$$M[x_1^{a_1} x_2^{a_2}] = \frac{a_1!}{(a_1 - a_2)!} r_{12}^{a_2} + \frac{a_1!}{(a_1 - a_2 + 2)!} \frac{a_2(a_2 - 1)}{2!!} r_{12}^{a_2 - 2} \\ + \frac{a_1!}{(a_1 - a_2 + 4)!} \frac{a_2(a_2 - 1)(a_2 - 2)(a_2 - 3)}{4!!} r_{12}^{a_2 - 4} + \dots \dots \dots (32).$$

J'écris enfin les moments jusqu'au sixième ordre, en employant la notation

$$\beta_{a_1 a_2 \dots a_p} = M[x_1^{a_1} x_2^{a_2} \dots x_p^{a_p}].$$

$$\beta_2 = 1;$$

$$\beta_{11} = r_{12}.$$

$$\beta_4 = 3;$$

$$\beta_{21} = \beta_{12} = 3r_{12}, \beta_{22} = 2r_{12}^2 + 1;$$

$$\beta_{211} = 2r_{12}r_{13} + r_{23};$$

$$\beta_{1111} = r_{12}r_{34} + r_{13}r_{24} + r_{14}r_{23}.$$

$$\beta_6 = 15;$$

$$\beta_{21} = 15r_{12}, \beta_{12} = 12r_{12}^2 + 3, \beta_{22} = 6r_{12}^2 + 9r_{12};$$

$$\beta_{411} = 12r_{12}r_{13} + 3r_{23}, \beta_{221} = 6r_{12}^2r_{13} + 6r_{12}r_{23} + 3r_{13}, \beta_{222} = 8r_{12}r_{13}r_{23} + 2r_{12}^2 + 2r_{13}^2 + 2r_{23}^2 + 1;$$

$$\beta_{2111} = 6r_{12}r_{13}r_{14} + 3r_{12}r_{24} + 3r_{13}r_{24} + 3r_{14}r_{23}, \beta_{2211} = 2r_{12}^2r_{34} + 4r_{12}r_{13}r_{21} + 4r_{12}r_{14}r_{23} + 2r_{13}r_{14} \\ + 2r_{23}r_{21} + r_{34};$$

$$\beta_{21111} = 2r_{12}r_{13}r_{14}r_{25} + 2r_{12}r_{14}r_{25} + 2r_{12}r_{15}r_{34} + 2r_{13}r_{14}r_{25} + 2r_{15}r_{13}r_{24} + 2r_{14}r_{15}r_{23} + r_{23}r_{45} + r_{24}r_{35} \\ + r_{25}r_{34};$$

$$\beta_{111111} = r_{12}r_{34}r_{56} + r_{13}r_{24}r_{56} + \dots (15 \text{ termes}).$$

Les moments jusqu'à l'ordre 4 à deux variables ont été calculés par M. Pearson et puis par M. Soper qui a donné une formule générale pour les moments à deux variables; *Biometrika*, t. ix, 1913, p. 101. (On doit remarquer cependant que sa formule (xxxii) est atteinte d'une erreur typographique.)

Le cas de trois variables a été traité par M. Wicksell dans "The general characteristics of the frequency function of stellar movements, etc." Lund 1915, p. 11.

Enfin M. Isserlis a déduit notre formule (28) pour le cas $2^k = 4$ ou de quatre variables dans *Biometrika*, t. xi, 1916, p. 189.

III. Formulae for determining the Mean Values of Products of Deviations of mixed Moment Coefficients in two to eight Variables in Samples taken from a limited Population.

By L. ISSERLIS, D.Sc.*

A. Let p_{12}, p_{34} denote the product moment coefficients, referred to a *fixed origin* in a sample of size n extracted out of a population of size N , there being *four* independent variables x_1, x_2, x_3, x_4 . The mean value of p_{12} in many samples is P_{12} , the corresponding product moment coefficient for the sampled population. Let $dp_{12} = p_{12} - P_{12}$, denote the deviation of the moment coefficient of the sample from its mean value, then

Mean value of $dp_{12} dp_{34}$ in many samples

$$= \frac{\chi}{n} (p_{1234} - P_{12}P_{34}),$$

where

$$\chi = \frac{N - n}{N - 1},$$

and p_{1234} is the product moment coefficient with respect to the four variables.

[* Dr Isserlis sent me a paper containing the results of the present note with others accompanied by proofs in the course of 1916. It has been impossible to publish that paper so far, but it is only fair to him, having regard to the fact that other investigators are now entering this field, to publish his formulae in association with the memoirs printed in this part of *Biometrika*. EDITOR.]

This result gives many particular cases if we identify two or more of the variables. For instance

$$\text{Mean value of } dp_1 dp_2 = (p_{12} - p_1 p_2) \frac{\chi}{n},$$

where p_{12} is the second moment coefficient with regard to x , i.e. μ'_{12} in the usual notation for one variable and so forth. Also

$$\text{Mean value of } (dp_1)^2 = \frac{\chi}{n} (p_1 - p_1^2).$$

B. Similarly if there are 6 independent variables $x_1, x_2, \dots, x_5, x_6$.

The mean value of $dp_{12} dp_{24} dp_{56}$

$$= \frac{\chi\chi'}{n^2} \{p_{123456} - p_{1234} p_{56} - p_{2456} p_{12} - p_{1256} p_{24} + 2p_{12} p_{24} p_{56}\},$$

where

$$\chi' = \frac{N - 2n}{N - 2}.$$

Particular cases are

$$\text{Mean value of } (dp_{12})^2 = \frac{\chi\chi'}{n^2} (p_{12}^2 - 3p_{12} p_1 p_2 + 2p_1^2 p_2^2),$$

or

$$\text{Mean value of } (d\mu'_{12})^2 = \frac{\chi\chi'}{n^2} (\mu'_{12}^2 - 3\mu'_{12} \mu'_1 \mu'_2 + 2\mu_1'^2 \mu_2'^2),$$

and Mean value of $dp_{12} dp_{23} dp_{31} = \frac{\chi\chi'}{n^2} [p_{123} p_{23} - p_{12} p_{23} p_{31} - p_{23} p_{31} p_{12} - p_{31} p_{12} p_{23} + 2p_{12} p_{23} p_{31}]$.

C. Let there be 8 independent variables $x_1, x_2, \dots, x_7, x_8$.

The mean value of $dp_{12} dp_{24} dp_{56} dp_{78}$

$$\begin{aligned} &= \frac{\chi\phi}{n^2} [p_{1234} p_{5678} + p_{1256} p_{2478} + p_{1278} p_{2456} - p_{1234} p_{56} p_{78} - p_{5678} p_{12} p_{34} \\ &\quad - p_{1256} p_{24} p_{78} - p_{2478} p_{12} p_{56} - p_{1278} p_{24} p_{56} - p_{2456} p_{12} p_{78} + 3p_{12} p_{24} p_{56} p_{78}] \\ &\quad + \frac{\chi\phi'}{n^2} [p_{12345678} + p_{1234} p_{5678} + p_{1256} p_{2478} + p_{1278} p_{2456} - p_{123456} p_{78} - p_{123478} p_{56} \\ &\quad - p_{125678} p_{34} - p_{245678} p_{12}], \end{aligned}$$

where

$$\phi = 2 - 4\chi' + 3\chi'' + 4\chi'/n - 6\chi''/n,$$

$$\frac{\phi'}{n} = -1 + 3\chi' - 2\chi'' - 3\chi'/n + 4\chi''/n,$$

and

$$\chi'' = (N - 3n)/(N - 3).$$

When the sampled population is infinitely great

$$\phi' = \chi = \chi' = \chi'' = 1, \quad \phi = 1 - 2/n.$$

As a particular case,

$$\text{Mean value of } (dp_1)^4 = \frac{\chi\phi}{n^2} (3p_1^4 - 6p_1^2 p_1^2 + 3p_1^4) + \frac{\chi\phi'}{n^2} (p_1^4 + 3p_1^2 p_1^2 - 4p_1^2 p_1^2),$$

or in the notation usual for a single variable,

$$\text{Mean value of } (d\mu'_1)^4 = \frac{\chi\phi}{n^2} (3\mu_1'^4 - 6\mu_1' \mu_1'^2 + 3\mu_1'^4) + \frac{\chi\phi'}{n^2} (\mu_1'^4 + 3\mu_1'^2 \mu_1'^2 - 4\mu_1' \mu_1'^2).$$

When the sampled population is normal the results of (A), (B) and (C) can be immediately expressed in terms of the correlation coefficients $r_{12}, r_{13}, \dots, r_{78}$, by means of the formulae established on p. 138 in the current issue of this journal.