XIII. General method for determining the orbits of comets

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Coal-soil. I cannot particularize, at present, the immediate stratum, in which the specimens principally occur, through the coal-measures; but I have had specimens of it in argillaceous gritstone from Alfreton; in ironstone from Wingerworth, and in the coal shale and the coal itself, from near Buxton. I have not yet observed it in that peculiar siliceous gritstone, found in the coal-measures, called by the miners *crowstone*, although this stone abounds with other vegetive *reliquia*.

Phytolithus corticiradix & *Phytol. compressiradix* T. 9. Millstone grit stratum? Chapel-en-le-Frith. The specimens were found in a wall about $\frac{1}{2}$ a mile from the town, on the Manchester road. I have not ascertained the quarry, from which the stone was brought. Coal-measures, in argillaceous grit; in a small quarry about $1\frac{1}{4}$ mile from Leek, a few yards left of the first Toll-bar on the road to Buxton, in considerable quantity (shale-grit?).

XIII. General Method for determining the Orbits of Comets.

By M. LAPLACE.

[Concluded from p. 49.]

HAVING thus the true perihelion distance, and the true instant of the passage of the comet by this point, we shall conclude from them the other elements in this way.

Let j be the position of the node which would be ascending if the motion of the comet were direct, and ϕ the inclination of the orbit: we shall have the six following equations,

$$\begin{aligned}\text{tang } j &= \frac{\text{tang } \varpi \sin \zeta' - \text{tang } \varpi' \sin \zeta}{\text{tang } \varpi \cos \zeta' - \text{tang } \varpi' \cos \zeta} \\ \text{tang } j &= \frac{\text{tang } \varpi \sin \zeta'' - \text{tang } \varpi'' \sin \zeta}{\text{tang } \varpi \cos \zeta'' - \text{tang } \varpi'' \cos \zeta} \dots (e) \\ \text{tang } j &= \frac{\text{tang } \varpi' \sin \zeta'' - \text{tang } \varpi'' \sin \zeta'}{\text{tang } \varpi' \cos \zeta'' - \text{tang } \varpi'' \cos \zeta'} \\ \text{tang } \phi &= \frac{\text{tang } \varpi}{\sin (\zeta - j)} \\ \text{tang } \phi &= \frac{\text{tang } \varpi'}{\sin (\zeta' - j)} \\ \text{tang } \phi &= \frac{\text{tang } \varpi''}{\sin (\zeta'' - j)} \dots (e');\end{aligned}$$

We may choose ad libitum among these formulæ: but it will be most correct to employ those whose numerators and denominators are greatest.

Suppose, for example, that we employ the two formulæ (e) and (e'): it is visible that the tangent of j may equally belong to the two angles j , and $180^\circ + j$, j , being the smallest of the positive angles to which it can belong. In order to determine which of the two to choose, we shall observe that ϕ and $\tan \phi$ ought to be positive, and thus, $\sin(\xi'' - j)$ ought to be of the same sign with $\tan \omega''$; this condition will determine the angle j , and this angle will be the position of the ascending node if the motion of the comet be direct; but if this motion be retrograde we must add 180° to it, in order to have the position of this node.

The hypotenuse of the rectangular spherical triangle of which $\xi'' - j$ and ω'' are the sides, is the distance from the comet to its ascending node in the third observation; and the difference between that hypotenuse and ν'' is the interval between the node and the perihelion, reckoned on the orbit.

Let us apply these results to the second comet of 1781, whose perihelion distance and instant of passage by this point we have already determined by a first approximation. For this purpose, we shall use some observations of the 9th of October 1781, and 17th of November and 20th of December of the same year: these observations give

Mean time at Paris.

1781. 9th Oct. at 16 ^h 50' 0'',	$\alpha = 124^\circ 27' 42''$,
	$\theta = 0^\circ 11' 40''$,
17th Nov. at 8 ^h 29' 44'',	$\alpha' = 306^\circ 57' 32''$,
	$\theta' = 44^\circ 17' 12''$,
29th Dec. at 6 ^h 6' 30'',	$\alpha'' = 306^\circ 17' 59''$,
	$\theta'' = 17^\circ 34' 25''$,

We have moreover

$$\begin{aligned} C &= 197^\circ 13' 44'', & \log R &= 9,998864, \\ C' &= 235^\circ 55' 43'', & \log R' &= 9,994602, \\ C'' &= 269^\circ 20' 35'', & \log R'' &= 9,992748. \end{aligned}$$

This being done; we shall form a first hypothesis, in which the perihelion distance will be, as we have found above, equal to 0,958359, and the instant of the perihelion passage took place on the 29th of November, at 18^h 10' 34''; we shall find in this hypothesis

$$\nu = -61^\circ 18' 3'', \quad \nu' = -18^\circ 7' 12'', \quad \nu'' = 29^\circ 8' 15'',$$

which gives

$$U = 43^\circ 10' 41'', \quad U' = 90^\circ 26' 18'';$$

we shall afterwards find

$$\begin{aligned} \xi &= 77^\circ 5' 50'', & \xi' &= 37^\circ 26' 37'', & \xi'' &= 346^\circ 49' 52'', \\ \omega &= 0^\circ 10' 34'', & \omega' &= 18^\circ 6' 32''\frac{1}{2}, & \omega'' &= 27^\circ 12' 57'', \end{aligned}$$

from which we extract

$$V = 42^\circ 53' 2'', \quad V' = 90^\circ 9' 22'',$$

dividing

$$m = 17' 49'', \quad n = 16' 56''.$$

The series of the values of ϵ , ϵ' , ϵ'' , visibly indicates a retrograde motion.

We shall afterwards form a second hypothesis, in which, by preserving the foregoing instant of the perihelion passage, we shall increase the perihelion distance by 0,003. We shall find in this hypothesis

$$m' = -33' 53'', n' = -12' 54''.$$

Lastly, we shall form a third hypothesis, in which, by preserving the same perihelion distance as in the former, we shall vary by $0^d 25$ the instant of the perihelion passage, which will thus be fixed on the 29th of November at $12^h 10' 31''$; this hypothesis will give

$$m'' = 48' 16'', n'' = 27' 13'';$$

we shall extract from these values the two following equations,

$$3102 \cdot u - 1829 \cdot t = 1069,$$

$$1760 \cdot u - 617 \cdot t = 1016,$$

which gives

$$u = 0,881406, t = 0,910400 :$$

hence we conclude

the true perihelion distance = 0,9609951,
and the true instant of perihelion passage, the 29th of November at $12^h 42' 46''$ mean time at Paris.

In order to ascertain if these elements are very accurate, we may calculate the corresponding values of m and n , and see if they are null or very small: now we shall find that in the present case these values amount only to a small number of seconds; for the corrected elements give, for instance, for the first and last observation,

$$\begin{aligned} v &= -60^\circ 56' 37'', & v'' &= 29^\circ 19' 22'', \\ \varpi &= 10' 33'' \frac{1}{2}, & \varpi'' &= 27^\circ 11' 56'' \frac{1}{2}, \\ \epsilon &= 77^\circ 2' 22'', & \epsilon'' &= 346^\circ 38' 53''; \end{aligned}$$

from which we extract

$$U' = 90^\circ 15' 59'', \quad V' = 90^\circ 16' 3'' \frac{1}{2},$$

and consequently $m = -4'' \frac{1}{2}$. These elements being very accurate, we shall extract from them by means of the formulæ (e) and (e') the position j of the ascending node, and the inclination of the orbit, and we shall find

$$\text{Place of the ascending node} = 77^\circ 22' 55''.$$

$$\text{Inclination of the orbit} = 27^\circ 12' 4''.$$

In order to determine the place of the perihelion, we shall observe that $\epsilon'' - j = 269^\circ 15' 58''$, from which we extract $269^\circ 20' 50''$, for the distance from the comet to its node, reckoned on the orbit, at the instant of the third observation: on adding to this distance the anomaly v'' which the comet has traversed by a retrograde motion since its perihelion passage, we shall have $298^\circ 40' 12''$, for the distance from the perihelion, reckoned on the orbit, to its ascending

node: by adding afterwards to this distance the longitude of this node, we shall have for the position of the perihelion on the orbit, $37^{\circ} 6' 7''$, or more simply still $16^{\circ} 3' 7''$. By collecting all these elements, therefore, we shall have for the true elements of the orbit of the second comet of 1781,

Perihelion distance 0.960995 i.

29th Nov. 1781. Mean time at Paris of the } 12^h 42' 46"

perihelion }

Place of the perihelion of the orbit . . $16^{\circ} \ 3' \ 7''$.Place of the ascending node $77^{\circ} 22' 55''$.

Inclination of the orbit $27^{\circ} 12' 4''$.

The motion of the comet is retrograde.

The supposition of the parabolic motion of the comets is not rigorous; it is even infinitely little probable, considering the infinite number of cases which give an elliptic or hyperbolic motion, relative to those which determine the parabolic motion. A comet moved either in a parabolic or hyperbolic orbit will only be visible once; we may suppose, therefore, with probability, that comets which describe these curves, if some of them exist, have long ago disappeared, so that nowadays we only observe those which, being moved in re-entering orbits, are incessantly brought back to intervals more or less large, in the regions adjoining the sun. If they have been observed with precision, and the visible arc of their orbits be considerable, we may determine with tolerable accuracy, by the following method, the time of their revolution.

For this purpose let us suppose that we have four excellent observations which embrace nearly the whole visible part of the orbit; and that we have already determined by the foregoing article, the parabola which nearly satisfies these observations. Let v, v', v'', v''' , be the anomalies corresponding to these observations; r, r', r'', r''' , the corresponding vector radii; take also

$$v' - v = U, v'' - v = U', v''' - v = U''.$$

This being done, we shall calculate by the foregoing article, with the parabola which nearly represents these observations, the values of U , U' , U'' , and those of V , V' , V'' ; take therefore

$$U - V = m, \quad U' - V' = m', \quad U'' - V'' = m''.$$

We shall afterwards vary by a very small quantity the perihelion distance in this parabola; take then

$$U - V = n, U' - V' = n', U'' - V'' = n'.$$

We shall afterwards form a third hypothesis, in which, by preserving the same perihelion distance as in the first,

we

we shall vary by a very small quantity the instant of the perihelion passage: take in this hypothesis,

$$U - V = p, \quad U' - V' = p', \quad U'' - V'' = p''.$$

This being done, with the perihelion distance and the instant of the passage of the comet by this point, found in the first hypothesis, we shall calculate the angle v and the vector radius r , in the supposition of a very eccentric ellipsis; so that by calling e the relation of the eccentricity of this ellipsis to its half great axis, the difference $1 - e$ is equal to a very small quantity, for instance to $\frac{1}{10}$. In order to have the angle v , in this supposition, it will be sufficient to add to the anomaly v , calculated in the parabola, a small angle, the sine of which is

$$\frac{1}{10} (1 - e) \cdot \text{tang } \frac{1}{2} v \cdot (4 - 3 \cdot \cos^2 \frac{1}{2} v - 6 \cdot \cos^4 \frac{1}{2} v);$$

The new anomaly v being thus known, we shall substitute it in the equation

$$r = \frac{D}{\cos^2 \frac{1}{2} v} \left(1 - \frac{1 - e}{2} \text{tang}^2 \frac{1}{2} v \right)$$

which is the expression of the vector radius in a very eccentric ellipsis: by this means we shall have the corresponding vector radius r . We shall calculate in the same way $v', r'; v'', r''; v''', r'''$; from which we shall extract U, U', U'', V, V', V'' ; in this case take

$$U - V = q, \quad U' - V' = q', \quad U'' - V'' = q''.$$

Finally, let us call u the number by which we ought to multiply the supposed variation in the perihelion distance, in order to have the true one; t the number by which we ought to multiply the supposed variation in the instant of the perihelion passage; and δ the number by which we ought to multiply the value supposed for $1 - e$; we shall form the three equations

$$u \cdot (n - m) + t \cdot (p - m) + \delta \cdot (q - m) + m = 0,$$

$$u \cdot (n' - m') + t \cdot (p' - m') + \delta \cdot (q' - m') + m' = 0,$$

$$u \cdot (n'' - m'') + t \cdot (p'' - m'') + \delta \cdot (q'' - m'') + m'' = 0.$$

We shall have, by means of these equations, the values of u , t and δ , from which we shall extract the true perihelion distance, the true instant of the passage by this point, and the true value of $1 - e$. Let D be the perihelion distance, and a the semi great axis; we shall have $a = \frac{D}{1 - e}$;

from which it is easy to conclude, that the time of the revolution of the comet will be equal to the number of sidereal

years expressed by $\frac{D^{\frac{3}{2}}}{(1 - e)^{\frac{3}{2}}}$. We shall have as in page

85, the inclination of the orbit and the positions of the ascending node and of the perihelion.

When the arc observed of the orbit of a comet is considerable, and particularly when it extends beyond 90° of anomaly, where the ellipticity begins to become perceptible; it will be desirable that we should have four observations made with all the precision which we ought to expect from modern astronomy, taking care to verify the position of the stars to which we refer the motion of the comet.

Whatever may be the precision with which we strive to make these observations, they will always leave some uncertainty as to the time of the revolution of comets: the most accurate method for determining it, is to compare the observations of a comet in two consecutive appearances: the resemblance of the elements of the two parabolic orbits determined by these observations, will make known the identity of the comet, and we shall have, by the difference of the instants of the perihelion passage, the time of its revolution and its grand axis. It was thus that the period of the comet observed in 1531, 1667, 1682, and 1759, was determined; a period which is a little unequal, on account of the attraction of the planets, as M. Clairaut has shown, by subjecting to analysis the perturbations experienced by this comet from Jupiter and Saturn.

XIV. Account of some remarkable Cases of Venereal Infection.

To Mr. Tilloch.

SIR, PERMIT me to communicate some medical facts which lately occurred in my practice; and as there is not any sufferer upon whom the circumstances can reflect dishonour, or whose delicacy can suffer by the relation, an insertion of the entire matter in your instructive Magazine may convey original information to professional men, and will oblige

Your obedient servant,

ROBERT HEALY, M.B.

On the 23d of July 1810, I was requested by a respectable friend to see his wife, who complained of great and general debility, loss of appetite, with violent pain of the head. She was nursing. The child seemed very healthy. She said she was attacked with hæmorrhoids about three weeks after her lying-in, which was on the 11th of May 1810. She had had small glandular swellings in her groins, which had subsided;