"Proof of the Generality of certain Formulae published for a Special Case" by Mr. Blakesley

This content has been downloaded from IOPscience. Please scroll down to see the full text.

1890 Proc. Phys. Soc. London 11 114

(<http://iopscience.iop.org/1478-7814/11/1/314>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 137.149.200.5

This content was downloaded on 10/09/2015 at 12:16

Please note that [terms and conditions apply](#).

possible to multiply such coils, and vary their turns and position so as to meet any case, if desirable. The method is merely indicated here.

From what I have said, it will not surprise those who have followed me that in questions of power I recommend that quotations should be made of mean current², as indicated by dynamometers, and that attempts at giving mean current in amperes, with reversing currents, should be given up. What a quantity of pains has been taken to make voltmeters give indications proportional to volts! The merit of a difference of potential is as its square, and that of cells as the square of their E.M.F. Give me twice the E.M.F., you quadruple my power of doing work by its means. To reduce readings to give the square root of mean square is doubly wrong. It is a ridiculous attempt to reach a useless quantity, and, further, gives one the trouble of squaring back again.

XIII. *Proof of the Generality of certain Formulæ published for a Special Case by Mr. Blakesley.* By Prof. W. E. AYRTON, F.R.S., and J. F. TAYLOR*.

I.

IN May 1888 Mr. Blakesley described before this Society† a very interesting method of testing the power given to the primary coil of a transformer based on the employment of three dynamometers. The method, however, really only requires two alternate current ammeters and one dynamometer with two coils. The ammeters are placed respectively in the primary and secondary circuits of the transformer, while the two coils of the dynamometer are electrically separated from one another, one being placed in the primary and the other in the secondary circuit of the transformer, as shown in the figure.

Mr. Blakesley in his paper assumed that the three instruments were not graduated directly; that is to say, that constants had to be employed in each case to reduce the arbitrary

* Read February 27, 1891.

† Proc. Phys. Soc. ix. p. 286; Phil. Mag. [5] xxvi. p. 34.

scale-readings to absolute measure. We will, on the contrary, suppose the instruments to be graduated as they ought to be,

so that if D_p , D_s , and D_{ps} be the readings of the three instruments, they are equal respectively to the square root of the mean square of the primary current, the square root of the mean square of the secondary current, and the mean product of the two currents.

With this definition Mr. Blakesley proved geometrically that the watts given to the primary coil of the transformer were equal to

$$pD_p^2 + \frac{P}{S} sD_{ps} ;$$

where p and s are the resistances, in ohms, of the primary coil and the whole secondary circuit respectively, and P and S are the numbers of windings in the primary and secondary coils of the transformer.

The formula is a simple one, and the values of the expressions in it are fairly easy to obtain experimentally. The proof of the formula, however, as given by Mr. Blakesley, was based on the following assumptions:—

1. The variations of the primary and secondary currents are harmonic.
2. The variation of the magnetism of the core is harmonic.
3. The magnetic stresses produced in the iron core by the currents in the primary and secondary coils are directly proportional to the ampere-turns in these two coils.
4. Each turn in each coil embraces at any moment the same number of lines of force.
5. The secondary circuit outside the transformer is non-inductive.

II.

In the spring of last year, 1890, Mr. Wightman, one of the third-year students of the Central Institution, showed that an analytical method for measuring the efficiency of a transformer, which had been described in one of the lectures at the college, could by a slight transformation be employed to prove the generality of Mr. Blakesley's formula given above.

The proof is quite simple, and shows that the formula in question is true whatever function the currents or the magnetism be of the time, and whatever amount of hysteresis or magnetic lag may exist. In fact the proof is independent of Mr. Blakesley's assumptions Nos. 1, 2, and 3, mentioned above.

We have delayed the publication of this proof until we had used the method for a lengthy series of experiments on a transformer kindly lent us by Mr. Mordey, and which has occupied the students for many months. But, in order that others like ourselves should be able to use Mr. Blakesley's formula with confidence, and without having any longer the fear that if the sine law were not true, or if much hysteresis existed, their calculations made by means of this formula from experimental results might be very wrong, we communicated nearly a year ago to Mr. Blakesley and to others interested in the matter the fact that we had proved mathematically that the two ammeters and one dynamometer method of measuring power was generally true.

Let V_p be the P.D. in volts at the terminals of the primary-coil at any moment t .

Let A_p be the primary current, in amperes, at the same moment.

Let A_s be the secondary current, in amperes, at the same moment.

Let n be the number of lines of force passing through one convolution at that moment.

Then

$$V_p = pA_p + \frac{1}{10^8} P \frac{dn}{dt},$$

$$sA_s = \frac{1}{10^8} S \frac{dn}{dt};$$

$$\therefore V_p = pA_p + \frac{P}{S} sA_s.$$

$$\therefore A_p V_p = pA_p^2 + \frac{P}{S} sA_p A_s.$$

If T be the time of one complete cycle,

$$\frac{1}{T} \int_0^T A_p V_p dt = \frac{p}{T} \int_0^T A_p^2 dt + \frac{P s}{S T} \int_0^T A_p A_s dt.$$

But the expression on the left-hand side is the mean watts given to the primary coil, and the expression on the right-hand side is simply

$$pD_p^2 + \frac{P}{S} sD_{ps};$$

D_p , as already explained, being the reading of the alternate current ammeter in the primary circuit, and D_{ps} the reading of the dynamometer having one coil in the primary and the other coil in the secondary circuit.

The following is another general proof of the same formula:—

The watts given to the primary coil of a transformer are spent partly in heating the primary coil, and partly in doing work against the back electromotive force set up by the varying magnetism of the core.

The watts spent in heating the primary coil are of course pD_p^2 ; while the watts spent in doing work against the back electromotive force are at any moment $A_p \times$ the back E.M.F.

The E.M.F. generated in the secondary circuit by the same variation of magnetism of iron is at any moment sA_s ; therefore the back E.M.F. in the primary coil at the same moment must be $\frac{P}{S} sA_s$.

Consequently the total watts given to the primary coil are

$$pD_p^2 + \frac{P s}{S T} \int_0^T A_p A_s dt;$$

$$pD_p^2 + \frac{P}{S} sD_{ps}, \text{ as before.}$$

III.

This expression, as Mr. Blakesley points out, may be written as follows :—

$$pD_p^2 + sD_s^2 + s \left\{ \frac{P}{S} D_{ps} - D_s^2 \right\} ;$$

and since $pD_p^2 + sD_s^2$ are the watts employed in heating the primary coil and the whole of the secondary circuit, it follows that

$$s \left\{ \frac{P}{S} D_{ps} - D_s^2 \right\}$$

are the watts employed in heating the core on account of hysteresis or magnetic lag ; a result now proved true independently of all assumptions as regards the sine law, or the magnetic stress being directly proportional to the ampere-turns, &c.

IV.

Since the portion of the secondary circuit outside the transformer is non-inductive and has a resistance s' say, the watts developed in it are of course $s'D_s^2$. Consequently the efficiency of the transformer is

$$\frac{s'D_s^2}{pD_p^2 + \frac{P}{S} sD_{ps}}$$

V.

Using the various assumptions already referred to, Mr. Blakesley arrived, by means of a geometrical proof, at a formula for measuring the mean square of the P.D. at the terminals of the primary coil by means of the two ammeters and the dynamometer.

The following *general* proof of this formula is very much simpler than the proof for only a special case which Mr. Blakesley gives, and furnishes a very good example of the fact that sometimes it is more easy to give an analytical proof which is true independently of any assumptions about the harmonic law, &c. than to give a geometrical proof which is only true when these suppositions hold.

$V_p = pA_p +$ the back E.M.F. at the moment

$$= pA_p + \frac{P}{S} sA_s ;$$

$$\therefore \frac{1}{T} \int_0^T V_p^2 dt = \frac{p^2}{T} \int_0^T A_p^2 dt + \frac{P^2 s^2}{S^2 T} \int_0^T A_s^2 dt + 2 \frac{P}{S} \frac{ps}{T} \int_0^T A_p A_s dt ;$$

that is, the mean square of the P.D. at the terminals of the primary coil equals

$$pD_p^2 + \frac{P^2}{S^2} s^2 D_s^2 + 2 \frac{P}{S} ps D_{as},$$

which is the formula given by Mr. Blakesley.

From the preceding it follows that Mr. Blakesley's expressions for the watts given to the primary coil of a transformer, for the efficiency of the transformer, and for the mean square of the P.D. at the terminals of the primary coil, are true irrespectively of any assumptions as to the functions the E.M.F.s, the currents, or the magnetic flux are of the time as well as of any assumptions as to the presence or absence of hysteresis or magnetic lag.

This being the case, the application of this two ammeters and dynamometer method for measuring power in other cases than those already treated of is worthy of careful consideration.

XIV. *On the Variation of Surface-Tension with Temperature.*

By Prof. A. L. SELBY, M.A., University College, Cardiff.*

MENDELEJEFF speaks of an ideal liquid as characterized by two conditions :—

- (1) $V_t = V_0 / (1 - kt)$, V_t being the specific volume at $t^\circ \text{C}$.
- (2) $T_t = T_0 (1 - at)$, T_t being the surface-tension at $t^\circ \text{C}$.

I believe that the following proof shows that all liquids satisfy Mendelejeff's second condition.

Let unit mass of liquid have a constant volume, but variable surface S , and temperature t .

* Read March 20, 1891.