermal oscillations going on in these remote sidereal systems, though forever beyond the range of direct telescopic exploration, are undeniably similar to the tidal oscillations familiar to *Newton*, *Laplace*, *Airy* and *Darwin* in the oceans and rivers of our terrestrial globe, and duly recorded since the age of *Galileo* in the curves of sunspots investigated by *Schwabe*, *Wolf*, *Spoerer*, *Faye*, *Newcomb* and other astronomers.

The possibilities opened up by these unexpected but verified lines of research are as gratifying as they are limitless, and seem to me beyond mortals wonderful! They present to us an impressive picture of the continuity and unbroken order of nature; and again emphasize *Newton's* doctrine that the laws found to hold true upon the earth are to be extended to the heavens and applied to the solar system and sidereal systems throughout the immensity of space.

In the author's paper on the Dynamical Theory of the Globular Clusters etc., (Proc. Am. Philos. Soc., 1912), it is pointed out that the globular clusters have been built up by the downpour of meteoric matter upon the component stars. Thus the prediction made nine years ago is now proved by some of the most impressive phenomena yet brought to light

in the sidereal systems of high order first studied by Sir *William Herschel*.

For over forty years the explorations of this unrivaled man made known the immensity and profundity of the star-depths. He wondered especially at the nature of the globular clusters, scattered along the path of the clustering stream of the Milky Way; and did not fail to notice that the symmetrical accumulation of brightness, increasing to a perfect blaze of starlight towards the centre, is a proof of the globular form of these dense masses of stars.

To the wonders perceived by *Herschel*, *Bailey* has added the amazing discovery of hundreds of variables in the interior of the globular clusters; and *Shapley* has recently supplemented the record with a systematic determination of the distances of these splendid systems.

It was justly pointed out by *Fourier*, in his historical eulogy on the author of the *Mécanique Céleste* (June 15, 1829), that the successors of *Herschel* and *Laplace* would witness the accomplishment of the great phenomena whose laws these celebrated astronomers discovered. At remote epochs the spectacle of the heavens will be changed, yet nothing can diminish the glory of the inventor who alone is able to assign the cause of natural phenomena.

If therefore we are able to assign to resisted wave action the wonderful periodic fluctuations of the starlight of the individual variables noticed in *Herschel's* glorious globular clusters, we shall bring to light a geometrical law which will verify the prediction of *Fourier*, whose rigorous analysis, involving sextuple integration between infinite limits, alone is adequate to deal with the general theory of wave motion.

It is a most impressive fact that the present theory explains:

1. The sun's equatorial acceleration, and the preservation of this acceleration from age to age.
2. The periodicity of the corona, with its great equatorial extension near the time of sunspot minimum, when the principal gust of the meteoric downpour occurs.
3. *Spoerer's* law of progress for spot distribution, the development beginning in high heliographic latitude and moving towards the equator with the advance of the sunspot cycle.
4. The mean period of the sunspot cycle, in 11.178 years, which can be calculated more accurately from the motions of Jupiter and Saturn than it can be found by the direct observation of spots after records extending over 311 years, from their discovery by *Galileo*, in 1610.
5. The form of the curve of frequency of sunspots, which by comparison with the curves in *Airy's* great treatise

On Tides and Waves, 1845, is shown to be identical with that of the tidal curve of a resisted wave.

6. The light curves of thousands of variable stars, which are explained also by the theory of gravitational and thermal waves under resistance.

7. The secondary maxima in the curve of frequency of sunspots and in the light curves of variable stars, which heretofore have proved utterly bewildering to astronomers and geometers.

8. The connection between the sunspot frequency curve and the light curves of variable stars — both types of curves being geometrically identical and depending on the modified oscillation incident to the resistance to wave motion.

9. The secular acceleration of the earth established by researches on ancient eclipses, showing that the sun's mass increases, and thus confirming the theory of a meteoric downpour.

10. The direction of the rotations of the spots in the zones north and south of the solar equator.

11. The Great Saros or theoretical restitution period in 88.9 years, which is confirmed by the sunspot curves of *Wolf* and other investigators during the past 176 years.

12. World-wide meteorological disturbances, with decennial droughts due to the increase of solar radiation, as witnessed for the greater part of a century and especially illustrated by the excessive heat experienced in all countries during the summer of 1921.

In the New Theory of the Aether, recently published, we have referred all the physical forces of the universe to wave-action in the aether. And now we find that free and forced waves in matter, fluid and thermal oscillations in the photospheres of the stars, depending on the gravitational action of other bodies, play a much greater part in the phenomena of the universe than we have heretofore believed. Accordingly, this general wave-theory of the universe, follows from the original development begun by *Newton*, for the tides of our sea and the sound waves of the air, and cannot fail to give us the deepest insight into the laws of nature.

The prompt and generous recognition lent to this investigation by several friends deserves the author's grateful remembrance, and thus he mentions especially Captain *Edward L. Beach*, U. S. N., Commandant at Mare Island, and Commander *L. M. Cox*, U. S. N., the eminent civil engineer; besides his associates Mr. *L. Tiernan*, Mr. *W. S. Trankle*, and above all Mrs. *See*, as contributing to the early completion of these researches.

Starlight on Loutre, Montgomery City, Missouri, 1921 Aug. 22.

T. J. J. See.

Discovery of the Cause of the Mira Variable Stars.

Postscript:

1. The day I mailed this paper to the editor, (Sept. 21, 1921) I received AN 5116 and 5117, containing important articles on variable stars by *Heiskanen* and *Ludendorff*. *Heiskanen* gives a well defined curve of S Vulpeculae, AN 5116, p. 63, which shows clearly that this star has a rapid rise, followed by a much more gradual decline, with distinct secondary maximum, and therefore depends on the

same cause as the Cepheid-variables, namely: meteoric showers on the photosphere, undergoing gravitation and thermal oscillations like the tidal curve of a resisted wave.

2. I had neglected to treat of the Mira stars, because I had not seen a good light curve of Omicron Ceti. Here it is, and the record tells the same story as in the case of S Vulpeculae, namely: gravitation and thermal oscillations following the tidal curve of a resisted wave.

Fig. 13.

The rapid rise to maximum, more gradual decline to minimum, with distinct secondary maximum about half way along the downward slope assimilates Mira not only to δ Cephei, as *Ludendorff* imagines, but also to the curve of the sunspot frequency much more distinctly than we have heretofore dared to believe.

3. No doubt the Mira stars generally will be found to have similar connections and to depend on the very cause here outlined. It is no objection to the meteoric theory that the periods, and amplitudes are somewhat variable, in the Mira stars; because we see above what happens in the case of our sun, with an »oscillation« in the spot period, and similar causes no doubt will operate among the stars generally. When the periods are very regular, as in some of the cluster variables, the orbital motion of the companion is so powerful as to be the chief influence, and an »oscillation« in the period is eliminated.

4. It is well known that the spectral changes of Mira show atmospheric ignition, as if the absorbing gases of the photosphere were aflame from the downpour of meteors, with internal disturbances incident to this conflagration and tidal rush of the flaming fluid. The star does not exhibit

orbital motion, but neither does our sun, owing to the smallness of the masses of Jupiter and Saturn. Thus Mira probably has comparatively small bodies active in precipitating the meteors, and the combinations of two or more of them will explain the »oscillations« in the period as well as the ignition of the star's photosphere.

5. It is interesting to note that the earliest observations of Mira by *David Fabricius* at Resterhave, East Frisia, were made in August, 1596, and thus just 325 years ago. This seems a long period to wait for an explanation of any celestial phenomenon, yet if the explanation is valid, the compensation is to be found in the establishment of a true law of nature.

6. Finally, attention may be called to the desirability of giving the light curves in the form shown above, Figure 13. In the absence of this form, the physical meaning of the fluctuation of the star light is difficult to recognize; but with this unsymmetrical oscillation and secondary maximum exhibited to the eye, the tidal curve of a resisted wave becomes as obvious as in the light curve of δ Cephei, or the curve of sunspot frequency.

1921 Sept. 22.

T. J. J. See.

T. J. See. Cause of the Sunspots and their Periodicity.

Airy's illustrations of the effects of resistance upon the motion of waves, and of the tide waves in the river Thames.

T. J. J. See. Cause of the Sunspots and their Periodicity.

Corresponding tidal curves on different parts of the Severn (513)

Tidal curve at Hung Road

62. Observed semimenstrual inequality of Height of High Water for London (549)
Solar Hours of Moon's Transit

Airy's illustrations of the tidal curves in the river Severn, and near the Isle of Wight.

T. J. J. See. Cause of the Sunspots and their Periodicity.

Curves representing the Variations (A) in Light, (B) in Radial Velocity of ζ Geminorum (Campbell).

a. Illustrations of the light curves of the variable stars η Aquilæ and δ Cephei, in comparison with the sunspot curve (1867-77) as found by *Ellis* of the Royal Observatory, Greenwich.

b. Light curves of the Geminid-variables U Vulpeculæ, ζ Geminorum, and V Puppis.

T. J. J. See. Cause of the Sunspots and their Periodicity.

Light curves of Cluster-variables, and of SS Cygni, and S Arae, which are essentially similar.

T. J. J. See. Cause of the Sunspots and their Periodicity.

Fig. 6. Illustrations of the types of curves examined by the author before adopting the tidal curve of a resisted wave as representing the order of nature.

Fig. 7. Illustration of the unsymmetrical distribution of comet orbits and meteoric swarms about Jupiter's orbit, which authorizes the use of the term $\Delta\pi (\sigma_0/\sigma_\mu) \sin[\beta - \alpha(t-t_0)]$ in the formula for the sunspot periodicity.

Fig. 10. Illustration of the escape of water vapor, molecular weight 18, to the higher regions of the atmosphere, which is relatively so free from dust, and condensation therefore so difficult, that when a drought in a large inland country once becomes established, it is apt to be prolonged till the larger seasonal changes produce such commotion in the earth's atmosphere as to bring on rain.

T. J. J. See. Cause of the Sunspots and their Periodicity.

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1. The corona during the total solar eclipse of May 28, 1900, drawn by *W. H. Wesley* from photographs by *E. W. Maunder*. This was near the spot minimum of 1901.5, and the corona therefore already much extended at the equator under the increasing downpour of meteors.
2. Composite photograph of sun's disc and limb, taken by *W. J. S. Lockyer*, 1904, July 19.
3. (a) The great cyclonic spot of May 5, 1858, drawn by *Secchi*; and (b) a stereoscopic view of solar atmosphere above a group of Sunspots, Aug. 7, 1915, by *Hale*.