SOME DIFFICULTIES OF THE MATHEMATICS
TEACHER.

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It is perhaps a hazardous thing to begin a paper on mathematics with a Scripturè reference. But I give my word of honor that a text does not, in this case, mean a sermon, perhaps the reader will let me remind him of that doughty old Hebrew warrior, Caleb. It will be remembered that during the conquest of Canaan he came to Joshua asking for a certain mountain as his inheritance. "I am an old man," he said, "but I am as strong as ever. As my strength was then (in my prime), even so is my strength now, for war, both to go out and to come in. Now, therefore, give me this mountain, for (and here lies the point of my reference) *for* thou heardest in that day how the Anakims were there and that the cities were great and fenced." (Josh., 14:11-12). The giants and the fenced cities that we, as teachers, must oppose—these form my theme. But we are to survey our enemies with the spirit of this old fighter in our hearts, prompting each of us to say: "Give me this work, not in spite of its obstacles, but just because its obstacles are so peculiarly great. Let me have the stern joy of overcoming foemen worthy of my steel."

To some of us the giants in the realm of mathematics seem peculiarly tall and terrible, the fenced cities more numerous than elsewhere and less open to assault; or, to drop a figure, that may seem to partake of the mock heroic, the difficulties of the mathematics teacher seem especially large in number and serious in nature. To state two or three of these difficulties, to suggest a few means of overcoming them, is all that this paper can hope to do.

One of the fundamental problems which we must face has to do largely with arithmetic. Algebra and geometry have, it is true, a utilitarian, every-day value to the worker in physical science, pure or applied. But to the ordinary child their worth lies mainly in the thought training they afford. Hence the teacher of algebra and geometry has a single definite object set before him. With arithmetic, the case is different and more complex; both utility and culture must be included in the object. The child must know arithmetic to earn his bread and butter; yet the study

of arithmetic is largely wasted if he does not find in such study a training in abstraction and generalization, in clear thinking, in close reasoning, in exact expression, in patient attention to detail. How are two objects, utility and culture, to be best combined and balanced in thought and in actual accomplishment?

An absolute answer cannot, of course, be given to this question. But a few simple and helpful considerations may be named. *First*, as to utility:

a. The average child will need a comparatively small amount of arithmetic for his daily needs.¹ Numbers larger than billions, for example, and decimals smaller than thousandths, will not enter into his computations; his compound numbers will be covered by a few tables and will rarely include more than two units. And so on through the whole range of topics. It is the teacher's business to know how few fundamentals there are in arithmetic viewed from the utilitarian standpoint and to see to it that the pupil masters, at any cost, these few fundamentals.

b. Most teachers agree that those topics in arithmetic which give positively false ideas of modern business life should be omitted—*e. g.*, Alligation, True Discount, problems in Stocks and Bonds involving unused quotations or a fractional number of shares. Some would carry the process of excision still further, cutting out most of commercial arithmetic, on the ground that the child is too immature to grasp its truths in the grades, while he will learn what he needs to know of these, and learn it quickly when he is confronted later with the actual conditions of the business world. This is the view taken by the Committee of Fifteen. It has much to commend it, especially when one remembers how few grade teachers have that first-hand knowledge of business needed by the instructor in commercial arithmetic.

c. However much we may differ on the second point, we are a unit on the third, *viz.*, that the child who completes the eighth grade should be quick and accurate in computation and expert in the solution of simple every-day problems. And we are equally a unit in saying that such accuracy and speed in the fundamental operations are rare. Possibly the main trouble is that we modern teachers give too much time to the more difficult applications of arithmetic and are afraid of drill in pure number and in simple solutions. There is nothing degrading in drill and nothing dis-

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¹ See *Teaching of Elementary Mathematics* (D. E. Smith), p. 21.

tasteful to the child when the drill is properly conducted. It leads directly to the desired result and we should have more of it.

To sum up: In making arithmetic a useful instrument in the child's hands, we must omit those topics which give him false business ideas, see that he masters the topics that he will surely need, see that he is quick and accurate in every-day problem solving and computation; whether we should emphasize business arithmetic or not is an open question.

Second, as to culture:

a. Some teachers hold that the subjects of arithmetic, as affording material for culture, are more or less on a par; others hold that the less the direct use of a topic the greater its culture value. In either case, modern arithmetic is so rich in content that even the most ardent culture advocate can readily dispense with the obsolete business forms and customs condemned by the business man.

b. To say that the phase of the study we present must fit the growing minds of our children seems a truism. Yet often we so forget, or misapply, this truism as to deaden our pupils' powers of thought. From children in the lower grades, for example, we have no right to expect logical reasoning. To train eye and hand, to strengthen the memory, to secure a correct insight into the nature of number, to develop through counting the power of abstraction, to cultivate the mathematical imagination—these must be the chief aims of the primary teacher. Gradually the children may be led to pass by inference from what is given to what is required, and to separate into steps the reasoning employed. Later still they may be led to make such inference and analysis unaided by questions. Then, and not till then, are they ready to give a formal explanation of a problem. Even in the higher grades care has to be taken that too severe exercises in logic are not demanded—*e. g.*, such a long chain of reasoning as that involved in the Euclidian method of finding the greatest common divisor.

c. While logical thinking on the student's part is of supreme importance, *independence* and *exactness* of thought are close seconds. The ability to discover truth for himself (from observation, experiment, study of the text or inference), the alert and critical attitude in class, the habit of formulating his own rules rather than taking them ready made—for these attainments on

the part of the pupil the teacher must constantly strive. Against that pathetic haziness of vision, that rooted habit of inaccuracy so often found among first year students in the normal school, the warfare must be equally earnest. Few definitions should be learned, but these few should be so carefully framed as to find no contradiction in the broader conceptions of the higher mathematics, and they should be expressed with the utmost accuracy possible. At every point satisfaction with anything short of the *exact* truth should be discouraged.

To sum up: In order to get the full culture value of arithmetic the child needs to study no topic condemned by the business man; the transition from arithmetic as an art to arithmetic as a science and an exercise in logic is to be gently made; independence and exactness of thought are to be emphasized next to logical reasoning.

The problem of reconciling utility and culture is, as we have seen, the special problem of the arithmetic teacher. But there is another fundamental difficulty which extends through elementary and secondary work and even into the college—the difficulty of proper division of emphasis between content and form, thought and expression. Only a few weeks ago I was comforted when a university dean to whom I had poured out my perplexity on this point answered me as follows: “The difficulty you mention is one that always troubled me in my teaching, and I never felt that I fully succeeded in securing the appreciation of formal statement.”

The professor to whom I have referred went on to say: “I think I did succeed in securing an interest in the subject and I did succeed in developing good geometric minds—to my way of thinking, much more important. The student must get the inspiration of the subject first and then every effort should be put forth to secure a fair degree of elegance in expression.” While admitting that the attitude taken by the professor may be entirely proper in the college class, my experience leads me to feel that form should receive a larger degree of emphasis in the secondary school. The importance from the business standpoint of accuracy in computation has been mentioned. But business enterprises need not only good computers, but boys and girls with the habit of using good English, with a feeling for clear expression and for orderly arrangement. From the standpoint of culture, form should be em-

phasized, because it reacts on the pupil's thought. Inaccurate and slovenly habits of expression foster inaccurate and slovenly habits of thought. On the other hand, every teacher is familiar with the clarifying influence on his own thought of the honest attempt to express himself clearly in class. After even a few months' experience with average students, the mathematics teacher has a rich collection of errors in form—mechanical mistakes in computation, misspelled words, pronouns whose antecedents are lost in the misty past, equations untruthful in form, *e. g.*, $(5 \times 3 + 2 - 5 = 12)$, given as $5 \times 3 = 15 + 2 = 17 - 5 = 12$, reduction which implies the change from degrees to hours merely in division by fifteen, algebraic problems half stated, geometrical theorems inaccurately phrased and proved in haphazard fashion. Even if the examples assigned have to be made simpler, it is well worth while to insist upon entire mechanical accuracy; even if less ground can be covered, it is worth while to fight for correct spelling, good grammar, accurate memorizing, orderly arrangement, adequate and truthful symbolic statements.

Those of us who love mathematics as mathematics and feel that "beauty is its own excuse for being" fail sometimes to give due consideration to a third fundamental problem—the problem of so presenting the subject to our pupils as to give them a maximum of interest and joy in their work. Some students like the abstract character of the exact science, but to others this comparative remoteness from human interests is a positive drawback. The rigor of mathematical reasoning, also, tends to discourage many a student who has not formed the habit of working intensely or of thinking to a point. How shall we make the subject warm with human interest? How tempt our pupils to go deep enough into the work to feel its charm?

One of the best means at our disposal is, of course, the correlation of the work in mathematics with the work in other branches, notably nature study and manual training. Many primary teachers now make first grade number work purely incidental and instrumental; undoubtedly the interest of pupils in the higher grades may be greatly quickened by a partial extension of the same method to their work in arithmetic and geometry. The high school teacher, perplexed over the correlation of mathematics and physics, is watching with interest the success of some recent attempts to construct simple physical apparatus and perform

simple physical experiments in connection with grade arithmetic.

Another means of awakening interest in mathematical work is sometimes available—viz., the correlation of the study with some work the pupils is doing or hopes to do. Possibly all of us know the story of the youth, who hitherto dull in mathematics, suddenly developed an almost alarming interest in the geometry of the circle, and read this portion of his text with astonishing rapidity, because he had decided to make a balloon and found that a knowledge of circular sectors, etc., was needful in its construction. I have myself known of high school boys whose interest in mathematics was due to their intention of becoming engineers.

Another and comparatively little used means of adding human interest to the study is to make free use of the history of mathematics. The study of the decimal system becomes charming when we trace the origin of the system to the habit of counting on the fingers, and find in the number words of savage tribes the evidence of such counting. In one South American tribe, e. g., the word for five means "a whole hand," the word for ten, "two hands," the word for eleven, "one to the foot," the word for sixteen, "one to the other foot," etc.* My classes have always been interested in the struggle of early mathematicians with fractions. The Egyptians, for instance, went through with what seems to us a fearful amount of labor in reducing each fraction to the sum of several fractions, each with the constant numerator one. The Babylonians, equally unable to grasp the simultaneous variation of numerator and denominator, usually kept the latter constant and sixty or some power of sixty.† Geometry, from Pythagoras with his mystic brotherhood to Pascal with his use of original investigation to allay the pain of toothache, fairly bristles with historic incidents and famous problems. The applied mathematics of physics, astronomy and engineering affords the same kind of material for advanced students. "I am sure," says one mathematician, "that no subject loses more than mathematics by any attempt to dissociate it from its history."

To correlate the study with allied subjects or interests, to present its topics in their historical setting—these are helpful means of securing interest in the pupil. There is another means, often more potent, but difficult to analyze and difficult to acquire

*1. Fine's *Number System of Algebra*, p. 83.

†2. Ball's *History of Mathematics*, pp. 3-4.

—the drawing and constraining force of the teacher's personality. We can look back to our own mathematics teachers and see some elements of such a personality—love for the subject and a sense of the latter's dignity, an enthusiastic conviction of the nobility of the teacher's work, transparent honesty, intellectual and moral, earnestness of character, courtesy and patience springing from genuine respect and love for the students. But in the actual building up of such a character no one can help us. This part of our problem we must solve alone, or, at our peril, leave unsolved.

NOTE ON AN "INDEPENDENT CRITICISM."

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Some good points are made in Professor Greenwood's paper in the March number of this magazine. Another is exemplified in Wentworth, 1, p. 1 and the first of his "Solid Geometry." In 492 he repeats the usual definition of a plane involved in 1, p. 1. In 493 he defines "determined." In 494 he says: "For a plane may be made to turn about any straight line AB in it." He gives a picture of several planes cutting in a straight line AB. On this assumption of his Theorem 1, "If two planes cut each other, their intersection is a straight line," he proves the determined principle. If we call these Y and X, he proves X by Y, then Y by X. This is the method of nearly all "the texts in common use." All the "folding proofs of plane geometry are subject to the same assumption. All this could be avoided if after the usual definitions of a plane and parallels the following theorem and proof, needing no figure, be given.

Theorem. If two planes have three points which are not in the same straight line in common, they have all their points in common.

Let M and N be two planes having three such points, A, B and C in common, and let P be a point in either.

To prove P in both planes.

Proof. Draw through P, any straight line that will cut both AB and CB. Let it cut AB in H and CB in K. Then

A and B being in both planes, H is in both planes.

C " B " " " " K " " "

H " K " " " " P " " " Q. E. D.