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He has also introduced a celluloid ring below the sphere, which on being raised presses the sphere against a similar ring above, thus preventing any damage to the integrating apparatus when the instrument is not being used.

Note.—At the request of Herr Coradi I add the statement that the idea of the new integrating apparatus, consisting of a sphere with two recording-wheels at right angles to each other, is not his own, but is due to Herr Max Küntzel, of Charlottenhof, near Königshütte in Silesia. Herr Küntzel invented the arrangement for an instrument designed to determine the coordinates of the vertices of a polygon, and submitted his design to Herr Coradi for the construction of such an instrument.

IX. *Harmonic Analyser, giving Direct Readings of the Amplitude and Epoch of the various constituent Simple Harmonic Terms.* By ARCHIBALD SHARP, B.Sc., Wh.Sc., A.M.I.C.E.*

LET the curve (fig. 1) be that represented by the equation $y=f(x)$, the scale of abscissæ being such that the period is 2π . Suppose a wheel W to roll on the paper

FIG. 1

(fig. 2), and to be connected with a tracing-point P (fig. 1) in such a manner that as P moves uniformly in the direction

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OX the axis of the wheel W turns uniformly in a horizontal plane, and the distance rolled through by the wheel during any short interval is equal to the corresponding displacement of the tracer P in the direction OY. If the axis of the rolling wheel W makes one complete turn while the tracing-point P moves over one complete period of the curve (fig. 1), the point of contact of the rolling wheel will describe a curve $Opp'R$ (fig. 2).

Let OY' (fig. 2) be the initial direction of the plane of the

FIG. 2

rolling wheel, *i. e.* corresponding to zero abscissa of the tracer (fig. 1). Let P be any point on the curve fig. 1, p the corresponding point on the curve fig. 2. Let P' and p' be two corresponding points infinitely close to P and p respectively. In fig. 1 draw $P'P_1$ and PP_1 parallel to OY and OX respectively, and in fig. 2 draw $p'q$ and pq respectively at right angles and parallel to OY' .

Then

$$pp' = P_1P' = dy,$$

$$p'q = pp' \sin x = \sin x dy,$$

$$pq = pp' \cos x = \cos x dy.$$

Draw RR' perpendicular to OY' (fig. 2), R being the position of p corresponding to $x=2\pi$. Then

$$RR' = \sum p'q = \int_0^{2\pi} \sin x dy,$$

$$OR' = -\sum pq = -\int \cos x dy.$$

In the Fourier expansion

$$y=f(x)=A_0+A_1 \sin x+A_2 \sin 2x+\dots+A_n \sin nx+ \\ +B_1 \cos x+B_2 \cos 2x+\dots+B_n \cos nx+\dots, \quad \left. \vphantom{y=f(x)} \right\} \quad (1)$$

$$\left. \begin{aligned} A_1 &= \frac{1}{\pi} \int_0^{2\pi} y \sin x dx = \frac{1}{\pi} \int \cos x dy = -\frac{1}{\pi} \text{OR}' \\ B_1 &= \frac{1}{\pi} \int_0^{2\pi} y \cos x dx = \frac{1}{\pi} \int \sin x dy = -\frac{1}{\pi} \text{RR}' \end{aligned} \right\} \quad (2)$$

Also $f(x)$ may be expanded in the form

$$A_0+C_1 \sin (x-\alpha_1)+C_2 \sin (2x-\alpha_2)+\dots+C_n \sin (nx-\alpha_n), \quad (3)$$

A_n , B_n , C_n , and α_n being connected by the relations

$$\left. \begin{aligned} A_n &= C_n \cos \alpha_n \\ -B_n &= C_n \sin \alpha_n \end{aligned} \right\}, \quad \dots \quad (4a)$$

or

$$\left. \begin{aligned} C_n^2 &= A_n^2 + B_n^2 \\ \tan \alpha &= -\frac{B}{A} \end{aligned} \right\} \quad \dots \quad (4b)$$

From (2) and (4b) it is evident that OR (fig. 2) is equal to πC_1 , and the angle Y'OR is equal to α_1 .

If now the axis of the rolling wheel W makes n turns while the tracer P moves over one complete period of the curve (fig. 1), the corresponding values of OR and the angle Y'OR will be $n\pi C_n$ and α_n respectively.

Various arrangements of mechanism are suggested for connecting the rolling wheel with the tracer so as to satisfy the above conditions; the following seems the most suitable, as it can be adapted for an instrument to give more than one simple harmonic constituent term for one tracing of the curve.

The motion of the rolling wheel relative to the paper is compounded of two simple movements:—(a) a pure rolling, the distance rolled being equal to dy the element described by the tracer P; (b) a motion of rotation, the point of contact of the wheel with the paper being the centre of rotation, and the angle turned through from the initial line being proportional to x the abscissa of P. The relative motion will, therefore, be the same if the wheel be rolled along a straight line *fixed* in the instrument, while the *paper* is made to turn, the centre of rotation of the paper being the point of contact of the wheel with it which is continually varying in position. Fig. 1 represents diagrammatically the mechanism. The curve

to be analysed is drawn on a flat sheet of paper and placed on a drawing-board. The carriage FF, which forms the base of the instrument, is supported by an axle with two equal wheels w_1 and a third wheel w which roll on the paper, the direction of motion of the carriage being OX. A disk d_1 mounted on a vertical spindle is driven by a pair of bevel wheels by the axle w_1 . A long key on the upper surface of this disk fits into a groove on the under surface of a disk d_2 , which is thus free to move in a straight line relative to disk d_1 . A groove on the upper surface of disk d_2 at right angles to that on its lower surface has a key from the lower surface of disk d_3 resting in it. Thus the disk d_3 always turns with disk d_1 , although any point on disk d_3 may be made the centre of rotation; the three disks being kinematically equivalent to Oldham's coupling for the transmission of motion between two parallel shafts. The keys and grooves would be replaced, in an actual instrument, by wheels and rails, in order to diminish frictional resistance.

The tracing-point P is mounted on a smaller carriage f , which is free to run in the direction OY relative to the main carriage F. This smaller carriage carries also the rolling wheel W which rolls on the disk d_3 . The rolling wheel W should be spherical in form, and of as small diameter as possible, so that its surface of contact with the paper on disk d_3 approximates to a point. The friction between wheel W and disk d_3 is great enough to prevent any relative sliding. As the tracer P moves over the curve (fig. 1) the point of the wheel W will describe on the disk d_3 the curve Opp'R (fig. 2). To ensure that, as the tracer P is moved in the direction OY, the wheel W will roll on the disk d_3 the same distance and not displace it relative to disk d_1 , a wheel W' of the same diameter as W is mounted on the same spindle and rolls on a fixed portion of the carriage FF. If W' be compelled to roll, W must roll on the disk d_3 an equal amount.

The actual shape of the curve Opp'R (fig. 2) is of no importance, the initial and final points being all that are required. A needle or pencil n may therefore be carried at any convenient part of the carriage f , and the initial and final positions O and R marked by it. The direction of the initial line OR will be recorded on the disk d_3 by making two marks with the needle n as the tracer P moves along the line OY (fig. 1).

The gearing must be such that the disk d_3 turns once while the tracer P describes one complete period of the curve. If now pairs of equal wheels w_2 w_2 , w_3 w_3 , ... of diameters

$\frac{1}{2}, \frac{1}{3}, \dots$ of w_1 , be made to roll on flat rails lying on the paper, the values of $c_2, a_2, c_3, a_3, \dots$ are obtained in succession, one pair of coefficients for each tracing of the curve.

This instrument has the advantage over any Harmonic Analyser previously designed that it gives directly the quantities—amplitude and epoch—of each simple harmonic term which are required; all other instruments, as far as I am aware, giving the coefficients A_n and B_n , from which C_n and a_n are calculated.

It is remarkable that no adjustments have to be made before using the instrument, the initial position of the disk d_3 having no influence on the curve $Opp'R$ described on it. There is no part of the instrument which demands *excessive* accuracy of construction. The accuracy and delicacy of the instrument depends on the accuracy with which the line OR and angle Y'OR can be measured, and will be quite as great as that with which the original curve fig. 1 is drawn.

In some cases there will be a danger that the disk d_3 may not be large enough to contain the complete curve $Opp'R$ (fig. 2). If the rolling wheel W is about to roll off the disk, a mark should be made with the needle n , and keeping the tracer P in the same position, the disk d_3 should be moved by hand into any other convenient position relative to disk d_1 , a new mark made with the needle, and the movement of the tracer P may then be proceeded with. The final line OR can then be easily built up from its separate parts.

Since writing the above I have designed an inversion of the mechanism described above giving a simple compact instrument, which I may have the pleasure of describing later on.

X. *Remarks on Prof. Henrici's Paper made by Prof. PERRY, F.R.S., in which he describes a Simple Machine which may be used to develop any Arbitrary Function in Series of Functions of any Normal Forms*.*

I CONGRATULATE Prof. Henrici, first upon his success in these Analysers, with which I shall presently form a practical acquaintance when the latest of them yet constructed reaches me from Zurich, second on the admirably clear way in which he described them to us.

I have had no experience with the hatchet, that simplest of all planimeters; but with regard to the Robertson-Hyne

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