



555. Remarks on Note 508: Brocard Points for a Quadrilateral

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In this case the acceleration can be measured by the change of velocity at any instant, and it therefore is $\frac{v^2}{r}$ directed towards the centre of the circle.

CYMRIC.

554. [K¹. 1. a.] *Area of a Triangle in Terms of the Coordinates of its Angular Points.*

In the ordinary text-books of Analytical Geometry the determinantal form, into which the expression for the area is translated, plays no rôle in the investigation. The following method is free from this objection.

Let $A(x_1y_1)$, $B(x_2y_2)$, $C(x_3y_3)$ be the angular points of the triangle.

Assuming that $ax+by+c=0$ represents a straight line, we have for the line through A and B ,

$$\begin{aligned} ax+by+c &= 0, \\ ax_1+by_1+c &= 0, \\ ax_2+by_2+c &= 0; \end{aligned}$$

whence, eliminating a, b, c ,

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0,$$

the equation to AB .

The perpendicular, p , from C to AB is, by the ordinary rule,

$$\begin{aligned} p &= \frac{\begin{vmatrix} x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}}{\sqrt{(y_1-y_2)^2+(x_1-x_2)^2}} \\ &= \frac{\begin{vmatrix} x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}}{\div AB}. \end{aligned}$$

Thus, the area $= \frac{1}{2} \cdot p \cdot AB$

$$= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

The assumption that the linear equation represents a straight line is often justified by showing that the area of the triangle formed by any three points on the locus is zero. For this mode of proof the following may now be substituted.

Let $P(x_1y_1)$ be any selected point on the locus of $ax+by+c=0$, and $Q(x_1+h, y_1+k)$ any other point on the locus.

Then $ax_1+by_1+c=0$

and $a(x_1+h)+b(y_1+k)+c=0$,

whence $ah+bk=0$ or $\frac{k}{h} = -\frac{a}{b}$.

But $\frac{k}{h} = \tan \theta$, where θ is the inclination of PQ to the x -axis. Thus, this angle is the same for all positions of Q . The locus is consequently a straight line.

AUSTRAL.

555. [K¹. 8. a.] *Remarks on Note 508: Brocard Points for a Quadrilateral.*

In a brilliant series of articles (pp. 202, 217, 241, 265) in *Mathesis*, vol. v. 1885, Prof. Neuberg has given a large number of properties of the harmonic

quadrilateral in a circle, including the theorem obtained independently by Mr. F. G. W. Brown. A neater formula for ω is found, viz.,

$$\operatorname{cosec}^2 \omega = \operatorname{cosec}^2 A + \operatorname{cosec}^2 B,$$

where A, B are consecutive angles of the quadrilateral.

By Geometry, or the Complex, or Grassmann's methods, other results may be obtained.

If I be the intersection of BD, AC ; M, N their mid-points, then a Brocard point X lies on the circles MAD, MBC, MNI .

If λ denote the ratio $MA : MD (= MD : MC)$,

$$\cot \omega = \frac{1}{2} \left(\lambda + \frac{1}{\lambda} \right) \operatorname{cosec} \angle AMD \text{ (or } \hat{B}).$$

$$MX \sin B = \sin(B + \omega) - \lambda \sin \omega$$

$$= \frac{1}{\lambda} \sin \omega - \sin(B - \omega).$$

If BD be a fixed diameter of the circle and A, C vary, the locus of X is a lemniscate.

If AB be fixed, $\hat{ACB} = \hat{ADB} = \gamma$; then

$$\cos A \cos B = \cos(A + B + 2\gamma),$$

and, if P be the intersection of AD, BC , the locus of P is a parabola, ellipse or hyperbola, according as $\gamma = <$ or $> 45^\circ$.

R. W. GENESE.

556. [R. 2. b.] c.g. of a Quadrilateral Lamina.

Let $A_1A_2A_3A_4$ be a quadrilateral lamina of uniform density, and let the masses of the triangles $A_2A_3A_4, A_3A_4A_1, A_4A_1A_2$ and $A_1A_2A_3$ be $3m_1, 3m_2, 3m_3$ and $3m_4$ respectively, so that $m_1 + m_3 \equiv m_2 + m_4 = M$, where M is one-third of the mass of the quadrilateral.

Drawing the diagonal A_2A_4 and replacing the triangles so formed by equivalent masses at the vertices, those for the quadrilateral are m_3 at A_1, m_1 at $A_3, m_1 + m_3$ at A_2 and A_4 . Taking an identical quadrilateral and drawing the diagonal A_1A_3 , the equivalent masses for the quadrilateral are also m_4 at A_2, m_2 at $A_4, m_2 + m_4$ at A_1 and A_3 . Superimposing these quadrilaterals, we have a new quadrilateral of mass $6M$ and with the following set of equivalent masses:

$m_2 + m_3 + m_4$ at $A_1, m_3 + m_4 + m_1$ at $A_2, m_4 + m_1 + m_2$ at $A_3, m_1 + m_2 + m_3$ at $A_4,$

i.e. $2M - m_1$ at $A_1, 2M - m_2$ at $A_2, 2M - m_3$ at $A_3, 2M - m_4$ at A_4 .

The two sets of negative masses m_1 at A_1, m_3 at A_3 and m_2 at A_2, m_4 at A_4 are each equivalent to a negative mass M at the intersection of the diagonals, for this point divides each diagonal in the inverse ratio of the areas of the triangles on each side of the other diagonal.

Hence the centre of gravity of the quadrilateral lamina is the same as that of four masses, each one-third that of the lamina, placed at the vertices, together with an equal negative mass placed at the intersection of diagonals; and the distance of the c.g. from any line is $\frac{1}{3}(x_1 + x_2 + x_3 + x_4 - x_5)$, where the x 's are the distances of the above points, in order, from the given line.

E. H. SMART, M.A.

557. [J. 2. f.] Probability and Athletic Sports.

If more than 6, and less than 13, runners enter for a race, it is usual to arrange them by lot in two heats: the first three in each heat to start in the final. If there are more than 12 runners, but less than 19, they run in three heats, the first two in each heat being chosen for the final, which contains in any case 6 runners, supposed to be the best 6.