ART. XXIV.—*On the best Mode of presenting, in a popular form, the Theory of the Tides, with suggestions for constructing illustrative apparatus*; by WILLIAM DENNIS, Philadelphia, Pa.

It is remarked by Sir J. Herschel that “many persons find a strange difficulty in conceiving how they (the tides) are produced;” and Mrs. Somerville goes so far as to say (*Physical Sciences*, C. 13), that among those classed as astronomical problems this “is by far the most difficult and its explanation the least satisfactory.” This latter statement is perhaps rather broad as it stands, but if it were limited somewhat, so that the singularity of the phenomenon and the importance and familiar interest that attach to it and to its effects should be taken into account, it would scarcely require further qualification. It can hardly be denied that an intelligent comprehension of this subject is rare even among those to whom the causes of most natural phenomena are familiar, while to the great majority of intelligent people it is altogether a mystery. It seems, therefore, worth while to enquire whether the difficulties complained of have been reduced to a minimum, or whether they be not in part owing to defects or errors in the usual mode of presenting the explanation.

Having had occasion, in the preparation of a new elementary treatise on astronomy, to consider this subject attentively, as well as to examine the explanations commonly given, I have been compelled to conclude that no small portion of the obscurity and perplexity commonly supposed to belong to this subject arises from the want of a proper consideration and statement of the conditions and circumstances under which the causes producing the phenomenon act. If a learner be told, (and for whom are explanations intended if not for learners?) that the waters of the ocean are raised by the moon’s attraction, his first idea, in many cases, will be that they are lifted up *by main strength*, as it were, the force of gravity being *overcome*,¹ and having no where observed any similar effect of the moon’s attraction, he cannot conceive how this can be. Nor will it tend in any degree to lessen his perplexity if he shall see it stated, (as he may,) that according to Newton’s calculations, the *disturbing* power of the moon’s attraction on the surface of the earth is less than a ten-millionth part of the force of gravity, and that of the sun’s attraction not even half as great as it. It is therefore important to show, by a preliminary explanation, that the waters of the

¹ An idea akin to this must exist, it would seem, in the minds of those authors who speak of the *lateral* attraction of the moon at a given place after or before its passing the meridian of that place; as if this *disturbing* force, so minute at its greatest, and in respect of this lateral action, so greatly reduced by its very oblique direction, or else by the near approach of the place in question to the *mean distance* could ever produce any appreciable effect whatever in that way.

ocean, in their general figure and outline, are in a state of perfect equilibrium or perfectly balanced, so that, in view of this, and of their vast extent and perfect freedom of motion, they may be compared to a scale-beam in perfect equipoise, suspended in the most delicate manner, and *several thousand miles in length*. To omit this would be much the same as if one should state, in proof and illustration of the attraction of gravitation, that a weight or ball at the side of a mountain had been observed, (referring to the Schellien experiment,) to be drawn towards the mountain by its attraction, leaving the learner to suppose that the weight was placed on a table or other level surface instead of being suspended by a long thread or wire.

In explaining this condition of equilibrium the most obvious course will be to refer to mere hydrostatic equilibrium, in which any portion of these waters may be regarded as exactly balanced by any other contiguous portion, each being maintained at its level by the pressure of the other which supports it: consequently if this pressure be, in the case of either portion, lessened or increased *in the least degree*, that is to say, if the force of gravity, to which this pressure is due, be in any degree counteracted or added to by any other force in one of these portions and not in the other; the lighter portion will immediately give way and be buoyed up by the heavier which will of course simultaneously sink: and this would be an explanation sufficient for the purpose. But as this statement, though true, is not the whole truth, it may be well to go a step further. The waters of the ocean do not maintain their general figure and outline under the influence of gravity alone. On the contrary it is well known that by the centrifugal force generated by the earth's rotation on its axis they are kept at a higher level or greater distance from the centre on other parts of the globe than at the poles, this elevation amounting at the equator where it is greatest to about 13 miles. They are therefore exactly suspended or poised between these two forces, namely, the force of gravity and the centrifugal force just mentioned, and any other force that should in the least degree add to or counteract the influence of either of these forces would at once cause a change in the figure of these waters. While therefore it is properly the hydrostatic equilibrium existing between the different portions of the waters themselves that is disturbed by the action of the forces that produce the tides, the statement just made may serve to show more clearly how far these waters are, in their normal condition, from lying as a *dead weight* in the depressions of the earth's surface that contain them.

Again it is stated in a familiar way, (see *Chambers Hand-B. of Astron.*, *Bouvier's Famil. Astron.*, et al.,) that the tide on the side of the earth towards the moon is owing to the waters there being attracted by it more than the mass of the earth because they are

nearer, while the tide raised at the same time on the opposite side of the earth results from the earth being drawn away from the waters there because they are more remote than the mass of the earth and are thus "left behind," or "left heaped up;" and then we are told that at full moon, when the attractions of the sun and moon are *opposite* in direction, they conspire to produce spring tides in the same manner as at new moon when their attractions coincide in direction. Now as it is not easy to see how a body can be drawn away so as to leave any thing behind in two opposite directions at the same time, these statements appear quite inconsistent and are well calculated to confuse and perplex. It is therefore important and indeed indispensable to the communication of an intelligible view of this phenomenon to explain, as before remarked, the conditions and circumstances, or, to express it more definitely, the relations and dependencies existing among the bodies concerned in it: a course at once so natural and so needful that it seems remarkable that it should not have been more generally and more fully adopted.

As the earth is *held* to its curved path around the sun by the attraction of that body acting in opposition to the centrifugal force generated by its rapid motion, in the same manner that a heavy ball or weight attached to the end of a cord and whirled around the head is held or restrained by the cord, we may regard it as suspended between these two forces, and if a ball be merely suspended by a cord it will be a fair illustration of its condition; the force of gravity or weight of the ball standing in place of the centrifugal force in the case of the earth, and the tension of the cord representing the restraining force of the sun's attraction which at each instant holds the earth to the place in its orbit which it occupies. But as this attraction of the sun diminishes rapidly with an increase of distance it is plain that it cannot hold *all parts* of the earth alike or equally, the nearest part being about 8,000 miles less distant than the most remote, while of course it holds *the whole as a mass* as it would hold it were it at the *mean* distance of all the parts, that is, at the distance of the centre. Consequently on the nearest part or side towards the sun the attractive force being greater than the mean, there will be a small *excess* over what is sufficient to hold *this part* to its place in the orbit, and this excess acting upon the surface waters there in opposition to the force of gravity renders them specifically lighter and the exact equilibrium before described is immediately disturbed: these waters will therefore rise somewhat while those that are so situated as to be unaffected by this disturbing influence will sink simply from the *giving way* of those which having become lighter yield to their superior pressure. Again on the opposite side of the earth or that most remote from the sun, the attractive or restraining force will be less

than the mean and therefore not quite equal to the centrifugal force and here accordingly there will be an excess of this latter force: but on this side it is this centrifugal force that acts in a direction opposite to that of gravity, and this *excess* of it will consequently disturb the equilibrium of the surface waters here in precisely the same manner as in the other case.

Referring now, for illustration, to the suspended ball before mentioned, let us suppose it to be a hollow globe one or two feet in diameter, of a quite flexible material, as India rubber, having an opening about half an inch in diameter at the top and also at the bottom: let the principal suspending cord pass through the top so as to be attached at the centre to the intersection of two stiff horizontal wires, which are placed at right angles to each other and the extremities of which pass *loosely* through small openings in the sides of the globe. Passing this cord over a pulley and attaching a weight, so adjust the weight that it shall be sufficient to support the *middle* horizontal zone or segment of the globe. Let there be two other cords with pulleys and attach one to the top of the globe and the other to the bottom, the latter passing down through the opening in the top; then attach to the former a weight *somewhat more than* sufficient to support the top part of the flexible globe and to the latter a weight *not quite* sufficient to support the bottom part. Now it should be remembered that in this illustration the force of gravity or weight of the globe stands in place of the centrifugal force generated by the earth's motion in its orbit, and the tension of the cords, in place of the sun's attractive force varying at different distances: the cord attached to the wires at the centre may then represent the mean attractive force of the sun at the mean distance, as by this the globe is mainly held to its place, while the greater of the other two weights, being a little more than sufficient to support the part upon which it acts, will represent the attractive force acting upon the part of the earth nearest the sun, and the smaller weight, not quite sufficient for the same purpose, will represent the attractive force acting upon the opposite or most remote side. The globe being flexible, it is evident that the top part will be drawn up somewhat by the *excess* of the weight connected with it, while the bottom part will sink from the *deficiency* of its counterpoise and the whole will be elongated vertically and *for that reason* contracted horizontally. In like manner is the earth, in its yielding portions, the waters, elongated in one direction, that is, in the direction of the attracting or restraining force of the sun, and for that reason contracted in the direction at right angles to that.

Such is a statement of the general principle upon which the attraction of the sun (or of the moon) tends to produce a tidal

elevation on two opposite sides of the earth with intermediate depressions; and I prefer taking the case of the sun for this statement because the relations upon which this phenomenon depends are in general better understood or at least are more familiar to most as they exist between the earth and the sun than the same sort of relations between the earth and the moon.

Indeed, the sun being the great central power which controls the general motions of both the earth and the moon, there seems to be a manifest propriety in commencing with its influence, and, for the same reason, it will doubtless seem to many at first sight that it should be the chief and controlling cause of the tides. But this is not the case. The disturbing influence of the attracting body depends, not on the difference simply between the distance of the centre of the earth from it and that of the nearest or most remote side, but on this difference as *compared with the whole distance*. Now this difference in the case of the moon is about one-sixtieth part of the whole distance, but in the case of the sun it is less than a twenty-thousandth part, and hence it happens that while the attractive force of the moon is quite small compared with that of the sun, its *disturbing influence* is about two and one-half times that of the larger body.

Accordingly it is found that the moon exercises a *controlling* power over the tides, and the influence of the sun appears only in *modifying* the effects which it produces. We have now therefore to consider the case of the earth and the moon and to apply to it the principle already explained.

If it were a tide upon the moon that we are to explain, the complete analogy between this case and that of the earth and the sun would be at once recognized, the revolution of the moon around the earth being a fact as familiar to most as that of the earth around the sun. But action and reaction being always equal, while the earth *holds* the moon to its course or orbit in opposition to the centrifugal force, the earth itself is in like manner attracted and held by the moon to an extent proportioned to its inferior size or mass, and the consequence is that both bodies revolve about their common centre of gravity, that is, about a point between them which is as much farther from the centre of the moon than it is from the centre of the earth as the earth is heavier than the moon. That this point is comparatively near the centre of the earth, even nearer than (or within) the surface, in no way affects the operation of the causes which produce the phenomenon under consideration; but it is important to guard against what seems to be a very common misconception of the revolution of the earth about this point. This material error consists in supposing that point in the earth where this centre lies, or which coincides with this centre at any instant, to be *stationary* as regards this revolution, while the other parts of the

earth revolve about it; whereas, it is a revolution of the centre of the earth around this common centre of gravity, every point in the surface or elsewhere having a *corresponding motion*: it is not a revolution of the *parts* of the earth about a point within it which remains fixed, but a revolution of the earth *as a mass* about a certain point lying between its centre and that of the moon. This being understood, the case of the earth and the moon may be regarded as precisely analogous to that of the sun and the earth as already explained: in performing its revolution in this narrow orbit, (its diameter is about 5,400 miles,) the earth is held to a curvilinear motion in opposition to the centrifugal force by the moon's attraction, in the same manner that it is held to its larger orbit by the sun's, and there is consequently a corresponding *excess* of the attractive force over the mean and therefore over the centrifugal force on the side of the earth towards the moon, and a corresponding *deficiency* of attraction and consequent excess of centrifugal force on the opposite side. Hence, there must result, as in the former case, a disturbance of the equilibrium of the surface waters—a tendency in those on these two sides to rise to a higher level, and a consequent depression of those portions not affected by this disturbing influence. The illustrative apparatus, described in reference to the sun's action, is evidently equally applicable to this case. If it be thought desirable to render the representation more exact by exhibiting a *greater* effect than that produced for the case of the sun, it will only be necessary to increase the difference between the weight attached to the top and that connected with the bottom by reducing the latter and adding to the former.

It has been already stated that, in consequence of the comparative nearness of the moon to the earth, its attraction is the chief and *controlling* cause of the tides, but the sun, although its distance is 400 times as great, has yet sufficient influence to modify materially the effects which the smaller body tends to produce: it remains to show how these two causes operate in combination, or, in other words, how the effect produced by the moon as principal is modified by the action of the sun as accessory. Taking, first, the case of the two bodies being in conjunction at new moon, it is easy to see that, as they are both on the same side of the earth, their attractions have the same general direction and must therefore conspire to produce the same effect which will of course be an exaggerated one: hence the spring tides that are observed about the time of new moon. Nor is what has been already stated in relation to the earth's being held to its place by the sun's attraction in any way inconsistent with this result. It is still held by the sun, and with the same force, but the attraction of the moon is *added* to that of the sun and it is thus not merely *held*, but actually drawn out of the course which it would

have taken under the influence of the sun's attraction alone and brought for the time somewhat nearer to the sun.

Again, when the two attracting bodies are on opposite sides of the earth at full moon, it will be found that the result should be the same; for the earth is still held, and its two opposite sides *unequally* held, by the sun's attraction, which thus produces an elongation, or a tendency to elongation, in the manner before described: at the same time it is drawn, and these two sides *unequally* drawn, in the opposite direction by the moon's attraction; and as the side that is held least firmly by the sun is the one most attracted by the moon, and *vice versa*, this action of the moon will also tend to produce a further elongation in the same direction. Hence, these separate actions combine and produce an increased result, which appears in the spring tides belonging to the full moon. Nor can the separate *influences* of the sun and moon, (as it is important to note,) in any degree counteract or interfere with each other. Their *effects* may be combined, opposed, or mutually modified, according to the circumstances of each particular case, but the tendency of each of these attracting bodies, so far as the phenomenon under consideration is concerned, is to produce its own specific effect, and this *tendency* is in no way materially affected by the position or action of the other at the same time. To be satisfied of this, we have only to observe that the earth is at all times and under all circumstances held suspended by the sun's attraction without regard to the moon's position or action upon it. While thus held or suspended, the moon by its attraction draws it somewhat out of the course it would otherwise pursue, bringing it now a little nearer to the sun and then taking it a little farther from it, at one time hurrying it onward and again retarding it somewhat, (movements necessarily resulting from that revolution of the earth about the common centre of gravity of itself and the moon before explained); but this *by-play* between the earth and the moon in no way essentially affects the relation existing between the earth and the sun: the former unceasingly pursues its orbital course around the latter, which must therefore constantly hold it suspended by its attraction as already described.

The case in which the sun's and moon's attractions being in the same direction (at new moon) conspire to produce a greater effect, namely, the spring tides of new moon, seems hardly to require separate illustration. It is as if the power of one of these bodies were temporarily increased, which would of course produce an increased result. It may however be illustrated by a modification of the apparatus before described. Let the flexible globe be suspended by three weights with pulleys, representing as in a former case the sun's attraction, the weights being now connected with the several parts of the globe by *elastic cords* or bands, and

when the suspension is completed *clamp* the cords at the pulleys ; they will then represent the constant action of the sun and at the same time allow the whole globe or any of its parts to move to a limited extent under the action of another force. By means of three other cords with pulleys, attach three additional weights to the same points to act in the same direction and thus to represent the *added* attraction of the moon ; and by a proper adjustment of the second set of weights, and of the size or elastic force of the first set of cords, not only the increased elongation of the earth (the spring tides) belonging to this position of the other bodies, but its temporary approach to the sun, may be represented.

In the other case described, in which the attractions of the sun and moon are opposite in direction, (at full moon,) and still conspire to produce an increased result, some illustration may perhaps be more needful, and it is readily supplied by a further modification of the apparatus arranged as last described. Let the globe be suspended by the three elastic cords representing the sun's attraction, and clamped as before ; but as these act upward, and the moon's attraction is now supposed to be opposite in direction to that of the sun, it may be represented by weights attached to the globe *directly* and acting downward : attach, therefore, in this manner, the other three weights to the bottom, the top, and the centre, respectively, to represent the attractive force of the moon at those three distances, and as the lower side of the globe now represents the side of the earth toward the moon acting from below, as the sun from above, it is obvious that the weight attached to the bottom must in this case be greater than that attached to the top. Now it is evident that under this arrangement of the forces the elongation of the globe must be increased by the addition of the second set of weights, and in like manner does the combined attractions of the sun and moon, though opposite in direction, produce the increased effect observed in the spring tides of full moon.

At the first and last quarters of the moon the direction of the sun's attraction is at right angles to that of the moon, and its disturbing influence must therefore act upon the equatorial portions of the earth's surface intermediate to those acted upon by the moon at the same time. Now we have seen that the effect of the disturbing influence of these bodies is to render the surface waters upon which it acts specifically lighter, and thus to disturb the exact equilibrium that would otherwise exist ; these lighter portions being buoyed up to a higher level by those portions not so affected, which of course at the same time sink. But in the case we are now considering, while the moon's influence is producing the lunar tide on two opposite sides of the earth, the sun is at the same time acting upon the intermediate equa-

torial spaces and is thus diminishing to the extent of its influence the gravity of those very portions upon the weight of which the lunar tide in part depends.² The lunar tide will consequently be lessened by this action of the sun's influence, and hence the neap tides of the moon's first and last quarters. These neap tides may also be illustrated by the flexible globe; for if we have it merely suspended by three weights, as at first, but representing now the moon's attraction, and attach to two opposite sides two other weights with pulleys acting in a horizontal direction, these last may be regarded as representing the disturbing influence of the sun which acts *from the centre*, or in opposition to gravity on two opposite sides of the earth. Now these two weights, by drawing the two sides to which they are respectively attached nearer to their normal position or distance from the centre, or, in other words, by tending to elongate the globe in a direction at right angles to that of the moon's attraction, will lessen somewhat the vertical or lunar elongation.

In the intermediate positions, or between new moon and the first quarter, this quarter and full moon, and so on, the influence of the sun will vary from what it is at one of these extreme or turning points to what it is at the next; but to describe these changes—and the same may be said of numerous subordinate or collateral branches of this subject—does not fall within the scope of my present purpose; which was merely to point out a mode of presenting the essential features of the theory of the tides, calculated, as it seemed to me, to make it more easily and more fully understood than those in common use, and also to suggest some ocular illustrations.

² It will be observed that I say "in part," and speak above of "equatorial spaces," etc. There seems to be good reason to doubt whether in this connection one circumstance has been by any means properly attended to, namely, that the lunar tide may be raised in part, perhaps in some cases chiefly, by the pressure of those portions of the surface waters lying northward or southward of the central points of the moon's direct influence; these would be for the most part wholly undisturbed by the sun's influence. A lunar spheroidal wave is formed *theoretically* by pressure on *all* sides: the sun can materially interfere with its formation only on *two* (opposite) sides, namely, where the two antipodal centres of its influence approach, or fall upon, the edge of the wave. The theoretical proportion between spring and neap tides has been commonly, (not to say carelessly,) stated as that between the sum and *difference* of the two separate disturbing influences of the sun and moon, say as 7 to 3; but even theoretically this is manifestly incorrect, and, in point of fact, so far as I have observed, on the western shore of the North Atlantic, it does not approximate to the truth. Here, indeed, this northern and southern influence would seem to have freest scope, and in view of the eastern and western *boundaries* of this ocean, it seems not unlikely that it would be more consistent with the actual facts of the case to consider the neap tides *here* as the lunar tide *alone*, and the spring tides as the lunar and solar combined.