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XXXVI. Galvanometers.—third paper

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XXXVI. *Galvanometers.—Third Paper.*
By Prof. W. E. AYRTON, F.R.S., and T. MATHER *.

SECTION A.

Introductory.

UNDER the above title the authors, in conjunction with Dr. Sumpner, read a paper before the Physical Society in January 1890[†], and gave a list of sensibility-records relating to a large number of instruments of various types. To facilitate comparison between different instruments, the results were reduced to a common standard as regards scale-distance, length of a scale-division, periodic time of vibration, and resistance, the values chosen for this purpose being, respectively, a scale-distance equal to 2000 scale-divisions, periodic time 10 seconds, and resistance 1 ohm. The sensibilities for steady currents were expressed as the number of scale-divisions deflexion a current of one microampere would produce (assuming proportionality) under the standard conditions, and the sensibilities for quantity as the swing produced by the instantaneous discharge of one microcoulomb when the instruments were used ballistically. The resulting numbers give a measure of the relative sensitiveness of the various instruments when quickness of action is all important. For cases where time is of little consequence, and constancy of

* Communicated by the Physical Society: read May 13, 1898.

† Proceedings of the Physical Society of London, vol. x. p. 393; *Phil. Mag.* vol. xxx. p. 58.

zero (as depending on the controlling forces) is of prime importance, the results were modified by appropriately introducing the moment of inertia of the suspended systems.

Our reasons for choosing a scale-distance of 2000 scale-divisions and a period of 10 seconds as standard conditions, were, that these conditions represented more nearly than any other round numbers those under which sensitive galvanometers were generally used. For example, scales of half-millimetres used at about a metre distance were frequently met with, whilst scales of fortieths of an inch used at 3 to 4 feet distance were quite common. Further, nearly all the reflecting-instruments at the Central Technical College had long scales with divisions about $\frac{1}{24}$ of an inch placed at about 6 feet from the mirror. Since 1890, however, scales divided into single millimetres have come into much more general use, and may now be regarded as the standard type in physical laboratories. As the present tendency in galvanometer construction is to reduce the size of the needles and mirrors, and therefore the available light, it is probable that scale-distances will decrease rather than increase. We are therefore led to believe that a scale-distance equal to 1000 scale-divisions (rather than 2000) would be, on the whole, a more convenient standard to adopt. Another reason for proposing the change is to facilitate the international acceptance of a common system; for we learn from Prof. H. du Bois that our system thus modified is to be recommended for adoption at the next meeting of the Society of German Physicists and Physicians in September next at Dusseldorf. Doubtless some instrument-makers will object to the change; others, we are sure, will welcome it, for some have used the proposed standard themselves for some time. *Millimetres at a metre*, as the deflexion per microampere, is a convenient way of stating the "figure of merit" of sensitive galvanometers. Moreover, the conversion into circular measure is quite simple, 1 division at 1000 divisions distance corresponding with a movement of the suspended system through $\frac{1}{2000}$ of a radian. We would therefore suggest that in future all sensibilities be expressed in terms of this standard. In the new sensibility-records given in Table II. of the present paper we have adopted this standard; and to facilitate comparison of these records with those given in our 1890 paper, we have appropriately reduced the numbers given in the principal table of the latter paper, and embodied them in Table I.

As regards the standard periodic time, 10 seconds represented the average conditions under which sensitive galvanometers were used much more nearly than any other simple

round number ; hence we decided in 1890 to use this time as the standard period to which all sensibilities were reduced. So far as we can see, this is still the most convenient standard time, and it has therefore been retained in the present paper.

The reduction to a standard resistance of one ohm has proved of considerable value in comparing various instruments, and has been frequently used by others interested in the subject. The question as to whether the "square-root law," *i. e.*

Deflexion per microampere $\propto \sqrt{\text{resistance}}$,

or the two-fifths power law,

Deflexion per microampere $\propto (\text{resistance})^{\frac{2}{5}}$,

should be used in this reduction is still perhaps debatable. On the whole, we consider the two-fifths law more nearly true over wide ranges of resistance ; nevertheless we have given the principal results reduced according to both systems.

Explanation of Tables I. and II.

Column 1 (or T)	gives	periodic time of vibration in seconds when tested.
" 2 (or λ)	"	logarithmic decrement of motion when tested.
" 3 (or F)	"	deflexion in divisions per microampere (as tested) when scale-distance = 1000 divisions.
" 4 (or r)	"	resistance of coil in ohms.
" 5 (or M)	"	deflexion in millimetres per microampere when undamped period is 10 seconds and scale placed as in actual use of the instrument.
" 6 (or D)	"	deflexion in divisions per microampere when undamped period = 10 seconds and scale-dis- tance = 1000 divisions.
" 7 (or S)	"	swing per microcoulomb under same conditions as in last.
" 8 (or V)	"	volume occupied by convolutions of wire, in cubic centimetres (approximately).
" 9 (or I)	"	moment of inertia of the suspended system, in C.G.S. units (approximately).
" 10 (or DI)	"	deflexion in divisions (scale-distance = 1000 divisions) per microampere for constant con- trolling moments, and for a periodic time equal to $10\sqrt{I}$ seconds.
" 11 (or SI $^{\frac{1}{2}}$)	"	swing per microcoulomb under same conditions as in last.
Columns 12, 13, and 14	give	the deflexion per microampere and swing per microcoulomb, when the period is 10 seconds and resistance of each instrument is one ohm*.

* Column 12 is based on the assumption that the sensitiveness is proportional to the square root of the resistance, whilst columns 13, 14, 15, &c. assume that sensitiveness varies as the two-fifths power of the resistance. (See *Phil. Mag.* July 1890, p. 85 *et seqq.*, and *Proc. Phys. Soc. of London*, vol. x. p. 422.)

Columns 15 and 16 give the deflexion per microampere and the swing per microcoulomb, for the same controlling moment, the resistance of each instrument being one ohm.

„ 17 „ 18 give the deflexion per microampere and the swing per microcoulomb per cubic centimetre of coil, when the period is 10 seconds and the resistance of each galvanometer one ohm.

„ 19 „ 20 give the deflexion per microampere and the swing per microcoulomb per cubic centimetre of coil, for the same controlling moment, and the resistance of each instrument equal to one ohm.

Periodic time of any instrument is $10\sqrt{I}$ seconds.

Recent Improvements.

Since the above-mentioned paper was published considerable advance has been made in the construction of sensitive galvanometers, more especially in the direction of reducing the dimensions of the suspended parts. We therefore thought it desirable to prepare a supplementary list of records, giving the results obtained on more recent instruments. In passing, we may remark that German and American instruments of the suspended-magnet type show the greatest progress in the reduction of dimensions and consequent quickness of action, whilst the d'Arsonval, or moving-coil type, seems to have received its greatest development in England.

To make the present list more complete than the original one, we have inserted three additional columns, giving respectively the period of vibration of the suspended system when tested, the logarithmic decrement, and the actual sensitiveness of the instruments obtained in the test, reduced to standard scale-distance and scale-divisions. From the first two of these columns (Columns 1 and 2 of Table II., pp. 356, 357) the relative suitabilities of the various instruments for ballistic or deflexional observation can be seen; whilst the third column (Column 3, Table II.) shows the actual "figures of merit" of the galvanometers (reduced to standard scale-distance) in the condition under which they were tested. This latter column is of considerable importance in the case of instruments whose controlling couple cannot be readily altered—for example, in the case of ordinary moving-coil instruments.

In addition to the three columns above referred to, we have also given in Table V. (p. 364) a list of records relating to moving-coil galvanometers used as voltmeters.

The values of $D/r^{\frac{1}{3}}$ or $D/r^{\frac{2}{3}}$ in columns 9 and 10 of Tables I. and II. express the relative sensibilities of the various instru-

ments for constant period and constant resistance of coil, when the galvanometers are used as ammeters. The same columns give their relative merits as voltmeters, provided the resistance of the coil is equal to the resistance between the terminals, *i. e.* when the resistance of the connexions is small compared with that of the coil. This may be taken to be the case in instruments with fixed coils; but for moving-coil galvanometers it is by no means true, especially when the coils have low resistance, for in some instruments the resistance of the suspensions is greater than that of the coil. Further, as instruments of the d'Arsonval type cannot always have their period, and therefore their sensitiveness, easily varied, no reduction to constant period or constant resistance has been made in Table V., and the numbers in column 3 (F') Table V. give the actual sensibilities of the respective instruments as voltmeters when the scale-distances equal 1000 scale-divisions.

We may again point out (as in our original paper *) that the comparison of galvanometers by reduction to constant period, scale-distance, and resistance, is a purely electromagnetic comparison, and takes no account of optical magnification, which in some instruments is of a much more perfect nature than in others. It is also to be noticed that the Table II. contains some records of galvanometers having very short periods, *e. g.* instruments numbered 4, 25, 41, 42, 43, 46, and 47; and the results of the tests on these instruments have, for the sake of uniformity, been reduced to the same period, scale-distance, and resistance, as the galvanometers having longer periods. It must not, however, be supposed that these short-period instruments could easily have their periodic time lengthened to 10 seconds—indeed, in some cases this would be practically impossible. But although the values given in columns 5, 6, 7, 12, 13, 14, 17, and 18 for the above-mentioned short-period instruments cannot as yet be realized in practice, the ratios of these numbers to those in the corresponding column for any other instrument give approximately the relative sensibilities of the two instruments compared, when both have the same *short* periodic time. To make our meaning clearer we will compare instruments numbered 24 and 43 in Table II. Referring to column 13 we find the values of D/r^2 for these instruments to be 254 and 985 respectively, indicating that No. 43 has a “factor of merit” between three and four times as large as No. 24. This must *not* be taken to mean that No. 43 is *actually* more sensitive

* Phil. Mag. vol. xxx. pp. 83–84; and Proc. Phys. Soc. of London, vol. x. p. 420.

TABLE I.
1890 LIST OF SENSITIVE GALVANOMETER RECORDS (reduced to present standard).

Type.	Description.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
		T.	λ .	F.	γ .	M.	D.	S.*	V.	I.	DI.	$SI\frac{1}{2}$	$D \frac{1}{r^{\frac{1}{2}}}$	$D \frac{1}{r^{\frac{1}{2}}}$	$S \frac{1}{r^{\frac{1}{2}}}$	$\frac{DI}{r^{\frac{1}{2}}}$	$SI\frac{1}{2} \frac{1}{r^{\frac{1}{2}}}$	$\frac{D}{r^{\frac{1}{2}}V}$	$\frac{S}{r^{\frac{1}{2}}V}$	$\frac{DI}{r^{\frac{1}{2}}V}$	$SI\frac{1}{2} \frac{1}{r^{\frac{1}{2}}V}$
Thomson. One pair of coils.	Elliott's Tripod (F.T.C.) †	3.62	0.19	0.50	0.6	3.8	3.8	2.38	...	...	...	...	5	4.7	2.95	8.55	2.7	2.27	1.43	0.023	0.15
	Tripod (F.T.C.) †	2.55	0.33	6.65	50	102	102	64	...	0.04	40.8	12.8	14.4	21.4	13.5	2.83	17.8	2.27	1.43	0.023	0.15
	Elliott's Tripod, specially light needles † ...	20	...	30,800	3,860	7,550	7,700	4,850	125	0.01	77	485	124	283	178	2.83	30.8	3.95	2.47	0.040	0.25
	" " " " † ...	20	...	64,000	6,000	15,750	16,000	10,000	125	0.01	180	1,000	206.5	493	308	4.93	30.8	3.95	2.47	0.040	0.25
5.	Elliott's Glass Cylinder (F.T.C.) †	2.33	0.26	66	4,600	1,220	1,220	760	...	...	...	...	13	41.5	26	...	...	...	...	...	...
6.	Lattimer Clark & Muthhead (F.T.C.) †	1.33	0.20	95	20,000	8,700	5,350	3,360	...	...	...	...	38	101.5	64	1.34	7.35	3.4	2.15	0.045	0.025
7.	Elliott's Stock Pattern †	20	...	15,200	7,000	4,300	3,550	2,225	30	0.013	46	254	42.5	102.5	64.5	2.27	12.6	0.73	0.46	0.010	0.05
8.	" Large Coil Pattern †	20	...	30,400	12,300	7,680	7,600	4,775	240	0.013	99	545	68.5	175	110	2.3	12.7	0.74	0.46	0.010	0.05
9.	" " " "	20	...	43,600	30,000	10,710	10,900	6,850	240	0.013	142	780	63	176	110	2.3	12.7	0.74	0.46	0.010	0.05
10.	" Special Form †	20	...	80,000	30,000	20,000	20,000	12,500	250	...	...	...	115.5	324	202.5	...	...	1.34	0.30	...	...
11.	" Ayrtton & Perry Ballistic (Japan)	39.5	...	11,200	20,000	1,070	715	450	...	...	...	...	5.25	13.65	8.55	...	...	...	...	...	...
12.	Tangley's Balometer	40	...	2,000	20	125	125	78	...	...	...	...	28	38	23.5	...	...	...	...	...	...
13.	Simple Reflecting, Fig. 5 (C.T.C.)	5.8	...	59	26	347	175	110	72	0.042	7	22.5	33.3	47.2	30	1.96	6.1	0.66	0.42	0.028	0.08
14.	" " " "	5.7	...	380	2,457	1,375	1,080	680	72	0.042	45.6	142	21.8	47.8	30	2.00	6.15	0.67	0.42	0.028	0.08
15.	" " " "	5.7	...	547	6,410	2,720	1,680	1,055	72	0.042	71	218	21	50.5	31.5	2.13	6.5	0.70	0.44	0.030	0.085
16.	" " " "	20	...	3,440	2,086	880	860	540	72	...	...	...	16.8	40.5	20.5	...	...	0.56	0.36	...	...
17.	Bell Magnet Ballistic, Walder Bros. † ...	20	...	2,700	7,800	720	675	425	...	...	...	...	7.65	18.7	11.8	...	...	...	...	...	...
18.	" " " "	20	...	2,400	10,000	600	378	...	...	...	...	...	6.0	15.1	9.5	...	...	...	...	...	...
19.	Simple Reflecting, Fig. 5 (C.T.C.)	5.7	...	370	5,330	...	1,140	720	72	0.042	47.8	148	15.5	37.0	23.3	1.55	4.45	0.57	0.33	...	...

20.	Mudford, Fig. 6 (F.T.C.).....	4.0	...	288	700	1,720	1,455	880	...	...	...	56	107.5	68	4.91	10.8	0.97	0.42	0.0065	0.12
21.	" " (C.T.C.)..... [(C.T.C.)	7.9	...	500	686	1,680	800	505	88	0.084	67	147	30.5	59	37	4.91	10.8	0.97	0.0065	0.12
22.	" " Variable damping, Large coils	10.0	...	2,140	9,744	4,200	2,140	1,320	200	0.052	111	301	21.8	54	83.5	281	7.65	0.27	0.014	0.038
23.	High Insulation Galv., Fig. 7 (C.T.C.).....	10.0	...	27,100	380,000	50,000	27,100	17,000	236	0.01	271	1,700	45	162	101	1.62	10.1	0.69	0.007	0.043
24.	" " " Nalder Bros.†.....	20	...	62,800	350,000	15,700	15,700	9,850	...	...	...	...	28.6	95	59.5	...	...	...	...	...
25.	Rosenthal Non-Astatic, Fig. 8 (C.T.C.).....	9	0.13	10.4	44	229	12.7	8	0.29	1.5	19	9.8	1.91	2.8	1.75	4.2	2.15	9.65	6.05	14.5
26.	" " Astatic, Fig. 10 (C.T.C.).....	10.5	...	15.3	44	25	13.9	8.7	0.29	0.7	9.7	7.25	2.1	3.05	1.9	2.14	1.8	10.5	6.55	7.4
27.	" " Non-reflecting ".....	8	...	2.9	5.2	...	4.55	2.85	2.0	0.21	0.96	1.31	2.0	2.35	1.47	0.49	0.68	1.18	0.74	0.25
28.	Gray's Galv. (Pr. Roy. Soc. 1884, p. 287)...	22	...	155	30,000	32	32	20	51	50	1600	141.5	0.19	0.82	0.3	26	2.3	0.01	0.0006	0.05
29.	Ballistic " Figs. 11 & 12 (C.T.C.).....	14.4	0.035	65	4,090	54	81.5	19.6	38	17	535	82.5	0.5	1.13	0.7	19.5	3.0	0.03	0.02	0.055
30.	Threllfall's Galv. (Phil. Mag. Dec. 1859, p. 460)	80	...	120	15,650	2.90	1.87	1.18	72	...	...	...	0.015	0.039	0.025	...	...	0.00054	0.00034	...
31.	Jolin's High-resistance (C.T.C.).....	1.75	...	3.98	750	137	130	81.5	2.8	3.0	3600	448	4.75	9.2	5.75	27.6	3.15	3.3	2.05	9.9
32.	Carpenter's Ordinary.....	0.8	...	1.34	208	286	188	118	...	...	...	...	13	29.2	14	...	...	...	...	...
33.	Jolin's Low-resistance (C.T.C.).....	1.5	...	0.475	3.8	39	21	13.2	0.6	5.5	115	31	10.8	12.5	7.75	67	18.2	21	13	113
34.	Invariable Sensibility, Large type (C.T.C.)	2.6	...	6.1	21	180	90.5	57	0.96	13	1180	205	19.8	27	16.8	350	61	28	17.5	365
35.	Nalder's type fitted (Pole pieces in (C.T.C.)	0.63	...	0.335	274	145	84.5	53	0.2	1.5	127	64.5	5.1	8.9	5.8	13.5	6.85	44.3	28	67.5
36.	with invariable sensibility device " out "	1.55	...	2.91	274	209	121	76	0.2	...	...	...	7.3	12.8	8.1	...	...	...	...	...
37.	Ca. pentier's Milliammeter (C.T.C.).....	...	...	...	183	...	36.5	22.9	3.2	25	910	114	2.7	4.5	2.8	113	14	1.4	0.87	35
38.	Single-needle Post-Office pattern (C.T.C.)	2.5	...	5.95	800	...	95	60	...	...	...	...	3.85	6.55	4.1	...	...	...	...	...
39.	Suspended needle instrument with	...	...	...	400	...	151.5	95	22	0.2	30.3	42.5	7.35	13.8	8.7	2.75	3.88	0.63	0.40	0.125
40.	Coils 6 centimetres long outside.....	4.0	...	11.6	350	...	73	45.9	22	0.2	14.5	20.5	3.9	7.0	4.4	1.41	1.97	0.32	0.20	0.065
41.	" 2.7 " " broad ".....	16	...	10	64	...	62.5	39.4	22	0.2	12.5	17.5	7.8	11.9	7.5	2.38	3.83	0.54	0.34	0.105
42.	" 2.2 " " high ".....	...	...	19	4.3	...	7.4	4.65	22	0.2	1.48	2.05	3.55	4.13	2.6	0.38	1.14	0.19	0.12	0.035
43.	" 2.3 " " thick.....	9.6	...	...	4.3	...	17.6	11.1	22	0.2	3.52	5.0	8.5	9.8	6.2	1.96	2.25	0.43	0.28	0.09
44.	Winding 0.6 ".....	...	...	3.14	0.87	...	3.4	2.14	22	0.2	0.7	0.95	3.65	3.6	2.27	0.75	1.01	0.16	0.11	0.035
45.	Nalder's Detector nearly astatic†.....	...	...	...	0.87	...	8.35	5.25	22	0.2	1.65	2.35	9.0	9.0	5.55	1.75	2.5	0.41	0.25	0.075
46.	" " partially ".....	...	...	490	2,000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
47.	" " ".....	...	...	42	2,000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...

* The numbers in this column are calculated from those given in column 6 without allowing for damping.
† The constants of these instruments have been kindly furnished by the makers.
‡ These instruments were tested after being in use for some time and without their needles being remagnetized.

TABLE II.
SUPPLEMENTARY LIST OF SENSITIVE GALVANOMETER RECORDS.

[illegible]

[illegible]

* The numbers in this column are calculated from those given in column 6 without allowing for damping.

† The constants of these instruments have been kindly furnished by the makers.

§ The inventor of this galvanometer says its period was intentionally made long for a special purpose and that the period could be considerably reduced without lessening the deflection per microampere.

than No. 24, but that No. 43 *would be* more than three times as sensitive as No. 24 if the period of No. 24 was reduced to, say, $\frac{1}{10}$ of a second by increasing its controlling field, and the period of No. 43 increased to $\frac{1}{10}$ of a second.

Comparison as Ammeters.

On comparing Table II. of the present paper with the one published in 1890 (shown in Table I., pp. 354, 355), it will be observed that in columns 12 and 13 (which express the factors of merit of the various instruments when time is of importance) the values are on the average greatly increased. For example, the highest values in the 1890 table were given by one of Messrs. Elliott Bros.' Thomson instruments of special form (see line 4, Table I.), for which $D/r^{\frac{1}{2}}$ was 206.5 and $D/r^{\frac{2}{3}}$ 493, and these numbers were far above any others in the list made out at that time. These figures are now surpassed by no less than eleven instruments given in the present list (omitting the oscillograph records in lines 4, 25, 46, and 47); and three of the eleven instruments are of the moving-coil type.

The highest values in the present table (omitting the oscillographs previously mentioned) are those given by one of Prof. Paschen's instruments, line 21, Table II., and are 5800 and 8750 for $D/r^{\frac{1}{2}}$ and $D/r^{\frac{2}{3}}$ respectively. We therefore see that Prof. Paschen has produced an instrument which, at a given periodic time and of a given resistance, is about twenty times as sensitive as the best given in our previous paper. Other instruments of the moving-magnet type giving very high factors of merit are:—

- (a) Two Paschen instruments. (Lines 20 and 23 of table.)
- (b) Prof. Nichols' galvanometer*. (Line 11 of table.)
- (c) Smithsonian Institution instrument, designed by Wadsworth† and constructed by Messrs. Elliott Bros. (Line 15 of table.)
- (d) Prof. W. B. Snow's bolometer galvanometer‡. (Line 10 of table.)
- (e) Two galvanometers by Weiss§. (Lines 18 and 19 of table.)

To show at a glance the character of the improvement, the numbers for these instruments are abstracted from the main table and put in order of magnitude below.

* 'The Galvanometer,' by Prof. E. L. Nichols, p. 80.

† Phil. Mag., vol. xxxviii. p. 553.

‡ Physical Review, vol. i. p. 37.

§ C. R. vol. cxx. p. 728, and Journ. Phys. vol. iv. p. 212.

TABLE III.

Description.	Values of		
	r .	$D/r^{\frac{1}{2}}$.	$D/r^{\frac{2}{3}}$.
Paschen's Galvanometer	60	5,800	8,750
" "	40	4,280	6,150
Nichols' "	9.3	2,200	2,730
Weiss' "		750	
Wadsworth's "	86	675	1,050
Weiss' "		600	
Snow's "	140	470	770
Elliott's Galvanometer (from Table I.)	6,000	206.5	493

For the moving-coil instruments, the numbers are

Description.	r .	$D/r^{\frac{1}{2}}$.	$D/r^{\frac{2}{3}}$.
Ayrton-Mather Galvanometer	243	570	985
" " "	267	421	735
" " "	244	375	650
Invariable Sensibility Galvanometer } (from 1890 Table)	21	19.8	27

These increases in the factors of merit have been brought about chiefly by reducing the dimensions of the suspended parts, and the use of better magnet steel has probably contributed something to the improvement.

The three moving-coil instruments which have surpassed the highest values of $D/r^{\frac{1}{2}}$ and $D/r^{\frac{2}{3}}$ given in our 1890 table are of the narrow coil type, the cross-section of the winding being of the form described before this Society in 1890*, viz., two equal circles in contact.

To attain the results here shown (Table III.), the transverse dimensions of the coils have been made smaller than usual and very powerful permanent magnets employed. When we compare the records of these instruments with the best listed

* "On the shape of Movable Coils," &c., Proc. Phys. Soc. vol. x. p. 376.

in our 1890 table (particulars of which are given in the last line of the preceding table) the improvement is evident; for whereas the highest value of D/r^2 given by a moving-coil instrument in 1890 was 27, the present highest value is 985.

It is also interesting to notice that so far as absolute sensitiveness for current is concerned (*i.e.* the "millimetres at a metre" per microampere, unreduced for period and resistance) the progress made with moving-coil instruments during the last eight years has been considerable. In our 1890 Table the d'Arsonval of greatest absolute sensitiveness as an ammeter was the "Large invariable sensibility" instrument given in line 4 of the d'Arsonval section of that list (see line 34, Table I.). This galvanometer gave 6.1 divisions per microampere, its period being 2.6 seconds, and coil-resistance 21 ohms. In the more recent list, Table II., the above record is surpassed by no less than thirteen instruments out of the twenty-one tabulated, the most sensitive d'Arsonval galvanometer to current being one made by Messrs. Nalder Bros. & Co., which gives 760 divisions per microampere, its period being 12.9 seconds and resistance 1053 ohms (see line 37, Table II.).

On comparing column 15 of Table II. with the corresponding one in the 1890 Table, which gives the values of DI/r^2 , *i.e.* numbers proportional to the deflexion per microampere with *constant controlling moment**, we observe that no increase is perceptible. On the contrary, the numbers in the columns of the present table are on the average smaller than in the previous one. This is what might be expected from a diminution of the dimensions of the moving parts of the instruments. An examination of columns 16 in the two tables (values of $SI^{\frac{1}{2}}/r^{\frac{3}{2}}$) leads to a conclusion similar to that deduced from a comparison of columns 15, the only instance of increased values being in the three narrow-coil d'Arsonval instruments just referred to.

It is perhaps worth while again calling attention to the fact of the superior sensitiveness (as ballistic instruments) of galvanometers with suspended parts having small inertia, a fact which is clearly shown by a study of columns 9 and 14 of the tables.

Prof. R. Threlfall's "New" Galvanometer.

Since the paper was read our attention has been directed to a very sensitive galvanometer designed by Prof. R. Threlfall, M.A. (an account of which is given in the Phil. Trans. of the

* See paper on "Galvanometers," Phil. Mag. July 1890, and Proc. Phys. Soc. vol. x. p. 421.

Roy. Soc. 1896, vol. clxxxvii. pp. 80-97), and used by him and Mr. J. H. D. Brearley in their "Researches on the Electrical Properties of Pure Sulphur." This instrument is in many respects a remarkable one. Unfortunately the details of the sensibility tests given in the printed paper are hardly sufficient to enable us to make the reductions to constant period and scale distance, as is done in Table II., so we give the results separately below. The tests were made by observing the throw produced on reversing a small current through the galvanometer, the throws being read by a telescope at about 267 centimetres from the galvanometer. The telescope had a micrometer eyepiece and a glass scale in it divided to fifths of a millimetre.

Date.	Period in seconds.	Current in amperes.	Throw on reversal in micrometer divisions.	Amperes per throw of 1 division.
Oct. 17th, 1892.	14.5	2.74×10^{-11}	7.9	3.5×10^{-12} *
Sept. 1893	25	2.74×10^{-12}	19.3	1.43×10^{-13} †

The definition of the optical system was sufficiently good to permit the micrometer-scale being read to a fifth of a division, so that currents of one-fifth the magnitude of those given in the last column of the above Table could be detected.

Oscillographs.

Oscillographs, or instruments for showing the character of rapidly-varying currents or potential-differences, were practically unknown when our 1890 paper was read. They have become possible by the great reduction of dimensions of moving parts which has gone on continuously since that date. An oscillograph may be defined as a galvanometer of very short period. The short periodic time is usually obtained by using very small moving parts and a very strong control. In these instruments the reduction of size has been carried far beyond what has yet been attempted for ordinary scale-reading work. Mr. McKittrick's‡ galvanometer has a moving system whose moment of inertia is about one-millionth of a C.G.S. unit, or about one ten-thousandth the inertia of an ordinary *light* astatic needle fitted with a $\frac{3}{8}$ in. *light*

* Phil. Trans. vol. 187, line 1 of table vi. p. 95.

† Ibid. line 1 of table vi. p. 96.

‡ Trans. A. I. E. E. vol. xiii. Nos. 6 and 7.

mirror. The record of this instrument is shown in line 4, Table II., and line 25 gives the record of another oscillograph galvanometer by Messrs. Hotchkiss and Millis *. Both the above instruments are of the moving magnet type with soft-iron needles adopted for similar purposes by Blondel, Moler, and others. Two other oscillographs are given in lines 46 and 47 of Table II., which are of the moving-coil type, and were made by Mr. W. Duddell † of the Central Technical College.

For convenience of reference the principal numbers relating to oscillographs are tabulated separately below, reductions being made to a constant period of a thousandth of a second, instead of 10 seconds, and the deflexions stated in terms of an ampere, instead of a microampere.

TABLE IV.

Description.	t .	f .	r .	I approxi- mately.	$d/r^{\frac{1}{2}}$.	$d/r^{\frac{2}{5}}$.
Hotchkiss and Millis' galv., 1895	0.256	31	4.1	2.2×10^{-6}	228	263
McKittrick's galv., 1896	0.377	1400	270	1.0×10^{-6}	600	1050
Duddell's galv., 1897.	0.358	156	1.3	30×10^{-6}	1070	1090
" " 1898.	0.093	420	2.08	6×10^{-8}	33,000	35,600

Explanation of Table IV.

Column t gives the periodic time in thousandths of a second when tested.
 " f " deflexion in divisions per ampere as tested when scale-distance = 1000 scale-divisions.
 " r " resistance of instrument in ohms.
 " I " moment of inertia of the moving systems in C.G.S. units approximately.

Columns $d/r^{\frac{1}{2}}$ and $d/r^{\frac{2}{5}}$ give the deflexions per ampere when the periodic time is one thousandth of a second and the resistance of each instrument is one ohm. Column $d/r^{\frac{1}{2}}$ is based on the assumption that the sensitiveness varies as the square root of the resistance, whilst column $d/r^{\frac{2}{5}}$ assumes that sensitiveness is proportional to the two-fifths power of the resistance.

* Phys. Review, vol. iii. p. 49. In Phys. Rev. vol. iv. p. 128, Mr. Millis gives an account of another instrument of very short period. Unfortunately the data are insufficient to permit of comparison with other oscillographs.

† See "Oscillographs," Electrician, Sept. 10, 1897, p. 638.

On comparing Mr. Duddell's 1897 instrument with those of Messrs. Hotchkiss and Millis and Mr. McKittrick, we find its factor of merit, as indicated by $d/r^{\frac{1}{2}}$ (the expression most suited to instruments having comparatively low resistance), is decidedly greater than either of the other two, in spite of the fact of its wires being of phosphor-bronze instead of copper. It may also be pointed out that the moment of inertia of the moving parts of Mr. Duddell's 1897 oscillograph is very considerably greater than the inertias of the two first-mentioned instruments, being in the approximate ratios of 30 : 2·2 : 1, thus showing the great superiority of the moving-coil type for work of this nature, where short periodic time (or high frequency) and good sensitiveness are necessary. In his latest oscillograph Mr. Duddell has carried the reduction of dimensions much further than has previously been attempted, and has produced an instrument whose periodic time is less than a ten-thousandth of a second (frequency 10,680). Although its resistance is little more than 2 ohms, it actually gives 420 divisions per ampere at standard scale-distance, and its factors of merit, as represented by $d/r^{\frac{1}{2}}$ and $d/r^{\frac{2}{3}}$, are more than 30 times those of any instrument made previously.

d'Arsonval Galvanometers as Voltmeters.

In Table V. the actual sensibilities of various d'Arsonval galvanometers when used as voltmeters are given, reductions having been made to bring them to the same scale-distance and scale-divisions only.

As will be seen from the table, considerable advance has been made in recent years. Of the d'Arsonval galvanometers mentioned in our 1890 paper, the most sensitive instrument as a voltmeter we had then seen was one of the "invariable sensibility" type, having a very large magnet. Its record is given in line 4 of the d'Arsonval section of that table (see line 34, Table I.). The instrument had a period of 2·6 seconds, coil-resistance of 21 ohms, and total resistance of 57·5 ohms, and gave a deflexion of 0·105 division per microvolt when the scale-distance was equal to 1000 scale-divisions. Referring to Table V. of the present paper, we notice that in 1892 a narrow-coil galvanometer was constructed, having a period of 2·2 seconds, coil-resistance 13·2 ohms, and total resistance 24·8 ohms, which gave 1·31 divisions per microvolt, an increase of over 12 times in sensitiveness, with a shorter period.

In 1893 Messrs. Queen and Co., of Philadelphia, produced a galvanometer of 178 ohms resistance, which gave 1·13 divisions per microvolt, whilst in 1896 Messrs. Crompton

TABLE V.—List of d'Arsonval Galvanometer Records (as Voltmeters).

Number on list.	Description.	1	2	3	4	5
		T.	λ .	F'.	r .	r' .
26	Holder d'Arsonval (Rectangular coil)	0.8	...	0.020	26.7	27.2
27	Ayrton-Mather Ballistic (Narrow coil) C.T.C.	2.2	0.05	1.31	13.2	24.8
28	Damped (Pauli's No. 6)	4.0	2.6	0.158	300	304
29	Holder d'Arsonval (Rectangular coil)	1.42	1.77	0.136	2.22	2.86
30	Queen & Co., Philadelphia (Large narrow coil)	1893	...	1.13	...	178
31	Sullivan's "Universal" Galvanometer *	1895	...	0.224	...	1260
32	Hartmann & Braun's, Frankfurt †	1896	...	0.175	1600	...
33	Crompton's Large pattern (Circular coil)	1896	Small	1.86	103.3	143.3
34	" " (Camb. Eng. Lab.) (Circular coil)	1896	0.03	0.975	100	141
35	" " (W. R. Cooper, Esq.) (")	1897	0.018	0.51	62.6	68.5
36	" Midget pattern (Camb. Eng. Lab.) (")	1897	Small	1.12	57.6	59.2
37	Nalder Ballistic (Camb. Eng. Lab.) (Rectangular coil)	1897	0.04	0.715	105.3	1060
38	" Damped No. 6283 (Rectangular coil)	1897	0.264	0.108	29.5	33.4
39	Anderson's Galvanometer (Circular coil)	1897	2.63	0.025	...	257
40	" " (")	1897	1.62	0.12	...	32.4
41	Ayrton-Mather Speaking Galvanometer (A) (Narrow coil) C.T.C.	1897	...	0.0022	244	244
42	" " (B) (")	1897	...	0.0020	267	267
43	" " (C) (")	1897	...	0.0031	243	243
44	" New pattern (Narrow coil) C.T.C.	1897	0.07	6.55	22.2	35.1
45	" " " "	1897	...	17.7	1.9	5.75
46	Duddell's Oscillograph (Very narrow coil and electromagnet) ‡ ..	1897	...	0.0012	1.3	1.3
47	" " (")	1898	...	0.00020	2.08	2.08

* See Note § Table II.

† The constant of this instrument was kindly supplied by the makers.

‡ This instrument has its moving coil of phosphor bronze, whereas all the others are of copper or silver.

Explanation of Table V.

Column 1 (or T)			gives the periodic time of vibration of coil when tested (in seconds).
„ 2 (or λ)	„		logarithmic decrement of motion when tested.
„ 3 (or F')	„		deflexion in divisions per microvolt when scale-distance = 1000 scale-divisions.
„ 4 (or r)	„		resistance of coil of instrument in ohms at about 15° C.
„ 5 (or r')	„		resistance of instrument between terminals at about 15° C.

showed to the authors one of their high-grade instruments, having a total resistance of 143 ohms, which gave 1·83 divisions per microvolt, its period being 12·1 seconds (see line numbered 33, Table V.). Up to the end of 1896 the Crompton instrument was the most sensitive d'Arsonval volt-meter we had any record of, but its period was, for some purposes, inconveniently long.

Table VI. is an abstract of Table V. arranged chronologically, showing improvements since 1888.

TABLE VI.

Description.	Date.	Period in seconds.	Divisions per microvolt.	Resistance of coil in ohms.	Total resistance in ohms.
1. Invariable Sensibility Type (Large)	1888	2·6	0·105	21	57·5
2. Ayrton-Mather Narrow coil	1892	2·2	1·31	13·2	24·8
3. Queen and Co., Philadelphia	1893	Aperiodic.	1·13		178
4. Crompton High Grade.	1896	12·1	1·83	103·3	143·3
5. Ayrton-Mather Narrow coil	1897	5·84	6·55	22·2	35·1
6. Ayrton-Mather Narrow coil	1897	7·6	17·7	1·9	5·75

During 1897 several fairly low-resistance d'Arsonval instruments of the narrow-coil type were made at the Central Technical College for use with the Lorenz apparatus tested by Prof. J. Viriamu Jones and one of the authors for the McGill University, Montreal. Records of two of them are given in lines numbered 44 and 45 of Table V. and lines 5 and 6 of Table VI. From these it will be seen that one gave 6·55 divisions per microvolt, and the other 17·7 divisions per microvolt, the latter number being nearly ten times as great as the previous best. Further particulars of these two very sensitive instruments when used as ammeters are given in lines 44 and 45 of Table II.

Phil. Mag. S. 5. Vol. 46. No. 281. Oct. 1898. 2 D

Uniformity in recording Tests.

We would here mention that in some of the galvanometer records we have received, the time of a single swing from one side of the scale to the other has been given as "the period," and this has caused considerable trouble on account of the sensibilities under standard conditions, calculated out on the assumption that the "complete period" was meant, coming out abnormally high. We therefore think it important that in all records of tests the "complete period," *i. e.* the time between two transits of the spot across the zero *in the same direction*, should be given. We may also remark that in all cases the logarithmic decrement (or the decrement) under the conditions in which the period was tested, should be stated, so that the change of period caused by damping may be allowed for. Unless this be done, well-damped instruments, which are usually the most convenient, will be unfairly handicapped. In the case of d'Arsonval or other instruments, in which the periodic time cannot readily be altered by the user, it is important that the actual period of the vibration when tested should be given.

For a complete record the following particulars are required :—

- (1) Resistance of instrument*.
- (2) Periodic time when tested.
- (3) Divisions deflexion per microampere (or micro-amperes per division).
- (4) Scale-distance.
- (5) Length of one scale-division.
- (6) Logarithmic decrement under test conditions.
- (7) Moment of inertia of the suspended system.
- (8) Mass of suspended system.
- (9) Diameter of mirror.
- (10) Length of suspended magnets (perpendicular to axis of rotation).
- (11) Type of instrument.
- (12) Volume of coils.
- (13) Date of test.

Nomenclature relating to Dead-beat Galvanometers.

Some ambiguity at present exists in the use of the words "dead-beat" as applied to galvanometers. Sometimes it is used as synonymous with "well-damped" or "aperiodic," whilst at other times the adjective is applied to instruments having quick-moving systems irrespective of whether the decrement

* If instrument be of the moving-coil type the resistances of coil and suspensions should be stated separately.

of the motion is large or small. Since quickening the movement by increasing the control (*i. e.* decreasing the periodic time of vibration) of a given system lessens the decrement, the two uses of the words "dead-beat" are to some extent incompatible. Maxwell used the term for galvanometers in which the motion of the suspended system was "aperiodic," *i. e.* the system only passed once through the position of equilibrium before coming to rest. As this meaning of the word seems most rational, we think it desirable it should be retained, and that its use in connexion with instruments having rapidly moving systems whose motion is not aperiodic should be discontinued. The expressions "quick-moving" or "short-period" galvanometer might be used to denote this class of instrument.

Insulation of Coils and Terminals of Galvanometers, and the use of Price's "Guard-Wire."

When testing very high resistances, such as the insulation resistance of a short length of good cable, with a galvanometer, the difficulties which arise from leakage over the cable ends or from the galvanometer itself are well known; the importance of constructing galvanometers so as to ensure good insulation was referred to in our previous paper*.

Since that date Mr. W. A. Price† has suggested the use of a "guard-wire" which practically eliminates error due to surface leakage over cable ends. The method there described can be adapted to galvanometers, thereby making the question of *perfect* insulation of coils and terminals of far less consequence than formerly.

Mr. Price's method consists in winding a bare wire W about midway between the extremities of the pared ends of the cable as shown in fig. 1, and connecting the other

* See p. 71, Phil. Mag. July 1890.

† 'Electrical Review,' vol. xxxvii. p. 702 (1895). See also Appleyard on "Dielectrics," Proc. Phys. Soc. vol. xiv. pp. 257, 264, where possible error is evaluated.

end of the wire to H, the battery side of the galvanometer G.

As the current through the galvanometer is very small, the P.D. between the wire W and the conductor C is also very small, and the surface leakage between them is quite negligible if the surface of the dielectric between them is fairly clean and dry. The full P.D. exists between W and the sheathing or wet braiding S, but any leakage-current from one to the other does not pass through the galvanometer. In the same manner error due to leakage from the terminal J of the galvanometer may be eliminated.

It will be evident that there is no need to have the battery side of the galvanometer *perfectly* insulated : all that is necessary is that the insulation resistance of H from earth is large compared with the internal resistance of the battery.

Probably the simplest way of applying the "guard-wire" principle to a galvanometer is to put one terminal of the winding to frame and case, preferably of metal, and carry the other terminal from the frame on a support moderately well insulated therefrom. The whole galvanometer may be sufficiently well insulated from earth by putting short ebonite sleeves, or caps, on the tips of the levelling screws. This construction was carried out in the Ayrton-Mather galvanometer as made by Mr. R. W. Paul in 1892, an instrument of this class being shown before this Society on June 10th in that year *. By using the frame terminal as the battery side of the galvanometer, the frame acts as an efficient guard-wire. Another advantage of the construction here described is that electrostatic deflexion of the moving system is at the same time prevented, a matter of considerable importance when resistance tests with fairly large potential-differences are being made.

A similar construction is, of course, applicable to shunt-boxes.

In concluding this section of our paper, we desire to thank the various manufacturers and inventors who have kindly supplied us with information and data relating to their instruments, and also to acknowledge our great indebtedness to C. G. Lamb, Esq., M.A., of the Cambridge Engineering Laboratory, and W. R. Cooper, Esq., M.A., for supplying us with several important records by which we have been able to add materially to the completeness of our lists.

* 'Electrician,' vol. xxix. June 17th, 1892, p. 174.

SECTION B.

Limiting Sensitiveness of Thomson Galvanometers.

At the Oxford meeting of the British Association (1894), Prof. Schuster read a paper "On the Construction of Delicate Galvanometers," in which inquiry is made as to the smallest current that can be detected by a non-astatic instrument of 1 ohm resistance, and controlled by a field of strength 0.17 C.G.S. unit. The calculated minimum current depends on the resolving power of the mirror employed, the smallest angle observable being taken as $\frac{\lambda}{2d}$ where λ is the wave-length of light and d the diameter of the mirror. The result arrived at is that 1.5×10^{-8} amperes is the smallest current that can be detected under the assumed conditions.

Taking λ as 5.9×10^{-5} centimetres we find the minimum angle to be 2.95×10^{-5} radians or about 6.1 seconds of arc; this corresponds with 0.0593 division on a scale at 1000 divisions distance. Expressing Prof. Schuster's result in this way we get 3.9 divisions per microampere as the limiting sensibility of a non-astatic galvanometer of 1 ohm resistance when controlled by a field of 0.17 C.G.S. unit, the cavity within the coil being a sphere of 1 centimetre diameter. It is interesting to notice that to obtain a deflexion of 200 divisions per microampere (a number one might fairly expect to get from a good Thomson instrument having a period of 10 seconds and needles about a centimetre long) would necessitate the controlling field of a non-astatic instrument being reduced to less than one-fiftieth of the assumed value, or to 0.003 C.G.S. unit.

In his calculations Prof. Schuster had no need to take into account the period of vibration of the suspended system, but in the following treatment of the subject period is considered an important factor, and the specific magnetism of the needle is also taken into account. Further, the suspended system is supposed astatic, but this is not essential, for a non-astatic system with an adjustable control would lead to the same results.

The calculations in this section of the paper were made to ascertain whether some unusually high records of galvanometer tests which had been sent to us could possibly be correct. The answer was in the affirmative. A similar investigation had previously been made relating to some instruments of the d'Arsonval type, and this gave an answer in the negative. The calculated maximum sensibility came out about $\frac{1}{20}$ of the published value. Subsequent tests made on an actual instru-

ment gave a result about $\frac{1}{30}$ of the published value, or about two-thirds of the calculated maximum.

In treating the subject mathematically the following assumptions are made :—

- (1) Clearance between coil and end of magnets 1 millimetre.
- (2) Mirror and stem of negligible inertia compared with that of magnets.
- (3) Perfect astaticism of needle. (Thickness negligible.)
- (4) Coil unlimited in size and of best shape as voltmeter.
- (5) Thickness of insulation on wire negligible.
- (6) Control of suspension negligible.
- (7) Deflecting field equal to field at centre of coil.

Note.—All the assumptions except (7), and perhaps (1), will give too high a value for the calculated sensitiveness.

Let $2l$ = length of magnets.
 A = area of cross-section of magnets.
 B = induction density in the steel.
 Δ = density of steel.
 m = strength of pole.
 σ = specific magnetism of needles.
 τ = time of vibration (complete).

Then,

$$\begin{aligned}\text{Magnetic moment} &= 2ml, \\ &= M \text{ (say),} \\ \therefore M &= \frac{2ABl}{4\pi}; \dots \dots \dots (8)\end{aligned}$$

$$\begin{aligned}\text{Mass} &= 2A\Delta l, \\ \text{Moment of inertia } I &= \frac{2}{3}Al^3\Delta; \dots \dots \dots (9)\end{aligned}$$

$$\text{Specific magnetism} = \frac{B}{4\pi\Delta}; \dots \dots \dots (10)$$

$$\therefore \sigma = \frac{B}{98} \text{ approximately.}$$

For an astatic pair we have

$$\tau = 2\pi\sqrt{\frac{2I}{MH_1 - MH_2}},$$

where H_1 and H_2 are the controlling fields acting on the two magnets respectively;

$$\begin{aligned}\therefore H_1 - H_2 &= \frac{8\pi^2 I}{\tau^2 M} \\ &= \frac{8}{3} \frac{\pi^2 l^2}{\tau^2 \sigma} \dots \dots \dots (11)\end{aligned}$$

from (8), (9), and (10).

This gives the difference of the two fields for any periodic time of vibration, length of needle, and specific magnetism.

If θ be the deflexion produced by a current γ , then

$$\text{Controlling couple} = (H_1 - H_2) M \sin \theta,$$

$$\text{and Deflecting couple} = 2M\gamma G \cos \theta,$$

where G is the principal constant of the coil.

$$\begin{aligned} \text{Hence } \gamma &= \frac{(H_1 - H_2)}{2G} \tan \theta, \\ &= \frac{4\pi^2 l^2}{3\tau^2 \sigma G} \tan \theta \quad \dots \quad (12) \end{aligned}$$

from (11).

$$\text{Now } G = N \frac{1}{a^2 \beta^2} \left(\frac{1}{a} - \frac{1}{x} \right) \quad \dots \quad (13)$$

(see Maxwell, vol. ii. p. 325, 1st ed.);

$$\begin{aligned} \text{where } N &= 2\pi \int_0^\pi (\sin \theta)^{\frac{1}{2}} d\theta \\ &= 9.034. \end{aligned}$$

α the ratio of the radius of the wire to the maximum radius of the layer;

β the ratio of the distance between the centre of two adjacent wires to the radius of the wire;

a the maximum radius of cavity in coil;

x the outer radius of coil.

The resistance of the coil is given by

$$R = N \frac{\rho}{\pi a^4 \beta^2} \left(\frac{1}{a} - \frac{1}{x} \right), \quad \dots \quad (14)$$

ρ being the specific resistance of metal used in the winding;

$$\therefore \frac{G}{\sqrt{R}} = \frac{1}{\beta} \sqrt{N \frac{\pi}{\rho} \left(\frac{1}{a} - \frac{1}{x} \right)} \quad \dots \quad (15)$$

A little consideration will show that $\frac{G}{\sqrt{R}}$ gives the value of G for a galvanometer whose resistance is one ohm.

This is a maximum when $\beta = 2$ and $x = \infty$.

Since $a = l + 0.1 \quad \dots \quad \{\text{assumption (1)}\}$

we get

$$\begin{aligned} \gamma &= \frac{4\pi^2 l^2 \beta}{3\tau^2 \sigma \sqrt{N \frac{\pi}{\rho} \frac{1}{l + 0.1}}} \tan \theta \quad \text{from (12) and (15)} \\ &= \frac{4\pi^2 l^2 \beta}{3\tau^2 \sigma} \sqrt{\frac{\rho(l + 0.1)}{N\pi}} \tan \theta \quad \dots \quad (16) \end{aligned}$$

Now for 1 division deflexion at a scale distance of 1000 divisions, $\theta = \frac{1}{2000}$, so taking

$$\begin{aligned}\tau &= 10 \text{ seconds,} \\ \rho &= 1.64 \times 10^{-6}, \text{ and} \\ \sigma &= 50,\end{aligned}$$

corresponding with $B = 5000$ approximately, a high value for short permanent magnets, we get

$$\begin{aligned}\text{Current per 1 division} &= \frac{6.32}{10^{10}} l^2 \sqrt{l+0.1} \text{ C.G.S. units,} \\ &= \frac{6.32}{10^9} l^2 \sqrt{l+0.1} \text{ amperes} \quad (17)\end{aligned}$$

From this formula the following table has been calculated.

TABLE VII.

Half length of needles (l) in centimetres.	Amperes per division.	Divisions per Microampere.
0.05	6.12×10^{-12}	163,000
0.1	2.86×10^{-11}	35,000
0.15	7.16×10^{-11}	14,000
0.2	1.38×10^{-10}	7,250
0.3	3.60×10^{-10}	2,775
0.4	7.14×10^{-10}	1,400
0.5	1.22×10^{-9}	820

With these results before us it is interesting to examine how nearly the limits here given have been approached in actual instruments.

One galvanometer for which the dimensions of the magnets are known is that mentioned in line 9 of Table II. In this instrument the magnets are 0.82 centimetre long, therefore $l = 0.41$. From Table VII. an instrument with such a needle and wound to have a resistance of 1 ohm should give about 1350 divisions per microampere under the assumed conditions. On referring to Table II., line 9, we find the value of $D/r^{\frac{2}{3}}$ (the expression most suited for a high-resistance instrument with small coils) actually obtained to be 380, or little more than a quarter $\left(\frac{1}{3.56}\right)$ the calculated possible value.

Another comparison can be made in the case of Mr. McKittrick's oscillograph, which has a needle of almost microscopic dimensions. In this instrument $l=0.053$, and we see from Table VII., that we might possibly get 150,000 divisions per microampere when the resistance is one ohm, period 10 seconds, and scale-distance equals 1000 scale-divisions. Line 4 of Table II. gives the values 60,000 and 105,000 for $D/r^{\frac{1}{3}}$ and $D/r^{\frac{2}{3}}$ respectively. These numbers approximate more closely to the theoretical values than those for the Mudford galvanometer previously referred to, the probable reason being that the needle of Mr. McKittrick's instrument consists of a piece of soft iron placed in a very strong magnetic field. The specific magnetism would therefore be much greater than that of short permanent magnets in a weak field such as used in controlling the needle of the Mudford galvanometer. In all probability the value of σ for Mr. McKittrick's instrument would be two or three times as great as the value used in calculating Table VII.

SECTION C.

Long versus Short Period Galvanometers for Zero Methods.

For rapidity and accuracy of working a short-period instrument is certainly better than a long-period one, providing its sensitiveness is as great. It is interesting, however, to inquire whether, with the same galvanometer, it is more expeditious to use a strong or a weak control, provided the sensitiveness when strongly controlled is sufficient for the purpose. For although the spot will move to its maximum elongation, corresponding with a given want of balance, more quickly when strongly controlled, that elongation will be less than when a weak control is used. Hence it is possible that a given small displacement of the spot from the zero position may occur in a shorter time when the control is weak than when it is strong, because such displacement is then a smaller fraction of the whole.

First applying general reasoning to the problem, we may notice that the deflecting couple produced by a given want of balance is initially independent of the control, and as the inertia of the moving system is constant, the acceleration at the beginning of the motion is the same whatever the control. If f be this acceleration the space traversed by the spot in a short time t is $\frac{1}{2}ft^2$, and this is therefore independent of the control, consequently for small displacements strong or weak control makes no difference in the time it takes to see that a want of balance exists.

It is to be noted, however, that the weakly controlled system will take longer to return to zero, so that a longer wait is necessary before another test can be made. On the other hand, should the sensitiveness of the strongly controlled instrument be only just sufficient to get the desired accuracy, then half a period must elapse before the want of balance can be detected, whereas the weakly controlled instrument would have moved over a greater distance in that time. Taking the controls in the ratio of 4 to 1, and therefore periods as 1 to 2, the movement of the weakly controlled instrument will be double that of the other in the case here considered. In this instance therefore the long periodic time is advantageous because it will show a very small want of balance more quickly, but the advantage is somewhat discounted by the longer time occupied in the return to zero.

Undamped Motion.

To consider the matter in further detail let us first take the case of undamped simple harmonic motion. Assuming the ratio of the controls to be as m^2 to 1, the periods will be in the ratio of 1 to m .

Fig. 2.

Draw two circles touching at A, centres at B and C, and of radii in the ratio of 1 to m^2 . Draw lines BP and CQ making angles β and γ with AC and such that $\beta = m\gamma$. The distances from A of N and M, the projections on AC of P and Q respectively, will then represent the displacements from zero after a given time of the short and long period galvanometers respectively. From the figure (to scale) it will be seen that these are nearly equal when the time is, say, one-sixth of a period of the quick-moving system. And even when the time is one-fourth of the period of the strongly controlled instrument, the difference in the displacements is not very

marked, for in that time the short-period needle will have moved over the space AB, whilst the long-period needle will have traversed the distance AM'.

Again,

$$\begin{aligned} \text{AN} &= \text{AB}(1 - \cos \beta), \\ &= b(1 - \cos \beta) \quad (\text{say}), \end{aligned}$$

and

$$\begin{aligned} \text{AM} &= \text{AC}(1 - \cos \gamma), \\ &= c(1 - \cos \gamma) \quad (\text{say}); \end{aligned}$$

but

$$c = m^2 b,$$

and

$$\beta = m\gamma,$$

$$\therefore \text{AN} = b(1 - \cos m\gamma),$$

and

$$\text{AM} = m^2 b(1 - \cos \gamma).$$

$$\therefore \frac{\text{AM}}{\text{AN}} = \frac{m^2(1 - \cos \gamma)}{1 - \cos m\gamma}. \quad \dots \dots (1)$$

Expanding the cosines we get

$$\frac{\text{AM}}{\text{AN}} = \frac{m^2 \left\{ \frac{\gamma^2}{2} - \frac{\gamma^4}{24} + \frac{\gamma^6}{720} - \&c. \right\}}{\left\{ \frac{m^2 \gamma^2}{2} - \frac{m^4 \gamma^4}{24} + \frac{m^6 \gamma^6}{720} - \&c. \right\}}.$$

Dividing top and bottom by $\frac{m^2 \gamma^2}{2}$, we have

$$\frac{\text{AM}}{\text{AN}} = \frac{1 - \frac{\gamma^2}{12} + \frac{\gamma^4}{360} - \&c.}{1 - \frac{m^2 \gamma^2}{12} + \frac{m^2 \gamma^2}{360} - \&c.} \quad \dots \dots (2)$$

This, to the 1st degree of approximation, is unity, and only differs slightly from unity when $m\gamma$ is considerable, thus confirming the general reasoning above. Taking m equal 2, as in the figure, and γ equal to $\frac{1}{2}$, i. e. $28^\circ 6'$, we have

$$\begin{aligned} \frac{\text{AM}}{\text{AN}} &= \frac{1 - \frac{1}{48} + \frac{1}{5760} -}{1 - \frac{1}{12} + \frac{1}{360} -} \\ &= 1.06. \end{aligned}$$

Consequently the displacements differ by about 6 per cent. in a time equal to about one-sixth $\left(\frac{1}{2\pi}\right)$ of the period of the quick-moving instrument.

To put the matter in another way, we may determine the relative times after which the spots will have moved appreciably from the zero. A little consideration will show that these times will depend on the relation of the least visible movement to the maximum displacement produced by the want of balance it is necessary to detect. For example, if the sensitiveness of the short-period instrument is such as to give a fairly large movement of the spot for the given want of balance, then a perceptible movement will occur in a time which is a small fraction of its period, and this motion will take place in practically the same time as an equal motion of the spot of the long-period instrument. On the other hand, if the sensitiveness of the short-period instrument is only just sufficient to show the given want of balance when the spot reaches its maximum elongation, then half a period must elapse before the want of balance would be detected, and the more sensitive long-period instrument would have moved an equal distance in a shorter time. In this case, therefore, the more sensitive instrument would have a distinct advantage in quickness in showing the want of balance, but more time would be required for its return to zero before another test could be made.

To show the relative times under different conditions of required accuracy, and of different relative periods of similar galvanometers, we have calculated the following table:—

TABLE VIII.

Ratio of maximum displacement of short-period needle to least movement visible.	Ratio of Time taken by short-period galv. to move visibly longer " " " " "				
	when the sensitiveness of the longer-period instrument is m^2 times as great as that of the short-period instrument.				
	$m^2=2.$	$m^2=4.$	$m^2=16.$	$m^2=64.$	$m^2=10,000.$
1	1.41	1.50	1.55	1.60	1.63
2	1.06	1.09	1.10	1.10	1.11
3	1.034	1.05	1.053	1.054	1.055
4	1.025	1.04	1.045	1.047	1.048
8	1.011	...	...	...	1.021
10	1.01	...	...	...	1.019
20	1.005	...	...	...	1.007

The first column gives a measure of the sensitiveness of the short-period instrument in terms of the minimum possible

value consistent with the desired accuracy, *e.g.*, the 1 in the first column indicates that the instrument is *only just sensitive enough* to indicate the maximum permissible error in balance ; the 2 in the first column denotes that the instrument is twice as sensitive as absolutely necessary ; and the 3 that it has three times the necessary sensibility &c. The numbers in line 1 below the various values of m^2 show the relative times required for the spot to move perceptibly, the time occupied by the slow-moving instrument in moving over the minimum perceptible distance being taken as unity. In column 2 line 1 we find the number 1.41, and this indicates that if the short-period instrument is only just sensitive enough to show the maximum permissible want of balance, then, doubling the sensitiveness by making the period $\sqrt{2}$ times as great, would enable the want of balance to be detected in a shorter time, the times being in the ratio of 1 to 1.41. Similarly for column 3 line 1, increasing the period to twice that of the short-period instrument and therefore increasing the sensitiveness four times would reduce the relative times to the ratio of 1 to 1.5. When m equals 100 (or the sensitiveness is increased ten thousand times, if such an increase were possible) the relative times become 1 to 1.63, showing a comparatively small extra diminution in time required to detect a want of balance, although the sensibility is very largely increased.

The numbers in line 2 show that when the short-period instrument is twice as sensitive as is absolutely necessary the gain in time required to detect the want of balance becomes less, the ratio of the times being 1 to 1.06 when m^2 equals 2, and only reducing to a value 1 to 1.11 when m^2 equals 10,000. Subsequent lines show that when the short-period galvanometer is several times more sensitive than is absolutely necessary to obtain the desired accuracy, the gain in quickness in detecting a small error by weakening the control becomes unimportant, and so the small gain will be more than neutralized by the longer time taken in the return to zero.

Aperiodic Motion.

If the motion of the suspended system be aperiodic, as in well-damped d'Arsonvals, the problem is rather more complicated.

When damping forces act on a vibrating system the equation of motion is

$$\frac{d^2\theta}{dt^2} + \frac{2B}{I} \frac{d\theta}{dt} + \frac{C}{I} \theta = 0,$$

$$\therefore \theta = A_1 e^{-(p+q)t} + A_2 e^{-(p-q)t},$$

where

$$p = \frac{B}{I}, \text{ and } q^2 = \frac{B^2}{I^2} - \frac{C}{I},$$

$$\text{or } q^2 = p^2 - n^2, \text{ where } n^2 = \frac{C}{I}.$$

$$\text{Let } \theta = \alpha \text{ when } t = 0.$$

$$\text{Also } \frac{d\theta}{dt} = 0 \quad ,, \quad t = 0,$$

then

$$\theta = \frac{\alpha}{2q} \epsilon^{-pt} \{ (p+q) \epsilon^{qt} - (p-q) \epsilon^{-qt} \}.$$

If we now lessen the control to $\frac{1}{m^2}$ its initial value, so that the period, if undamped, would be lengthened m times, the sensitiveness will increased m^2 times, and consequently the value of α is increased m^2 times. This change of C alters q , but p is not affected thereby. Let the new value of q be q' .

Under the altered conditions we have

$$\theta' = \frac{\alpha'}{2q'} \epsilon^{-p't} \{ (p+q') \epsilon^{q't} - (p-q') \epsilon^{-q't} \}.$$

We may also write θ in the form

$$\theta = \frac{\alpha}{2q} \{ (p+q) \epsilon^{-(p-q)t} - (p-q) \epsilon^{-(p+q)t} \},$$

and expanding

$$\begin{aligned} \theta &= \frac{\alpha}{2q} \{ (p+q) \left[1 - (p-q)t + \frac{(p-q)^2 t^2}{2} - \&c. \right] \\ &\quad - (p-q) \left[1 - (p+q)t + (p+q)^2 t^2 - \&c. \right] \} \\ &= \frac{\alpha}{2q} \{ 2q + (p^2 - q^2) \left[\frac{t^2}{2} [p - q - (p+q)] \right. \\ &\quad \left. - (p^2 - q^2) \left[\frac{t^3}{3} [(p-q)^2 - (p+q)^2] + \&c. \right] \right\} \\ &= \frac{\alpha}{2q} \{ 2q + (p^2 - q^2) \left[\frac{t^2}{2} (-2q) - (p^2 - q^2) \left[\frac{t^3}{3} (-4pq) + \&c. \right] \right\} \\ &= \alpha \{ 1 - (p^2 - q^2) \left[\frac{t^2}{2} - 2p \frac{t^3}{3} + \text{terms involving } t^4 \&c. p \text{ and } q \right] \} \\ &= \alpha \{ 1 - n^2 \left[\frac{t^2}{2} - 2p \frac{t^3}{3} + \text{terms } \&c. \right] \} \\ \therefore \theta - \alpha &= - \alpha n^2 \left[\frac{t^2}{2} - 2p \frac{t^3}{3} + \text{terms } \&c. \right]. \end{aligned}$$

Similarly

$$\theta' - \alpha' = -\alpha' n'^2 \left[\frac{t^2}{2} - 2p \frac{t^3}{3} + \text{terms involving } t^4 \text{ \&c. } p \text{ and } q' \right].$$

Hence the ratio of the two displacements is

$$\frac{\theta' - \alpha'}{\theta - \alpha} = \frac{\alpha' n'^2 \left[\frac{t^2}{2} - 2p \frac{t^3}{3} + \text{terms in } t^4 \text{ \&c. } p \text{ and } q' \right]}{\alpha n^2 \left[\frac{t^2}{2} - 2p \frac{t^3}{3} + \text{terms in } t^4 \text{ \&c. } p \text{ and } q \right]},$$

but $\alpha' = m^2 \alpha,$

and $n'^2 = \frac{1}{m^2} n^2,$

$$\therefore \alpha' n'^2 = \alpha n^2,$$

$$\therefore \frac{\theta' - \alpha'}{\theta - \alpha} = 1 \times \left[\frac{\text{terms in } t^4 \text{ \&c. } p \text{ and } q'}{\text{terms in } t^4 \text{ \&c. } p \text{ and } q} \right].$$

Thus, when $\frac{t^4}{4}$ and higher powers are small, the ratio is unity, consequently there is no difference in the rapidity of detecting want of balance when the sensitiveness of the short-period instrument is sufficient for the purposes of the experiment.

To find the ratio in the other cases we may observe that the general term in t^r is, when r is odd,

$$-\frac{t^r}{r} \left\{ (r-1)p^{r-2} + \frac{(r-1)(r-2)(r-3)}{3} p^2 q^2 + \dots + (r-1)pq^{r-3} \right\};$$

and when r is even

$$+\frac{t^r}{r} \left\{ (r-1)p^{r-2} + \frac{(r-1)(r-2)(r-3)}{3} p^{r-4} q^2 + \dots + q^{r-2} \right\}.$$

Decreasing the control decreases n^2 and therefore increases $q,$

$$\therefore q^2 = p^2 - n^2.$$

Hence q' is $> q,$

consequently the term in t^4 in the numerator of $\frac{\theta' - \alpha'}{\theta - \alpha}$ will be greater than the corresponding term in the denominator, and this indicates that the displacement of the slow-moving instrument gains on that of the quick-moving one as the time increases.

The above investigation teaches us that when we have a galvanometer whose control can be readily altered and whose sensitiveness can be easily made ample for the purpose of the test, then *for rapid working adjust the control so that the sensitiveness is, say, two or three times as great as is absolutely necessary for the desired accuracy.*