IV.—THE DISJUNCTIVE JUDGMENT.

By G. R. T. Ross.

THE main object of this paper is to try to determine the import of the disjunctive judgment and to find out the exact place which it occupies in the connected whole of logical thought.

The point which I wish to raise first is as to the question of the exclusiveness of the judgment. I wish to bring up certain arguments to combat the theory held by Mr. Bradley and Mr. Bosanquet when they declare that in the disjunctive judgment (A is either B or C), properly interpreted, the alternatives B and C are exclusive of each other. It is my intention rather to uphold the view of Mr. Keynes, who is quite as sure that the common proposition with 'either' is merely an alternative judgment and that B and C do not, according to the mere form of the thought, necessarily exclude each other. It seems to me that there are several considerations based on common logical usages which go to show that Mr. Keynes is right.

Mr. Bradley (*Principles of Logic*, p. 124) allows that the form of words "A is either B or C" may sometimes be used when we do not wish to deny that A may be both B and C, but he declares that, when using this expression, we leave out of sight the contingency that A may be both B and C, and finally asserts that, in such cases, our language is slovenly, implying that if we wished to be accurate we should say "A is either B or C or both B and C"—all the alternatives being exclusive. We might criticise this conclusion and ask how it is that B C can be exclusive of B and C when as a matter of fact it includes them.

If alternatives are necessarily exclusive of each other we shall have to make our judgment still more pedantic and say "A is either B alone or C alone or both B and C"—"He is either merely a fool or merely a rogue or both a fool and a rogue". The mere fact that in order to make our disjunction

exclusive we have to prefix "merely" to the terms, should indicate that naturally it is not exclusive.

It is, of course, matter of fact that, when we use the judgment symbolised by "A is either B or C," very frequently B and C are exclusive, but what I wish to maintain is that this is owing to the nature of those terms, and is not to be inferred from the form of the judgment. When we make a disjunction, we, more often than not, divide a genus into its species, and of course co-ordinate species are mutually exclusive; they would not be species unless they were exclusive, but frequently the predicate concepts are not of species but of attributes not known to be specific, and then we cannot say that they are exclusive of each other.

No doubt one of the aims of science is to divide genera into mutually exclusive species, just as it is to find a relation between subject and predicate in virtue of which we can say not only "All S is P" but "All P is S," i.e. to attain to the predication of property. Now, though in many cases when we assert that "All S is P" we know that "All P is S," no one maintains that we can infer from the form of the judgment that this is so. But, if logicians like Mr. Bradley consistently applied the principle they adopt in interpreting the disjunctive judgment, they would have to assert that the universal affirmative, properly employed, is simply convertible.

So much for the argument from consistency, but there are other considerations which go to show how inexpedient it is to treat the disjunctive judgment as necessarily exclusive.

It is generally admitted that the force of a disjunction can be partially rendered by hypotheticals. The hypotheticals do not exhaust its content, for disjunctive judgment is the union of hypotheticals upon a categoric basis. "A is either B or C" means that A is positively related to X the underlying identity of both B and C. Though our alternatives be B and not-B, the mere *nomen indefinitum* (in which case the disjunction is *a priori*, merely a particular instance of the law of excluded middle, and can be made, whatever the concept symbolised by B), there is still, I suppose, the underlying identity of Being, the X which pervades both B and not-B.

But commonly B and not-B are merely contradictories within a genus or limited "Universe of Discourse". In such a case the alternatives are more properly symbolised by $b \ x$ and $b' \ x$ as, for example, in this instance "triangles are either right-angled or not right-angled". The categorical assumption at the basis of this disjunction is that the triangle has

angles. But in this and all similar examples the disjunction proper, the relation asserted between the two predicate terms, is derived *a priori*, that is to say, it is deduced not from some principle peculiar to the science with which we are dealing, but from one of the *κοινὰ ἀξιώματα* common to all thought, in fact, in this case—disjunction—the law of excluded middle. When we say that triangles are either right-angled or not we are simply giving an instance of the *principium exclusi tertii*, which could be equally well illustrated by the predication of any other attribute appropriate to the science in question and its contradictory.

Now the hypotheticals into which it is claimed by the exclusivist theory that a disjunction can be analysed are "If A (which is assumed to be X) is not *b* it is *c*" (categorically, "All not-*b*¹ is *c* = *b'* ac") together with its equivalent, "If A is not *c* it is *b*," together with the other geometrically converse non-inferable hypothetical, "If A is *b* it is not *c*" (categorically "No *b* is *c* = *b* e c"). The first hypothetical shows that the disjunction is exhaustive, the second that it is exclusive.

But in the case we were discussing, that of a *a priori* disjunction, the alternatives are given as *b* and *b'*. So that our statement is analysable into the two jejune truths "All not-*b* is *b'* = *b'* a *b'*" and "No *b* is *b'* = No *b* is not-*b*". Obviously if the terms of our disjunction are given as *b* and *b'*, i.e., a positive concept and its corresponding *nomen indefinitum* (which, though it be restricted in range, is still indefinite as containing a possible endless internal plurality), then our judgment is both exhaustive and exclusive. But such a judgment is of no importance in itself and finds no place in the science within which the subject of predication lies. It is an empty truth and we do not get real disjunction

¹ I hope my symbols may not be misunderstood. Strictly the categorical judgments which should represent "A is either *b* or *c*" are "All A which is not *b* is *c*" and (on the exclusivist theory) "No A which is *b* is *c*," or perhaps "All A (agreed to be X) which is not *b* is *c*" and the corresponding negative. But for brevity the reference to the *ὑποκείμενον* A or AX may be left out, as it is, so to speak, a common factor in all the judgments and does not enter specially into our further reasoning. That part of the content of thought through synthesis with which fresh determinations are added to the subject is represented by the concepts *b* and *c*.

N.B. The symbols *a* and *c* refer nowhere to terms, but indicate the quantity and quality of the judgment as in Mr. Keynes's scheme. The symbols S and P which in his scheme represent the terms are inadmissible here, as the alternatives *b* and *c* are generally viewed equally as subjects (S) when viewed as subjects, and equally as attributes (P) when viewed as attributes.

until the alternative terms both stand for positive concepts, until we know what we mean by b' (in our illustration acute and obtuse angled), in which case our judgment is better symbolised by "A is either B or C" than by "A is either B or B'".

We have then to interpret what we mean by a real disjunction, one in which C is not by definition not-B. It is still apparent in those disjunctions that a categorical basis is assumed. When we say such and such a kind of flower blooms either in spring or in autumn, "blooms" is the predicate categorically asserted, and when we declare that some one is either a fool or a rogue, objectionable person is probably the basal quality common to both.

Now there are three possible suppositions as to the relation between b and a in real disjunctions.

(1) That both propositions hold, both "If A is not b it is c " and "If A is b it is not c " ($b'ac + bec$). This is the exclusivist theory and its supporters point to such judgments as "lines are either straight or curved," "organ pipes are either closed or open" in seeking evidences for their plea. The special type of such judgments as those last quoted, where it is apparent that both the independent assertions SaP and ScP are true, will be investigated later on.

(2) Then there is the possibility that b and c may be exclusive but need not between them exhaust the whole of the proximate genus, e.g., dog and wolf do not exhaust the Canidæ. Such a statement, however, is not a disjunction. Logicians are quite clear that the disjunctive judgment is at least exhaustive, that all not- b is c . If I say—"this species of fish is found either in lakes or rivers," when I know that it is found also in the ocean, I make a misleading statement.

(3) There is lastly the possibility that b and c may be merely alternatives not mutually exclusive, as, e.g., fool and rogue. Now if the first possibility is true and both the hypotheticals "If A is not b it is c " and "If A is b it is not c " are to be found in the disjunction, it will follow that "A is either B or C" and "A is either not B or not C" mean exactly the same thing; for the former is equivalent to—"If A is not b it is c " and "If A is b it is not c " (all not b is $c + no\ b$ is c) and the second becomes "If A is not not- b it is not c " and "If A is not- b it is not not- c "; these simplified come to—"If A is b it is not C" and "If A is not b it is c "—exactly the same pair of propositions as the other disjunction yielded.

This result can be shown in another way. According to the theory of complete exclusion both the *modus tollendo*

ponens and the *modus ponendo tollens* in the disjunctive syllogism are valid. But whether the major premiss be "A is either B or C" or "A is either not B or not C," we can by applying the same minor premisses get the same conclusion.

<p>(1) A is either B or C
A is not B
∴ A is C</p>	<p>(2) A is either B or C
A is B
∴ A is not C</p>
<p>(1) A is either not B or not C
A is not B
∴ A is C</p>	<p>(2) A is either not B or not C
A is B
∴ A is not C</p>

But surely it is clear that when we say "A is either B or C" we do not mean the same thing as "A is either not B or not C" and *vice versa*. If some one tells us he has seen a ghost, we may declare that he is either not sane or not telling what he believes to be the truth. But it would be rather perplexing for the person who uttered this to be assured that from his assertion could be inferred the other—that the ghost-seer was either sane or truthful. While admitting that sanity and truthfulness in a ghost-seer are exclusive of each other he would not desire to make any disjunction between these qualities. He means that no person who asserts that he has seen and talked with a ghost and is sane, is truthful, but not that all such who are insane are truthful. Many people may be neither sane nor truthful.

(Note.—Diagrams to represent the three possible cases of relation of *b* and *c*.)

- (1) $b'ac + bec$ both true.
- (2) bec alone true.
- (3) $b'ac$ alone true.

The above example is illustrated by (2) if *b* and *c* are taken to represent 'truthful' and 'sane' respectively. The disjunction is exhaustive between *b'* and *c'*, for all not *b'* is *c'* = all *b* is *c'* = no *b* is *c*.)

Again, since on the exclusive interpretation of disjunction "A is either B or C" and "A is either not B or not C" are equivalent expressions, it will be impossible to commit the fallacy of denial of the antecedent or affirmation of the consequent in dilemma.

The formula for a simple dilemma is :—

If A is B it is D
and if A is C it is D
But A is either B or C
<hr style="width: 50%; margin: 0 auto;"/>
∴ A is D

But as alternatives are exclusive, A is either not B or not C will be equivalent to A is either B or C.

If we take as an example of a minor premiss in a dilemma the statement which we find in Jevons (*Elementary Lessons*, p. 168) as conclusion of a destructive dilemma, that a person who speaks irreverently of Scripture is either not wise or not good and add as a major that we should cherish the company of one who is either wise or good, then on the theory of the equivalence of positive and negative disjunctions we should have to conclude that we should cherish the company of one who speaks irreverently of Scripture!

To avoid this result, those who hold to the position that a disjunctive judgment should express complete exclusion would have to maintain that the above negative disjunctions are carelessly worded; but, as has been pointed out before, no amendment would be satisfactory that stopped short of the very far-fetched formula: 'He is either merely not wise or merely not good or neither wise nor good'.

But it is quite unnecessary to adopt a form of words so remote from ordinary expression, for it can be shown that the disjunction serves all logical purposes (except indeed the establishment of a conclusion by the disputed *modus ponendo tollens* which will be discussed later on) if we interpret it as merely exhaustive without necessarily being exclusive, i.e., 'If A is not *b* it is *c*' is the only hypothetical necessarily implied in disjunction.

The minor premiss of a dilemma is always a disjunction, but this disjunction enters the argument only so far as it is exhaustive; furthermore, however we interpret the minor premiss, whether as exhaustive only or both exclusive and exhaustive, the conclusion of the argument, when disjunctive, is proved only in so far as it is exhaustive, never as exclusive.

Since hypothetical and disjunctive reasoning is true only in so far as it obeys the canons which were formulated first of all for the categorical syllogism, and since reasoning is perhaps clearest when reduced to that type, I propose to prove my last assertions by an analysis of a dilemma which reduces it to its categorical elements.

The simplest and normal type of complex constructive dilemma is:—

If A is B it is D = all b is d = bad

If A is C it is F = all c is f = caf

A is either B or C = if A is not b it is C = $b'ac$ (+ if A is b it is not c = bec)

∴ A is either D or F = if A is not d it is f = $d'af$

It will be shown that from the given premisses the conclusion cannot be made to yield if A is d it is not f = def . From the various premisses shown above we have to elicit a conclusion showing a relation between d and f . Take the minor $b'ac$; this gives by immediate inference $c'ab$, which combined with the first limb of the major, bad , gives as conclusion $c'ad$:

$$\begin{array}{r} bad \\ b'ac = c'ab \\ \hline c'ad \end{array}$$

Similarly $b'ac$ and caf give $b'af$. Again $c'ad = d'ac + caf$ give $d'af$, i.e., all not d is f = if A is not d it is f ; which is the same as 'A is either D or F,' interpreted only exhaustively.

And it can be easily seen that, however we interpret the minor, it is impossible to reach the conclusion def = no d is f ; for bec with bad will yield no universal conclusion, nor will it do any better when taken along with caf . Hence, as there is no other available premiss, there is no way of reaching the conclusion def , which we should have to do if 'A is either D or F' were exclusive.

A man at the top of a burning house may reason thus, 'If I jump from the window I shall break my leg and if I do not I shall be burned to death'. Between jumping and not jumping there is no middle course. These two alternatives exclude one another and exhaust all the possibilities (it is a case of *a priori* disjunction), but though the minor premiss is, in this case, both exhaustive and exclusive, the conclusion cannot be held to be so. It is—"I must either get my leg broken or be burned to death". But this can only be held to mean—"If I do not face the certainty of getting my leg broken I shall be burned to death"—and its equivalent—"If I escape death by burning it will be only at the cost of a broken leg". But the unfortunate man cannot be sure that the accident to his limb will procure him safety from burning (he might be scorched fatally even though he took the leap); nor can he promise himself that he will not break his leg before the flames consume him. It is obvious then that

the disjunctive judgment as used in dilemma is exhaustive only and not exclusive.

Now it is in this form of argument, I submit, that disjunction is most commonly employed. No doubt the dilemma is very frequently dialectical (in the Aristotelian sense) for the reason that an exhaustive minor premiss of the type All not-*b* is *c* is hard to substantiate even when *b* and not-*b* fall within a limited genus *x* and are not logical contradictories which between them exhaust all reality like *S* and not-*S*. "All non-parallel straight lines meet" is an example of a proposition of the kind which is immediate and does not require proof. (It may be called axiomatic but yet it is not *a priori* in the sense of being drawn from a principle superior to the science of Geometry itself; it is a peculiar immediate principle of that science.)

But it is very difficult to *deduce* a judgment of this kind. Generally speaking it is impossible to be sure that we have exhaustively divided a genus unless the *fundamentum divisionis* itself contain distinctions which can be exhaustively enumerated, *e.g.*, we can divide hawks into long- and short-winged, or organ pipes into closed and open, when those two distinctions, while contradictory to each other, are both positive and exhaust the possibilities as to the relation between the end of the pipe and the outside air.

Again, when the minor premiss of a dilemma is exhaustive either immediately or as a result of proof, then in those circumstances it is common for one or other of the limbs of the major premiss to be false or at least not to be a necessary truth.

For example our man in the burning house says that if he does not jump he is sure to perish by burning, but this being a predication concerning a finite individual is contingent; it can only be a statement of probability.

Though the dilemma is thus chiefly employed in 'dialectic' and many logicians almost omit to mention it on this account, still it is one of the chief arguments in which the disjunctive judgment is employed.

Apart from this, the proper place of disjunction in science is in division, which is not proof, but practically a special *μέθοδος* by itself. Any attempt to discover the properties of an object by *διαίρεσις* involves a begging of the question at each step in the division.

For example, if one begins by dividing all reality by dichotomy and at length arrives at a genus *A* opposed to not-*A* which again falls into divisions *B* and *C*, then if we independently know that the object of which we are treating

is not B we can say it is C, but only if we have already subsumed this particular thing under A; the proof must always rest on this assumption. This is one way of stating the objections to the value of the disjunctive syllogism. Mr. Bosanquet has indeed gone so far as to assert that the disjunctive syllogism is no inference, but that the whole of the inference is contained in the major premiss—the disjunctive judgment which is itself an inference. This is surely misleading, and I shall try to show that the disjunctive syllogism has a certain limited value in science but that the value of the inference lies wholly in the *modus tollendo ponens*.

The judgment which we are here considering may be, for want of a better name, styled the *divisive* disjunction. In matters of empirical knowledge it is, as Sigwart (*Logic*, i., p. 230, Eng. Trans.) points out, preceded by the divisive judgment—‘Some A is B and some A is C’. Both predicates are considered as actual or realisable.

It is what Mr. Keynes (*Formal Logic*, p. 232, third edition) refuses to regard as a true alternative, but merely an “alternative synthesis of terms”. Mr. Bosanquet however (*Logic*, i., p. 345) thinks it to be the “true or ideal disjunction”; it is a “generic judgment whose content is developed or interrelated by the aid of hypotheticals”.

It is, of course, different from the ‘disjunction of ignorance’. We may have judgment of both types about the same subject. Thus we may say ‘The triangle is either equilateral, isosceles or scalene’—a true divisive disjunction—and again, ‘the triangle is either an arbitrary fiction of the imagination or an objective determination of reality’. In divisive disjunctions the subject is used distributively or as a system of interrelated parts or functions; in disjunctions of ignorance, on the other hand, the subject is viewed non-distributively, i.e., as an individual in the strict sense. If ‘the signal light is either red or green’ is not a disjunction of ignorance, it must be a predication about the functions of the signal light and tell us that sometimes it shines green and at other times red.

Now Mr. Bosanquet’s objection to the disjunctive syllogism is that the minor premiss adds no content to the disjunctive major. But is this so? Surely we can specify in the minor under what conditions the signal shows either red or green. We may say “With the lever in the cabin so, the signal cannot be green, hence it is red”.

Or again, if our subject be a genus ‘A,’ we may say, ‘All A is either B or C,’ but in the minor we are not limited to this ‘A’ as a subject; surely we may specify our subject

and say species *a* is not B and hence it is C. The assumption came in, as we saw, in ranking species *a* under A; the *disiunctio* is presupposed, but apart from this there is genuine inference. What we have done is to limit our discussion to the genus or universe of discourse marked by A, but within this there is a genuine synthesis of content, *a* with *c*, and it is a mediated synthesis; hence it is, so far, a genuine inference.

The divisive disjunction is used chiefly perhaps in classification. Now classification depends upon the mutual exclusiveness of species, and the distinction of species in a genus depends, I shall try to show, on certain indemonstrable negations. If our disjunction is used in classification then the *modus ponendo tollens* is valid (though *a priori* and producing no conclusion in itself valuable), but a divisive judgment need not be one of classification, and hence may not permit of the employment of the *modus ponendo tollens*. I shall give an example of this latter class and try to show how such a judgment passes into a classificatory disjunction.

We may say that such and such a kind of fish is found either in streams or in fresh water lakes. This is more than the conjunctive proposition that the kind of fish in question is found both in streams and in lakes, for we have denied that it is found elsewhere (*e.g.*, the ocean) than in the two places mentioned. But our assertion does not deny that the very same kind of fish is found in both. In this case we may very well understand the reverse, nor does our statement imply that the same individual may not live now in a lake at another time in a stream.

Now, we may find that those individuals found in the streams have peculiarities distinct from the characteristics of the denizens of lakes. We may find that living in the different localities may accompany or may indeed have helped to produce differences of behaviour and structure in each case, and hence we may be able to divide our genus into two species or at least varieties of fish, according to the amount of difference between the two classes. The moment that we understand that the attributes river- and lake-inhabiting mark two different classes, then to the exhaustive judgment All not B is C we are able to add the exclusive statement No B is C. But this is a merely *a priori* judgment drawn from the general logical principle that co-ordinate species are distinct, which again is a corollary of the law that the individual cannot be predicated, for co-ordinate species are related to each other as individuals.

The knowledge that we have discovered two classes within our genus is dependent on the proposition that 'No B is C,' but not only on this but on the fact that B and C carry with them peculiar properties, so that in virtue of this statement as premiss we can go on to deny DEF, etc., which are properties of *c*, of B. It is obvious that 'No B is C' must be indemonstrable, *e.g.*, that no lake fish, belonging to genus A, inhabit the streams (= F), or though we may appear to prove it through the fact that they feed on a species of insect found only in lakes, we must have previously explained this fact by their being in the lakes. Or, to put the matter in another light, we may prove that No B is C by assuming that No B is D, but this negation has to be itself derived from 'No B is C'. We must select one of those negations as an ultimate premiss, and we should take as the terms of our ultimate those properties which seem to be the cause of the subsequent differentiations. The negation No B is C is indemonstrable (just as the definition, B is Ba, is indemonstrable).

Thus we see that when we have two terms B and C each involving along with it a number of attributes in such a way that by denying B and C of each other we are able to deny of the other the attributes involved in each, we are entitled to raise B and C into the distinguishing marks of species. When once this has been done it is merely going over old ground again to deny the properties of one class of the other. Now it is when B and C are regarded as specific differences that the *modus ponendo tollens* is valid, but when valid it is valueless. "Red fleshed trout are lake-dwellers, therefore they are not to be found in the streams." This is true, but is based upon the mutual exclusion of classes, whose distinction had already been justified by the difference of their properties.

There are cases of disjunction in which we are not specifying the various classes which fall within a genus and in which it would yet appear that we might draw a valid conclusion from them by the *modus ponendo tollens*. But in all such cases the mutually exclusive predicates must be known to be incompatible with each other in virtue of some principle antecedent and superior to the particular science which deals with the subject of predication. The predicates must be specific differentiations of a wider reality. Thus we may say, 'Planetary orbits fall either wholly inside or wholly outside the earth's orbit'. We can therefore infer, if we care to take the trouble for such a trifling purpose, that Jupiter's orbit, lying without that of the earth, cannot lie

wholly nearer to the sun than it. But our exclusive major premiss in virtue of which we drew our conclusion was founded on the fact that outside and inside are specific differentiations of spatial relations, that nothing which is out is in. The exclusive force of our disjunction is then indirectly *a priori* and built upon the logical principle of individuation, and is of no particular interest to the science of astronomy; the real force of the disjunction lies in its exhaustiveness, the denial that there are any planets with orbits intersecting that of the earth.

Contrast the statement—"Planets whose orbits lie between the earth and the sun are, when visible, to be seen either in the morning or in the evening". Now there is no general self-evident principle which states that a heavenly body visible in the morning is not, during the same period, visible in the evening. The incompatibility of these two predicates has to be deduced from the nature of the particular subject treated of—planets between the earth and the sun. But we may very well make and employ the above disjunction without possessing this proof, without knowing that Venus, when visible in the evenings, must rise after the sun and be lost in his light in the morning. Hence we see that the force of the disjunction lies in its exhaustiveness, while the exclusiveness, if present, depends upon the predicates involved in each case.

A classification is of value chiefly for its exhaustiveness. It is a predication of the properties of a genus taken concretely. As such it is an ideal of science and like the predication of property, that with which science may rest content. But for the practical life it is the starting-point of activity; it is the possession of this kind of knowledge which the practical life desires. We wish to know the whole number of types into which a genus is divided, so that we may have confidence when to expect to find the qualities common to the whole class. For example, we wish to know the complete tale of the ores of iron so that in our prospecting we may know what substances to collect and test if we are investigating the possibilities of iron mining in a certain district. It is not sufficient to know that the particular ore (B) is iron-producing (A) and that C, D and E fall under the same category. What we wish to be sure of is that the list is exhaustive, that all not-B is C D or E, and all not-C is D, E or B. The specific difference of each of the various sorts of minerals may be taken for granted, but to assert it is not the purpose of a judgment of this kind. Again, if we cannot identify a certain species with any of the members

into which a class is divided, we may infer that it has not the peculiar properties of this genus. Here once more we have in view only the exhaustiveness of the division. And with this I may conclude my argument, which goes to show that the function of the disjunctive judgment both in science and in practical reasonings is to be exhaustive and not necessarily exclusive.