V.—DISCUSSIONS.

THE ARGUMENT *A FORTIORI*.

WHENEVER logicians can be induced to think about their science they speedily contrive to show that a logical question has at least as many sides as there are logicians contemplating it. This and the fact that certain deserving sides of their question seem so far to have escaped the notice both of Dr. Mercier and of Mr. Pickard-Cambridge may serve as my apology for participating in the interesting little discussion about the arguments *a fortiori* in which they are engaged.¹ If I am lucky I may not only succeed in illustrating the many-sidedness of logical questions, but also in inducing them to compose their differences, and even in contributing to a more precise use of terms than has hitherto prevailed in Formal Logic.

As the matter stands it appears to me that effective discussion is rendered almost impossible by the vagueness and ambiguity of the notions of 'validity,' 'purpose of the argument,' and 'proof': they mean different things to different writers and are used more or less loosely by all. Let us begin therefore by inquiring how strictly the notion of a valid 'form' is to be taken.

(1) If it is taken quite strictly, the validity of arguments *a fortiori* must certainly be denied. No argument which is not in syllogistic form can be formally valid, and indeed even the syllogism is open to attack on the ground that it is not an adequate example of formal validity. For, as Mr. Alfred Sidgwick has shown, two true premisses validly combined in a syllogistic form may yet conduct to a conclusion which is in fact false, and so defeats the doctrine that a conclusion is necessarily true if it has true premisses and a 'valid' form. Arguments *a fortiori*, may indeed be true at any rate, but they cannot possibly be *valid* in the sense that their form guarantees actual truth to their conclusion: when they are true, their truth is 'material' and not 'formal,' i.e. it depends on the actual nature of the relation to which the reasoning appeals. This strict logical standpoint should not be overlooked, even though it is not as fashionable as it was and though Dr. Mercier and Mr. Pickard-Cambridge seem to agree in rejecting it.

(2) Both, having thus departed from the narrow path of orthodoxy, apply the term 'validity' to *material* implications. Of these

¹ In No. 96.

there are plenty, if one's standard of 'proof' is not too exacting, and if familiar reasonings based on a knowledge of these relations are therefore said to be 'proved'. Dr. Mercier goes further still. He is willing to allow a reference also to the 'purpose' of the argument, nay to insist on it, and to make 'validity' contingent upon that. To this, however, Mr. Pickard-Cambridge demurs, and he endeavours to show that while dependence upon a 'material' relation should not be held to detract from formal 'validity,' dependence upon purpose should. I venture to doubt whether either position is tenable. To a strict formal logician, if any such survives, both Dr. Mercier's position and Mr. Pickard-Cambridge's would be anathema; but without advocating such an excess of logical asceticism one may well summon both these gentlemen to reflect whether they have not entered on a primrose path of reasoning on which they will not find any logical stopping-place short of a complete denial of the old notion of formal validity.

To render them aware of their danger, it must first be pointed out to them that the illustrations they have been discussing are not real reasonings at all, but merely verbal forms in which various real reasonings may be expressed. In Mr. Russell's terminology, they are 'propositional functions,' to the terms of which values must be assigned, before we can get a judgment which can be true, or false. In my own, they are forms-for-meaning, but as yet their meaning is potential and they actually mean nothing. In the terminology of Formal Logic, they are all 'ambiguous,' and the question of what they *mean* has to be settled before we can say whether they are true or false.

In point of fact it is not very difficult in any of these cases to suggest senses in which the conclusion drawn would be true, and others in which it would be false, although in both cases the premisses *per se* may be true. For example, when we argue *A is greater than B, and B is greater than C ∴ A is greater than C*, it is tacitly assumed that these differences in magnitude are themselves of a sensible magnitude and relevant to the context of the actual argument. But if A were only microscopically larger than B and B than C, and if these differences were negligible, it would not actually be true that A was *greater* than C. For the purpose in hand it would be truer to say that A was *equal* to C.¹ Again there are many contexts in which the adage *πλέον ἡμῶν παντός* holds, and the half is greater than the whole. It is more *e.g.* to offer a woman your hand than your arm. Allowing therefore for the possibility of uses in which the conclusion 'A is greater than C' is false, though its premisses are separately and for other purposes true, how can one properly maintain that the

¹ We should probably say they were 'practically' equal, and so illustrate a 'theoretical' use of 'practical' which may be brought, respectfully, to the notice of those who insist on having an absolute chasm between 'theory' and 'practice'.

conclusion is necessarily drawn from the premisses? Surely the form of the reasoning is not as such either valid or invalid, though it admits both of the truth and of the falsity of the conclusions drawn in different contexts.¹

What then is the difference between the 'valid' and the 'invalid' forms? Chiefly that in the former case we are more prone to think of uses in which the conclusion as stated is true, while in the latter we think more readily of cases in which the conclusion is false, and find it harder to conceive the conditions under which it would correctly follow. But this difference is surely psychological and not logical, and it is quite possible to imagine conditions in which 'valid' inferences could be drawn from what would ordinarily be called 'invalid' forms.

Let us examine therefore the specimens of forms which Mr. Pickard-Cambridge regards as inherently 'invalid'; we shall find that they yield valid enough conclusions in certain cases. If in 'A is next to B and B is next to C' we presuppose the ordinary reference to lateral proximity we cannot indeed infer that 'A is next to C'; but we can successfully infer that 'A is next but one to C,' as Dr. Mercier contends. Nor is it clear how Mr. Pickard-Cambridge can deny the 'validity' of this inference. For it follows from the (material) relation of the spatial 'universal' assumed, and neither he nor Dr. Mercier has argued that the relation must be syllogistic. But of course the premisses need not assume this universal nor aim at describing a laterally linear relation, and, as Mr. Shelton pointed out (p. 528), 'next' is ambiguous, and if we are putting marbles together we may arrange them triangularly, so that the conclusion 'A is next to C' is true, because 'A is next to B and B is next to C'. Or again, from the point of view of a popular actress contemplating from the stage three of her admirers in the stalls, it may be quite natural to describe them as sitting 'C by himself in the front row, with A and B next (behind) him'. 'Nextness' would then be determined in the line of sight. Similarly I would proffer as a valid inference in the form 'A cheats B, B cheats C $\therefore$ A cheats C,' 'the Armenian cheats the Jew, the Jew cheats the Scotchman, so of course the Arinenian cheats the Scotchman'; this seems fully as 'valid' as any of these arguments *a fortiori*.

We get then this result: the general 'validity' of a form is no protection against the falsity of a conclusion drawn in it in special cases, the general invalidity of a form is no bar to the 'validity' of a conclusion drawn in it in special cases. There is no 'necessary connexion' therefore between validity of form and actual truth. This result is surely utterly subversive of the whole traditional 'ideal' of a 'valid form,' and no one who retains any respect for

¹ I have elsewhere shown that the syllogistic form itself appears to suffer from the same defect. Cf. my article in No. 89 on *Aristotle's Refutation of Aristotelian Logic* and *Formal Logic*, ch. xvi. §§ 7-10.

this ideal should endure the aspersion cast upon it without making the most strenuous efforts to rehabilitate it. For is not validity of form explained as a logical *mirage*, thrown by unperceived 'material' truth upon the reflecting surface of language?

But if we give up 'validity,' on what shall we say that actual truth depends? Dr. Mercier's suggestion that the 'purpose' of the argument has much to do with it certainly deserves consideration, especially as Mr. Pickard-Cambridge's objections do not seem to dispose of it. Mr. Pickard-Cambridge is confident (p. 537) that "whether your inference about certain relations follows or does not follow from the premisses cannot depend upon your object in attempting to draw it, or in starting the inquiry. It depends upon a difference between the relations themselves." The examples given above may I hope abate a little Mr. Pickard-Cambridge's confidence. For in all these cases the 'validity' of the reasoning certainly does seem to be intimately related to the object, aim, or purpose of the argument. If we aim at microscopic exactness, differences in the size of A, B and C become relevant which are negligible when a rough estimate will suffice. The reasoning about the 'next-ness' of A, B and C depends on how mathematically or colloquially we are purposing to conceive the notion of 'next,' and similarly that about 'cheating' depends on the context in which the term occurs.

Mr. Pickard-Cambridge also seems to labour under a certain misapprehension of the position he attacks. He supposes it to be implied that if 'validity' depends on the purpose of the argument "we are *entitled* to neglect the difference between the kinds of relation referred to and to look *only* to the purpose, *assured* that in any argument whose purpose is *parallel* to that of an argument which we already know to be valid, it will be valid to draw a parallel conclusion. But this expectation, of course, breaks down at once." Here I have ventured to italicise the terms which seem open to exception. The insertion of the word 'entitled' begs the question in dispute by again slipping in a reference to the ideal of a purely 'formal' validity which the reference to the purpose of the argument has implicitly repudiated. For in strictness it cannot be contended that the neglect of *any* difference can ever be a 'valid' process to which the logician is 'entitled'. It always has to be done, of course, in real reasoning, but it always involves a *risk*. What we neglected as irrelevant may always turn out to have been relevant and so may wreck our inference. Hence we can never be 'assured' in advance, when we embark on an argument, that it will bring us safely to port. In real reasoning, nothing venture, nothing have; the hope of truth is ever bound up with the risk of error. Again I can see no reason why Dr. Mercier should accept the '*only*' Mr. Pickard-Cambridge has *added* to his position. It is quite possible to think a knowledge of the purpose 'essential' to the knowledge of the value of an argument without assuming that this knowledge is all-sufficient. It may be part of

the 'essence' without being the whole of it. Lastly, the term 'parallel' seems to involve a metaphor the purport of which is obscure. In (Euclidean) geometry certain lines are said to be 'parallel' when they can be produced ever so far either way and yet never meet. But it can hardly be *this* property of parallels that Mr. Pickard-Cambridge wishes to indicate when he attributes 'parallelism' to arguments.

Having in these respects defended Dr. Mercier against Mr. Pickard-Cambridge's strictures, candour compels me to add that his phraseology does not seem to me to be altogether unexceptionable. His conception of 'validity' is not of course as formal as Mr. Pickard-Cambridge's, just as the latter's is not as formal as that of the strict formal logician, but still it seems doubtful whether he would admit that his phrase 'the purpose of the argument' stands in need of further definition. This purpose is of course a very real fact, though the old logic has ignored it; it becomes a perfectly definite thing whenever a form of words is *used*. It has to be ascertained before the *truth* of a proposition can be discussed, because it has to be ascertained before its *meaning* can be understood. But it may vary (within pretty wide limits) in the various cases of the use of the same form. The mere words of the reasoning therefore cannot be said to reveal '*the*' purpose of the argument, or indeed any of the many purposes to which they may minister. If it was judicious to refer to so important a point so briefly (which I doubt), Dr. Mercier's formula should at least have run 'for a purpose of the argument'.¹ That will not at any rate delude the logician into thinking that a 'form' can have only one meaning and can serve only one purpose, and that he need not trouble to ascertain what is the actual meaning of the 'judgments' he sets out to evaluate.

Lastly I would suggest, certainly to Dr. Mercier, and (with some trepidation) also to Mr. Pickard-Cambridge, that when they talk of 'validity' they really mean 'value'. But the topic of Value is still, alas, a *terra incognita* for logic, and indeed largely for all philosophy.

¹ I now see that in his reply in No. 97 Dr. Mercier has gone more fully into the logical importance of the purpose of the argument (pp. 87-89), and cordially agree with his account of this.

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