56. Note on the Simson Line

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Again,

$$\angle ABP = \angle BFG, BGF = \angle PGL;$$

$$\therefore AB \parallel GL, \text{ and similarly } BC \parallel FH,$$

hence (a), (b) [or Euc. I. 26], $\triangle s ABP, BCQ$ are congruent, and $AB = BC$. Hence $ABCDE$ is equilateral and also equiangular (Euclid I. 15), therefore, etc.

Euclid IV. 13 may be derived at once from IV. 14. For the chords AB, BC, CD, DE, EA are proved equal, therefore they are equally distant from the centre (Euclid III. 14). Hence a circle with centre O (found in 14) and radius equal to the perpendicular distance of O from any one of the chords will touch the other chords and be inscribed in the pentagon.

R. TUCKER.

56. Note on the Simson Line.

I. If LMN is the Simson line of a point P on the circumcircle of a triangle ABC , each segment LM, MN, NL is proportional to the join of P to the angle containing the segment.

Let AD be the diameter through A .

Then, since the triangles PMN, BPC are similar, $\frac{MN}{PM} = \frac{BC}{PC}$; and since the triangles PMC, APD are similar, $\frac{PM}{PC} = \frac{AP}{AD}$;

$$\therefore MN = \frac{BC}{PC} \cdot PC \cdot \frac{AP}{AD} = AP \cdot \frac{BC}{2R} \propto AP;$$

similarly for LM, NL .

II. If P' be a point on the circle such that $PAD = P'AD$, and $N'L'M'$ be the Simson line of P' , we have

$$M'N' = AP' \cdot \frac{BC}{2R} = AP \cdot \frac{BC}{2R} = MN.$$

III. If $N'L'M'$ be the Simson line of Q the end of the diameter through P ,

$$\begin{aligned} M'N'^2 &= \frac{BC^2}{4R^2} A Q^2 \\ &= \frac{BC^2}{4R^2} (4R^2 - AP^2) = BC^2 - MN^2, \\ \therefore MN^2 + M'N'^2 &= BC^2. \end{aligned}$$

Hence the sum of the squares of the segments of the Simson lines of the extremities of a diameter is equal to the sum of the squares of the sides, i.e. is constant.

IV. If $MN=s_1$, $NL=s_2$, $LM=s_3$, and dashes refer to $L'M'N'$, then from the relations

$$s_1' \pm s_2' \pm s_3' = 0, \quad s_1'^2 + s_1'^2 = a^2, \quad s_2'^2 + s_2'^2 = b^2, \quad s_3'^2 + s_3'^2 = c^2,$$

we get

$$\sqrt{a^2 - s_1'^2} \pm \sqrt{b^2 - s_2'^2} \pm \sqrt{c^2 - s_3'^2} = 0.$$

V. An easy method of drawing the Simson line of P is as follows:

AD the perpendicular on BC meets the circumcircle in G . PG cuts BC in E . Join P to the orthocentre H . Then the join of the mid-points of PE and PH is the Simson line of P .

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57. Criteria of Divisibility.

Let $\dots abcdef$ be any number,

and $1, x, y, z, \dots$ the successive remainders on dividing $10^0, 10^1, 10^2, 10^3, \dots$ by any divisor F .

Then $(\dots abcdef) \div F$ and $(f + ex + dy + cz + \dots) \div F$ have the same remainder.

If the remainder of the latter is zero, F is a factor of $\dots abcdef$.

Hence the following simple criteria:

Dividing $10^0, 10^1, 10^2, 10^3, \dots$ by 3, 9 the remainders are 1, 1, ...

..... 6	1, 4, 4, ...
..... 18	1, 10, 4, 4, ...
..... 15, 45	1, 10, 10, ...
..... 8	1, 2, 4, 0, 0, ...
..... 125	1, 10, 100, 0, 0 ...
..... 7	1, 3, 2, 6, 4, 5, ...
	or 1, 3, 2, -1, -3, -2,
..... 11	1, 10, ...
	or 1, -1, ...

Hence any number $\dots abcdef$ is divisible by

3 or 9 if $f + e + d + c + b + a + \dots$ is $M(3)$ or $M(9)$,

6 if $f + 4e + 4d + 4c + 4b + 4a + \dots$ is $M(6)$,

15 or 45 if $f + 10e + 10d + 10c + 10b + 10a + \dots$ is $M(15)$ or $M(45)$,

18 if $f + 10e + 4d + 4c + \dots$ is $M(18)$,

8 if $f + 2e + 4d$ is $M(8)$,

125 if $f + 10c + 100d$ is $M(125)$,

7 if $f + 3e + 2d + 6c + 4b + 5a + \dots$ is $M(7)$,

or if $(f + 3e + 2d) - (c + 3b + 2a) + \dots$ is $M(7)$,

11 if $f - e + d - c + b - a + \dots$ is $M(11)$.

II. If $1, x, y, z, \dots$ are the successive remainders on dividing

$10^0, 10^n, 10^{2n}, 10^{3n}, \dots$ by F ,

then any number is divisible by F if, on dividing the digits into groups (from right to left) of n digits, the sum of the products of the groups by $1, x, y, z, \dots$ is a multiple of F .

(a) Take $n=2$. The remainders for 11 are 1, 1, ...;

$\therefore \dots abcdef$ is divisible by F if $(ef) + (cd) + (ab) + \dots$ is $M(F)$.

[Ex. 13893; we have $93 + 38 + 1 = 132 = M(11)$.]

(b) Take $n=3$. The remainders for 13 are 1, 12, ..., or 1, -1, ...;

$\therefore \dots abcdef$ is divisible by F if $def - abc + \dots$ is $M(F)$.