COUNTERBALANCING RECIPROCATING ENGINES.

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THE perfect counterbalancing of the momentum of the reciprocating parts of an engine in an *horizontal direction*, is often a matter of considerable importance, especially when the engine is required to work at a high speed. It is seldom, however, that this is successfully carried out, except in horizontal engines, from the fact that the manner in which it can be accomplished is not at first sight apparent. We propose, in the present article, to give a complete discussion of the elements involved in this problem, and to deduce a simple and direct method by which the horizontal momentum of any engine, inclined or vertical, as well as horizontal, may be properly counterbalanced. We have intimated already, that it is only with reference to an horizontal direction that the subject is to be considered, since, in general, it is supposed that an engine is so firmly secured as not to be influenced by a force acting in a vertical direction, while it is to prevent all tendency to horizontal swaying that the counterbalance is applied.

Let us, in the first place, consider what it is that causes this tendency to motion in the direction of the centre line in an unbalanced engine. Disregarding, for the present, the effect of the connecting rod, we have the fact, that when the crank is at right angles with the centre line, the reciprocating parts are in motion with a maximum velocity. If uncontrolled by the crank-pin, they would, of course, continue to move forward in a straight line at the same velocity, and would oppose a resistance to any body tending to arrest their motion. Such a body is the crank-pin, and accordingly, during the last half of the stroke, a pressure is exerted upon that part, the amount of which will depend on the mass and velocity of the reciprocating parts. Having brought the parts to rest, it is now necessary to set them in motion again in the opposite direction, and to do this will require exactly as much work as was absorbed in arresting their progress before. If, for simplicity's sake, we regard the crank-pin as the motor, which we may do without changing in the least any of the facts involved, we see that the pressure on that point while bringing the parts to rest, and that while starting them into motion, is continually in the same direction, so that we have, during the whole of the

half revolution of the crank, from its position at right angles to the centre line, on one side, to the same position on the other side, a pressure exerted in the direction of the end of the stroke which lies in that half revolution. The same is, of course, equally true of both half revolutions of the crank from the dividing line at right angles to the centre line. The point which we wish to call attention to is, that it is not the velocity, but the *change of velocity*, that gives rise to pressure, and which, therefore, needs to be counteracted. In the case of an engine in which the power is taken off from the shaft on the other side of the pillar-block from the crank, as is almost universally the case, whatever pressure is applied to the crank-pin is transmitted directly to the pillow-block, and hence the effect of this alternating force arising from the changes in velocity of the reciprocating parts, in producing a bodily motion of the whole engine. Now, to prevent this in an horizontal engine, we place a weight equal in magnitude to that of the reciprocating parts opposite to the crank-pin, and at an equal distance from the centre of the shaft; so that the changes produced in its velocity *horizontally*, may at all times be equal to those occurring in the reciprocating parts, and therefore exactly neutralize their effect. As, however, this mass moves in a circle, it exerts a pressure at right angles to the centre line at all points in its revolution, except when it coincides with that line; but as in this case this is a vertical force, we do not regard it. We resolve the centrifugal force of the revolving weight into its two components, at right angles to and parallel with the centre line in this manner, because this gives us at once a key to all the effects produced by the counterbalance weight. The component parallel with the centre line is exactly neutralized in its effects on the stability of the engine by the action of the reciprocating parts; the other remains unaffected. We see already, therefore, that this method of counterbalancing would not answer for an inclined or vertical engine, since this latter force would in these cases have a direction approximating to the horizontal, and would therefore give rise to a motion exactly such as we wish to prevent. But before proceeding to examine these cases, let us investigate the peculiar motion of the connecting-rod, partaking as it does of both a reciprocating and a rotary motion, and see how this is to be provided for. It is evident that the end immediately at the crank-pin, will offer a resistance to any force or body which tends to vary its velocity in the direction at right angles to the centre line, in exactly the same manner as a body re-

volving with the crank, or say the crank-pin itself, and this will, of course, in all directions of the centre line, horizontal or vertical, be perfectly counterbalanced by a weight placed opposite to it. Our first problem, then, is to ascertain what proportion of the weight of the rod is to be so allowed for. If we let A B, Fig. 1, represent the connecting-rod, supposing it also stripped of all extra weight at either end, and considered merely as a plain bar, we may suppose it to be

divided into any number of small or elementary masses, of equal magnitude, m, m , &c. It is evident that the velocity of each of these, when the rod coincides with the centre line, is proportional to its distance from the centre, A, and the amount of work stored up in each, being as the square of the velocity, will be represented by $\frac{1}{2} m_0 \times 0, m_1 a_1^2, m_2 a_2^2$, &c. Now, in moving to the position, A B, in which they are at rest, in this direction each point passes through a distance proportioned to its distance from the centre, and consequently the resistance which it offers during this motion to the force tending to bring it to rest, will be represented by $\frac{m_m a_m^2}{a_m} = m_m a_m$, or, in other words, the pressure which it exerts will be proportional to the distance from the centre, A. These pressures are all sustained by the crank-pin at one end, and the slides at the other, and the proportion of each, which either of these bears, is in the inverse ratio of the distance of the moving masses from the sustaining point, or in the direct ratio of its distance from the opposite end of the rod. The pressure coming on the crank-pin, therefore, will be represented by $\frac{1}{2} m_0 \times 0 + m_1 a_1^2 + m_2 a_2^2 + \dots + \frac{1}{2} m_n a_n^2$, or, as the sum of all the elementary masses into the squares of their distances from the point, A. Calling the distance, A B, unity, and the sum of all the masses $\frac{1}{2} m_0, m_1$, &c., M, we shall have the sum of this series equal to $\frac{1}{3} M$. From this, we find that the pressure on the crank-pin in a direction at right angles to the centre line, resulting from the swing of a connecting-rod of uniform dimensions, would be the same as that of a mass of one-third the

weight of the rod situated at the crank-pin; and if we add to this the weight of the extra metal in the end of the rod at the crank-pin, we shall have the amount of weight which will be necessary to counter-balance this motion of the connecting-rod, supposing it to be placed diametrically opposite the crank, and at the same distance from the centre of the shaft.

We have observed that a part of the pressure thus generated in the connecting-rod comes upon the slides. The portion of the pressure exerted by each of the elementary masses, m , so sustained, being proportional to its distance from the end, B, while the total amount of each such pressure is proportional to its distance from A, we shall have for the combined pressure of all the masses, m , the sum of a series of the form,

$\frac{1}{2} m n p \times 0 + m(n-1)p \frac{1}{n} + m(n-2)p \frac{2}{n} + \dots \frac{1}{2} m(n-n)p \frac{n}{n}$, in which p = the resistance offered by an elementary mass, m , moving at a unit's distance from the centre, A, n , such units making up the length of the rod. The sum of this series we find to be $\frac{1}{6} M$, or the pressure on the slides due to the changing momentum of the connecting-rod will be equal to that which would be exerted on the crank, by a weight of one-sixth the magnitude of the plain part of the rod revolving with the velocity of the crank-pin. It is not necessary, in this case, to make any addition for the extra weight of the ends of the rod, since at both these points the mass is inoperative to produce pressure on the slides, as is shown from the fact that the extreme terms of our series are both equal to zero.

It is not possible to balance this part of the momentum of the connecting-rod by a weight applied to the shaft, since the opposing force so produced would not act in the same line with that which it would be required to counteract. It is, however, of little importance that we should do so, since, in comparison with the pressure coming on the same part from the force of the steam exerted through the inclined rod, it is insignificant, and it is noticed here merely for the purpose of bringing all the acting forces before our minds. We have now found, then, that a weight equal to one-third the body of the connecting-rod (if it be one of uniform dimensions) and the whole of the extra weight at the crank-pin, must be placed diametrically opposite to the crank-pin, and at an equal distance from the centre of the shaft, whatever be the inclination of the engine. This being done, we will proceed to inquire what will be the position and magnitude of the mass required to counterbalance the directly reciprocating

parts, and the remaining two-thirds of the connecting-rod. Let c , Fig. 2, represent the direction of the centre line of the engine making any angle, $c b e$, with the horizontal, and a a mass equal to the weight of these parts, and exactly sufficient, therefore, if placed opposite to the crank-pin, to counterbalance their momentum *in the direction $c c$* . Let us represent the weight of this mass by the length of the radius ab . Now, the effect of the centrifugal force of the mass, A , placed as we have supposed, will be at all times neutralized in the direction parallel with the line $c c$ by the motion of the reciprocating parts, but will, as we have already observed, exert an influence in a direction at right angles to that line, varying in amount according to the position of A , its intensity being proportional to the sine of the angle by which A is removed from the centre line. Its maximum effect, therefore, will be, when this angle is 90° , or when the weight is at e . We may resolve this force into two components, $b d$ and $c d$, the one horizontal and the other vertical, and equal respectively to the sine and cosine of the angle which the centre line of the engine makes with the horizontal, and we now wish to neutralize this horizontal component.

Draw the radius $b e$ in a vertical direction when the mass A is on the centre line. Now, if we place at the extremity of this radius a mass, B, equal to $b d$, we shall have no horizontal effect produced by it when A is on the centre line, but when A is at right angles to that line, B will be in an horizontal direction from the centre of the shaft, and will accordingly exert a force directly opposed and exactly equal to the horizontal component, $b d$, of the centrifugal force of A, and we should accordingly have the momentum of all the parts perfectly counterbalanced in that direction.

It now remains to combine the three weights which we have successively found necessary, and to find the position and magnitude of a single mass that shall exactly perform the duty of the three. To do this, lay off on the centre line the distance, $b h$, equal to the combined weight of A and the mass that we first found for counterbalancing the swing of the connecting-rod: on $b c$ lay off a distance, $b f$,

equal to the mass, B , and complete the parallelogram, $bfg h$. The diagonal, bg , will represent the direction and magnitude of the resulting weight. Were it not for the effect of the weight necessary to counteract the change of momentum of the connecting-rod at right angles to the centre line, the position of this final weight would always be in an horizontal direction from the shaft when the engine was on centre, since the line, hg , equal to the sine of the angle of inclination, would then fall from the point, a , situated on the circle; as it is, the required direction will always be somewhat nearer to the centre line.

We have thus far supposed the counterweight to act in the same plane in which the reciprocating parts move. But as the weight itself cannot be placed exactly in that plane, we must see how we can dispose it so as to produce the same effect as if so placed. If it is a crank-shaft engine, we can easily accomplish this by dividing the counterweight into two equal parts, and placing one on each side of the crank, equidistant along the shaft from the centre line of the engine; or if, for particular reasons, this is not practicable, then we must divide the weight into portions inversely proportional to the distance from the centre line, at which we are obliged to locate them. In the case of an overhung crank, we must proceed in a different yet similar manner. It will be necessary, in order to avoid the tendency to produce rotation that exists in the case of *two* opposite forces not acting in the same line, to provide a third force, and this we may do by placing a weight exactly opposed in direction to the counterweight, on the opposite side of the latter, from the centre line of the engine, and of a magnitude, as compared with that which we have already assigned to the counterweight, inversely as the ratio which its distance from the position of the counterweight bears to that of the counterweight from the centre line measuring along the shaft, and the counterbalance weight must now be augmented by an amount equal to this second weight.

The case of a double cylinder engine differs in no respect from those which we have already considered, since each set of reciprocating parts is to be balanced separately, and

the weights may then be combined in the manner already illustrated.

This, then, is the proper manner in which to counterweight an engine of any description, horizontal, inclined or vertical. In the former case, the weight, supposing it situated in the plane of the centre line, would fall opposite to the crank, and be equal in magnitude to the total weight of the reciprocating parts; in the latter it would be placed in the same position, but would only be equal to one-third the connecting-rod, as already determined.

There remains yet one fact to be noticed in connection with this subject, viz: the effect of the connecting-rod in producing more rapid motion of the reciprocating parts when they are in the portion of their stroke farthest from the crank-shaft, than when in that nearest to it. With a rod six times the length of the crank, the maximum velocity is absorbed and re-imparted in the former portion of the revolution, while the crank advances in the direction of the centre line through but five-sevenths the space through which it moves in the latter case, while the rate of motion when the crank is near the centre line is about forty per cent. greater on the one end than on the other. As the same amount of *vis viva*, therefore, has to be absorbed and imparted by a less forward motion of the crank-pin, the pressure coming upon that point must be greater in the same proportion at one end and less at the other. The result of this is, that if the engine were balanced for the mean speed of the piston, the counterbalance weight would be too little when the piston was at the end of its stroke from the shaft, and too much when at the other end. In both cases, therefore, there would be a force acting to urge the engine bodily in the direction away from the shaft. But this pressure being always in one direction, would not give rise to vibration, except, perhaps, to a very limited extent, as its intensity varied at different portions of the stroke. It is evident that any increase or diminution of the counterbalance weight, would only tend to produce inequality in the effect of this force on the two strokes. The writer lately witnessed a beautiful illustration of the fact now referred to, in the working of a high speed engine that was placed for trial on some planed iron surfaces, and not bolted down. After it had been running a few minutes, he observed that it had moved bodily backward several inches, and the present explanation thereupon suggested itself to him as the cause of the phenomenon.

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