*(Paper No. 3332.)***"Formulas relating to Saturated Steam."**

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IN presenting the following investigation of formulas relating to saturated steam, the Author desires to emphasize the importance of considering the properties of steam from the point of view of its "intrinsic energy," *i.e.*, the amount of heat-energy contained in the steam at any particular temperature. One pound of saturated steam at any given temperature contains a definite quantity of heat-energy, its amount being altogether independent of the cycle of operations through which the steam has passed before arriving at the temperature in question. It will be observed that the "total heat of the steam" at a certain temperature is an entirely different quantity, for it comprises (1) the amount of heat necessary to raise water from 32° F. to the given temperature, (2) the amount of heat necessary to overcome the molecular resistances in converting the water into steam, and (3) the amount of heat which must be expended in making room for the steam at the pressure corresponding to the given temperature. The first two parts represent heat-energy existent in the steam, and together constitute its "intrinsic energy," whilst the third part is simply a concomitant of the production of the steam, and does not represent energy contained in the steam. In the cylinder of a steam-engine the intrinsic energy of the steam at the point of cut-off represents its power of doing work, while its intrinsic energy at the point of release represents the heat-energy rejected from the cylinder. The importance of the second part, as representing, under ordinary working conditions, inevitable loss of heat at the point of release, will be emphasized later.

Since 1 lb. of steam, at any given temperature, has a definite amount of intrinsic energy, it appears reasonable to suppose that a formula can be obtained expressing it in terms of the temperature. Having been led to consider the relative values of the intrinsic or internal energy of steam and that of water at the same temperature, the Author found that the ratio of the intrinsic

energy of water (*i.e.*, the heat required to raise a given weight of water from 32° F. to the temperature t° F.) to the difference between the intrinsic energy of the same weight of saturated steam at t° F. and at 32° F. has a value whose rate of variation with the temperature is constant. If h be taken to represent the amount of heat, in British thermal units, required to raise 1 lb. of water from 32° F. to t° F., and if ψ and ψ_0 denote the intrinsic energy of steam, in British thermal units, at t° F. and at 32° F. respectively, this property may be expressed—

$$\frac{d}{dt} \cdot \frac{h}{\psi - \psi_0} = \text{a constant} \quad . \quad . \quad . \quad (1)$$

Table I. (pp. 412–413), in which the figures necessary for the calculations are taken from Rankine's Tables for saturated steam, shows that, between the temperatures of 41° F. and 428° F., the ratio of h to $\psi - \psi_0$ varies between 5.27 and 4.56, the decrease of this ratio moreover being proportional to the increase of the temperature. In the Table Δ is the difference between consecutive lines in the 4th column.

Let H represent the amount of heat, in British thermal units, necessary to raise 1 lb. of water from 32° F. to t° F. and evaporate it into steam at the latter temperature.

H_0 the heat, in British thermal units, expended in evaporating 1 lb. of water at 32° F.

P_0 and P the steam-pressures, in pounds per square foot, corresponding to the temperatures 32° F. and t° F. respectively.

p_0 and p the same pressures in pounds per square inch.

v_0 and v the volumes, in cubic feet, of 1 lb. of saturated steam at the same temperatures.

T_0 and T the absolute temperatures corresponding to 32° F. and t° F.

s the volume, in cubic feet, of 1 lb. of water.

and J , "Joule's equivalent."

$$\text{Then } \psi = H - \frac{P(v-s)}{J} \text{ and } \psi_0 = H_0 - \frac{P_0(v_0-s)}{J}.$$

From the Table the constant in equation (1) can be readily determined, and the equation becomes—

$$\frac{d}{dt} \cdot \frac{h}{\psi - \psi_0} = -0.00183 \quad . \quad . \quad . \quad (2)$$

Integrating this—

$$\frac{h}{\psi - \psi_0} = -0.00183 t + c,$$

where c is the constant of integration.

From the Table it is easily seen that $c = 5.34$, and from Rankine's Tables that $\psi_0 = 1037.92$.

The equation therefore becomes—

$$\psi = 1037.92 + \frac{h}{5.34 - 0.00183 t} \quad (3)$$

Or, representing $\frac{h}{5.34 - 0.00183 t}$ by μ ,

$$\psi = 1037.92 + \mu \quad (4)$$

This expression is, of course, the area of the entropy diagram below the line of constant volume for any given temperature of saturated steam, *Fig. 1.*

It will be noticed that the accuracy of the figures in the fourth column of Table I. is not impaired by the fact that Rankine took "Joule's equivalent" as 772, instead of the now generally accepted value of 778, since each term in the ratio involves this quantity, $v - s$ being calculated from the formula—

$$(v - s) \frac{dP}{dT} = \frac{JL}{T},$$

L being the latent heat of evaporation, in British thermal units, for temperature T .

To show the degree of exactitude with which the expression approximates to the figures derived from experiment, the values of ψ in the third column of Table II. (p. 414) are taken from Professor Peabody's Tables for saturated steam,¹ those in the fourth column being calculated from equation (3).

The greatest difference between the figures given in the last two columns is 0.3, *i.e.*, less than 0.03 per cent., and the sign of the difference is sometimes positive and sometimes negative.

¹ "Tables of the Properties of Saturated Steam and other Vapours," C. H. Peabody. J. Wiley and Sons, 1896.

The formula (3) is thus seen to have a definite physical interpretation, and, moreover, it is of the same order of accuracy as the

TABLE I.

t	$\psi - \psi_0$	h	$\frac{h}{\psi - \psi_0}$	Δ
° F.	Foot-Lbs.	Foot-Lbs.		
32	6	0		
41	1,318	6,948	5.27	0.06 2
50	2,665	13,896	5.21	-1 2
59	3,992	20,844	5.22	0 1
68	5,317	27,792	5.22	3 2
77	6,695	34,740	5.19	0 1
86	8,034	41,702	5.19	2
95	9,405	48,650	5.17	1
104	10,782	55,612	5.16	2
113	12,173	62,560	5.14	2
122	13,595	69,522	5.12	3 2
131	15,028	76,484	5.09	1 2
140	16,433	83,459	5.08	2
149	17,850	90,435	5.06	1
158	19,279	97,411	5.05	2
167	20,747	104,387	5.03	2
176	22,207	111,363	5.01	1
185	23,667	118,353	5.00	2
194	25,186	125,357	4.98	2
203	26,645	132,360	4.96	1
212	28,155	139,363	4.95	2
221	29,678	146,380	4.93	2
230	31,213	153,412	4.91	

formula $H = 1082 + 0.305 t$; it may therefore be used in investigating the properties of steam in place of the empirical formulas,

which, although they express a relationship between the variables, in most cases have no concrete meaning.

TABLE I.—(continued).

t	$\psi - \psi_0$	h	$\frac{h}{\psi - \psi_0}$	Δ
° F.	Foot-Lbs.	Foot-Lbs.		
239	32,791	160,429	4.88	3 } 2
248	34,314	167,460	4.87	1 } 2
257	35,888	174,505	4.85	2
266	37,470	181,564	4.84	1
275	39,025	188,637	4.83	1
284	40,641	195,711	4.81	2
293	42,257	202,798	4.79	2
302	43,889	209,885	4.78	1
311	45,520	216,986	4.76	2
320	47,168	224,087	4.75	1
329	48,800	231,216	4.73	2
338	50,540	238,358	4.71	2
347	52,292	245,501	4.69	1
356	53,910	252,658	4.68	1
365	55,599	259,829	4.67	2
374	57,346	267,013	4.65	2
383	59,106	274,198	4.63	1
392	60,869	281,394	4.62	2
401	62,632	288,634	4.60	1
410	64,407	295,874	4.59	1
419	66,172	303,128	4.58	2
428	67,974	310,381	4.56	

For example, this formula may be used in finding an expression for the specific volume of saturated steam in terms of the experimental quantities t , P and h :—

$$\text{From (3)} \quad \frac{P(v-s)}{J} = H - 1037.92 - \frac{h}{5.34 - 0.00183 t},$$

substituting

$$H = 1082 + 0.305 t,$$

$$v = s + \frac{J}{P} \left(44.08 + 0.305 t - \frac{h}{5.34 - 0.00183 t} \right)$$

$$\text{or} \quad v = s + \frac{J}{P} (44.08 + 0.305 t - \mu) \quad . \quad . \quad (5)$$

an expression for v in terms of t , P and h .

TABLE II.

Absolute Pressure. Lbs. per Square Inch.	Temperature of Boiling Point. °F.	ψ (British Thermal Units).	
		From Saturated Steam Tables.	Calculated from Equa- tion (3).
1	102.0	1051.2	1051.5
14.7	212.0	1074.2	1074.4
25	240.0	1080.6	1080.6
50	280.8	1089.9	1089.8
75	307.4	1096.0	1095.9
100	327.6	1100.7	1100.8
125	344.1	1104.6	1104.8
150	358.3	1108.1	1108.4
175	370.6	1111.2	1111.5
200	381.7	1114.1	1114.3
225	391.8	1116.7	1116.9
250	401.0	1119.2	1119.3
275	409.5	1121.5	1121.5
300	417.4	1123.9	1123.6

Next, to find the rate of increase of the intrinsic energy of saturated steam with the temperature:—

Let this quantity be represented by C . By differentiating equation (3)—

$$C = \frac{d\psi}{dt} = \frac{d}{dt} \cdot \frac{h}{5.34 - 0.00183 t}$$

$$\therefore C = \frac{(5.34 - 0.00183 t) \frac{dh}{dt} + 0.00183 h}{(5.34 - 0.00183 t)^2}$$

$$= \frac{1}{5.34 - 0.00183 t} \left(\frac{dh}{dt} + \frac{0.00183 h}{5.34 - 0.00183 t} \right).$$

$$\text{Or} \quad C = \frac{\mu}{h} \left(\frac{dh}{dt} + 0.00183 \mu \right) \quad . \quad . \quad . \quad (6)$$

$$\text{But}^1 \quad h = t - 32^\circ + 0.000000103 \{ (t - 39^\circ)^3 + 343^\circ \}$$

$$\therefore \frac{dh}{dt} = 1 + 0.000000309 (t - 39^\circ)^2.$$

Therefore when—

$$p = 14.7 \text{ lbs. per square inch, } C = 0.217 \text{ B.Th.U.}$$

$$\text{and when } p = 300 \quad " \quad " \quad " \quad , C = 0.262 \quad "$$

It is easy to obtain from equation (3) an expression for the amount of heat necessary to overcome the molecular resistances in converting water into steam. If ρ denote this quantity in British thermal units—

$$\psi = \rho + h$$

$$\text{or} \quad \rho = \psi - h.$$

Substituting for ψ —

$$\rho = 1037.92 - (4.34 - 0.00183 t) \mu$$

$$\text{or} \quad \rho = 1037.92 - \frac{4.34 - 0.00183 t}{5.34 - 0.00183 t} \times h \quad . \quad . \quad (7)$$

By taking the average value of the last term, and allowing for the slightly greater rate of increase of h compared with t , the expression may be written—

$$\rho = 1061.3 - 0.79 t \text{ (approximately)} \quad . \quad . \quad (8)$$

which is the form of expression given by Zeuner.

To recognize the importance of this quantity ρ , it is only necessary to write down the expression for the work done by 1 lb. of steam expanding adiabatically in a steam cylinder. Let this be represented by U —

$$\text{Then} \quad U = \frac{P_1 (v_1 - s)}{J} + \psi_1 - \psi_2 x - h_2 (1 - x) - \frac{P_b v_2}{J}.$$

The suffix 1 refers to the point of cut-off, and the suffix 2 refers to the point of release.

P_b is the back pressure, and $1 - x$ is the proportion of the 1 lb. of steam that liquefies during adiabatic expansion.

The first term on the right-hand side of the equation is the work done during the conversion of 1 lb. of water into 1 lb. of

¹ "The Steam Engine," W. J. M. Rankine, 9th edition, p. 246.

steam, the second term is the intrinsic energy of the steam at the point of cut-off, the third term is the intrinsic energy of the unliquefied portion of the steam at the point of release, the fourth term is the intrinsic energy of the liquefied portion at the same time, and the last term is the work done against back pressure.

It may be written—

$$\begin{aligned} U &= H_1 - (\psi_2 - h_2) x - h_2 - \frac{P_1 v_2}{J} \\ &= H_1 - \rho_2 x - h_2 - \frac{P_1 v_2}{J}. \end{aligned}$$

Of the negative terms, h_2 represents heat which may be utilized to raise the feed-water to the temperature of the exhaust, P_1 may be reduced nearly to zero by condensing, but the remaining term $\rho_2 x$, where $x = \left(\log. \frac{T_1}{T_2} + \frac{L_1}{T_1} \right) \times \frac{T_2}{L_2}$, which depends simply on the initial and final temperatures, represents inevitable loss as far as the engine is concerned, as it simply heats the condensation water, or is dissipated in the atmosphere in the case of a non-condensing engine.

Consequently ρ is seen to be important as representing unavoidable loss in the steam at the point of release; it can be reduced by expanding the steam as much as possible, but a definite loss occurs depending upon the temperature at "release."

$$\text{Just as} \quad \psi = H - \frac{P(v-s)}{J}$$

$$\text{so} \quad \rho = L - \frac{P(v-s)}{J}$$

And it is interesting to note that a relationship, of the form

$$\frac{d}{dt} \cdot \frac{\rho}{L} = \text{a constant, exists.}$$

It is easily found from the Tables that—

$$\frac{d}{dt} \cdot \frac{\rho}{L} = -0.000146.$$

Integrating this and introducing the constant—

$$\rho = L(0.956 - 0.000146 t) \quad . \quad . \quad . \quad (9)$$

In the following Table the values of ρ as calculated from formulas (7) and (9) are compared with the values given in Peabody's Steam Tables:—

TABLE III.

p .	ρ from Equation (7).	ρ from Equation (9).	ρ from Peabody's Tables.
Lbs. per Square Inch.	B.Th.U.	B.Th.U.	B.Th.U.
14·7	893·7	893·5	893·5
50	839·6	839·4	839·7
100	802·9	802·8	802·8
150	778·4	778·3	778·1
200	759·7	759·7	759·5
250	744·6	744·5	744·5
300	731·7	731·9	732·0

This at once suggests a simple formula for L by combining equations (8) and (9), when the following equation is obtained:—

$$L = \frac{1061 \cdot 3 - 0 \cdot 79 t}{0 \cdot 956 - 0 \cdot 000146 t} \cdot \cdot \cdot (10)$$

An equation for L involving only t to the first power.

The closeness of the agreement of the values calculated from this equation with the experiment values is shown in the following Table:—

TABLE IV.

p .	L from Equation (10).	L from Peabody's Tables.
Lbs. per Square Inch.	B.Th.U.	B.Th.U.
14·7	966·2	965·9
50	917·5	917·4
100	883·6	884·0
150	861·1	861·2
200	843·9	843·8
250	829·5	829·5
300	817·6	817·4

From equations (8) and (9) a formula for p in terms of t may be obtained, which is in a convenient form for calculation:—

$$\frac{P(v-s)}{J} = L - \rho$$

from (9)

$$= L(0 \cdot 000146 t + 0 \cdot 044)$$

$$= \frac{(v-s) T}{J} \cdot \frac{dP}{dT} (0 \cdot 000146 T - 0 \cdot 0233)$$

$$\therefore \frac{dP}{P} = \frac{dp}{p} = \frac{dT}{T(0 \cdot 000146 T - 0 \cdot 0233)}$$

Integrating and finding the constant by substituting $p = 100$ lbs. per square inch, when $T = 788.6^\circ$:—

$$\log p - \log 100 = \frac{1}{0.0233} \left(\log \frac{T - 159.6}{T} - \log \frac{629}{788.6} \right),$$

$$\text{or} \quad p = 100 \left(\frac{788.6}{629} \cdot \frac{T - 159.6}{T} \right)^{42.9}$$

$$\text{or} \quad \log p = 6.21305 + 42.9 \log \frac{T - 159.6}{T} \quad (11)$$

In Table V. are given the values of p calculated from this formula for various values of t .

TABLE V.

t .	p from Formula (11).	p Experimental Value.
$^{\circ}\text{F.}$	Lbs. per Square Inch.	Lbs. per Square Inch.
212.0	14.8	14.7
280.8	50.0	50
327.6	100.0	100
358.3	150.0	150
381.7	200.0	200
401.0	250.2	250
417.4	300.0	300

As a further proof of the accuracy of the foregoing formulas, the following shows that h may be calculated from equations (8) and (9) to a high degree of accuracy; it is not suggested that the expression for h so obtained has any further value than affording additional evidence of the reliability of the formulas in question—

$$\begin{aligned} h + \rho &= H - L + \rho \\ &= H - (L - \rho), \end{aligned}$$

substituting on the left from (8) and on the right from (9)

$$\begin{aligned} h + 1061.3 - 0.79 t &= H - L (0.000146 t + 0.044) \\ &= H - (H - h) (0.000146 t + 0.044) \end{aligned}$$

substituting for H —

$$\begin{aligned} &= (1082 + 0.305 t) (0.956 - 0.000146 t) + h (0.000146 t + 0.044) \\ &= 1034.4 + 0.134 t - 0.0000445 t^2 + h (0.000146 t + 0.044) \end{aligned}$$

$$\therefore h = \frac{0.924 t - 0.0000445 t^2 - 26.9}{0.956 - 0.000146 t}$$

$$\text{e.g. if } t = 240^{\circ} \text{ F., } h = 208.8 \text{ (tabular value } 209.1)$$

$$,, t = 401^{\circ} \text{ F., } h = 374.9 \text{ (tabular value } 374.7).$$

Having regard to the importance of the quantities ψ and ρ —the former as representing the heat-energy contained in 1 lb. of steam at any given temperature, and the latter as representing the inevitable loss of heat in a pound of steam at the point of release—the Author ventures to think that the formulas for these quantities—formulas (3), (7) and (9)—the accuracy of which is established by the foregoing applications, are of sufficient interest and importance to warrant their presentation to the Institution.

The Paper is accompanied by a tracing, from which the Figure in the text has been prepared.