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THE OPPORTUNITY OF THE TEACHER IN THE CLASS ROOM.¹

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A pupil receives many impulses to the study of our science, and his growth rests upon many determining causes. There may be, what is somewhat rare, his own native love of the subject, which will prompt him to independent study. There is the assigned home work, the required outside reading, the aid that he should receive from a good text-book; and there is the artificial stimulus of marks, of unannounced tests, and of the general review forced upon him by the approach of a semiannual judgment day. But greater and far more vital than any one of these, or than all of them put together, is the stimulus he receives from the class room hour.

The teacher has ordinarily from forty to fifty minutes at his disposal from three to five times a week, and, allowing for examinations, for perhaps thirty-two weeks in a year. In these precious minutes he has to deal with a score of boys, that he may help in their transformation, not necessarily into mathematicians, but into rational men. Truly he cannot reflect too often on how to use the time. So I trust that you will be patient if you have to hear some old-fashioned truths, some homely, oft-repeated truths. It pays even our public men, from time to time, to refresh their knowledge of such old-fashioned things as the Ten Commandments and the Declaration of Independence.

The opportunity of the teacher in the class room: this, then, is my theme. We will assume and omit from consideration the mechanical side, suppose the pupils seated, the roll called, the absentees noted, and, yes, the test applied as to whether the boys have their lessons—for this, too, belongs to the mechanical side. The vital relationship of the teacher to the pupil still remains, and it is of this mainly that I wish to speak. A word or so at the outset by way of digression with reference to preparation. It sounds like a commonplace to say that a teacher should be

¹Read before the Association of Mathematical Teachers of New England, December 9, 1911.

prepared for his work; and yet it is a matter of equally common observation that men and women teach, and continue to teach, without meeting such a requirement. Thorough equipment is fundamental, for only mastery can beget mastery. The mathematics teacher should know the subject with which he has to deal in outline and in detail. He should have searched its length and breadth, have reached to its heights, and sounded its depths. He should have gone to its bounds, yes, and beyond. The danger line is nearly always at the boundary. The teacher's experience is like that of many of the functions with which he has to deal; they behave in a reputable and orderly fashion within the region of their definition—but at the boundaries, beware! The only safe course is to fix the boundaries of one's knowledge far beyond the probable limits set by the demands of one's daily rounds. That introduces the factor of safety, as the bridge builder says. You remember that Kipling's polo pony, the Maltese Cat, won the match because of the demoralizing effect of boundary conditions on his opponents, and many a baseball or football game has been lost at the side lines.

And this preparation should not be a rigid fact, something that was finished far back in college or university days. It should be an active, continuous, vital process, not so much a preparation as a preparing. You know that in the administration of the German *gymnasium* it is expected that the *ordinarius* at least shall usually be engaged in advanced work; when possible, in actual research work. Many notable contributions to the advance of mathematical science have come from the labors of these teachers in secondary schools. But while mathematics has been thus enriched, such work has its chief value in its reaction upon the worker. Among other things—and this is not to be despised as a consideration—the difficulties of investigation in his own advanced work will make him more sympathetic with the attitude of the earnest student toward what is new to him in mathematics. It will enable him to realize that the pupil who cancels the “3” in $3 \sin \frac{\pi}{3}$ is not necessarily stupid or lazy. It will remind him that new symbols are a new language, and that one is never master of a language when he still feels the need of translation; but only when it has come to be a tool of thought.

But something more is requisite than such a general preparation as has been indicated. The teacher must prepare himself for each individual exercise. He must see it in perspective, de-

fine sharply in his mind what most needs emphasis, see its relation to what has gone before and what is to come after. No familiarity with the subject, no matter how extensive the experience upon which it may rest, will justify the teacher in neglecting such special preparation. A number of years since I was making a short stay in Göttingen, and dropped in at one of Klein's lectures. He was giving his elementary course in the differential and integral calculus. The lecture was the perfection of art. Subsequently, in a conversation with his assistant, I learned, that (according to Klein's own statement) he was putting more time on the course than when he had first given it, many, many years before. That one of the greatest, certainly the most famous mathematical teacher in the world should have thought it worth his while to lavish care and thought on youngsters who had just come up from the *gymnasium*—was illuminating and stimulating.

The stages of preparation behind us, let us now go into the class room and meet our pupils. What shall we do first? Perhaps assign the next lesson. At any rate we should not leave it to the fag-end of the period, or, worse still, till the bell has sounded the recessional. I have actually known teachers who shouted the lesson to their pupils as these were leaving the room. The assignment of a lesson is, in my judgment, one of the most important and serious things in connection with the class room exercise. Connection should be shown with past lessons and that of the day. It may be wise even to assign work for several days ahead, such assignments, of course, being open to modifications as the work progresses. Every writer or speaker knows the importance of getting into touch with his hearers, and one of the very best ways in the case of a connected series of lectures or exercises is by an outline review, a resumé of what has just been studied. Such a resumé should, in many cases, be asked for from some member of the class; but unless the response is entirely successful, the teacher should go over the outline again. Such connected reviews of large, or at least considerable portions of a subject are a source of one of the most admirable disciplines offered by mathematics. President Lowell said last October, at the meeting of the New England colleges and preparatory schools, that while many a student whom he had set at a reading task proved, when called upon, to know a great deal about particular paragraphs, few knew anything about the book. The teacher should make every effort to enlarge the boy's horizon, by ex-

plaining underlying principles and possible classifications of the facts of the science. These things should not, of course, be dogmatically stated (dogmatism has no possible place in our science), but like every conquest of the recitation hour they represent the joint effort of teacher and taught. Indeed, the work of the class room should be before all else coöperative. Did you ever read Dicey, "On the Teaching of English Law at Harvard"? It is a fine tribute, from a man who is himself a great teacher, to an institution of the first rank. A number of years ago, I had the privilege of visiting the school and hearing some of the lectures. I recall very definitely the hour with Dean Ames. It was called a lecture; but such in the usual sense of the term it most emphatically was not. The teacher began by stating briefly but clearly an issue that had been joined, involving a legal distinction. The matter once fairly launched, Dean Ames asked Mr. Hitchcock—I recall the name, as he was an Amherst graduate—to give his own opinion. Mr. H. was followed by Mr. A. and Mr. B., and others, who acquiesced in, modified, or attempted to controvert the previously expressed opinion. It was a series of recitations, bridged over by keen and incisive questions by the dean, every one calculated to inspire thought and to lead to a solution. When this process had gone on for a half hour, perhaps, the teacher closed the discussion somewhat in the way in which a judge sums up the arguments of the counsel in an actual case at law, and states whither the weight of the discussion must inevitably lead. In fine, the students in that class had by joint effort under the guidance of their teacher thought out the law.

This illustrates the fact that the basis of the highest class room efficiency in any subject is the creation in the pupil of the lively sense that under sympathetic, careful guidance he is rediscovering truth. I am aware that such a genetic method, such a method of rediscovering truth, is the intent of most of our best schools and colleges to-day. But if your experience resembles my own, you will confess that we are, all of us, ever falling from grace. It is so easy to err on one side or the other, to lecture, in the literal sense of that term—which is really to dogmatize, or rather to furnish the pupil with ready-made reasoning which he needs only to assimilate and give back in tests; or, on the other hand, to let the exercise degenerate into "hearing the lesson." I have recently read anew an article which appeared in the *Educational Review* in 1903 on "The American College, from the Viewpoint of an Undergraduate." It was written by a man only a

few years out of college, and I can assure you it would interest you along many lines. What impressed me especially in the article this time was its statement that the best teaching must appeal to the will of the pupil, and not merely to his intellect. Such a thought connects up with what has just been said. The highest work of the teacher is to arouse the boy to a habit of initiative, to render him active rather than passive. He must be stimulated to think for himself. His performances in the recitations and tests must be directed so that he will not merely or mainly give back what he has received, but add as well no small quantity of his own thought. If he cannot act solely by his own volition as the result of a self-determining process, he should at least react and not merely reverberate.

A number of years since, Amherst was fortunate in having on its staff for a half year a distinguished English scholar and writer, Professor Woodberry. After some weeks with a class in English literature, he gave his first test. I recall very vividly his despair when he read the papers. "Why," he said, "the men are merely firing back at me my own words." It was a process of regurgitation. His lament was that he had failed, so he thought, to make the men think independently. Our own subject offers, of course, a peculiar advantage in its limitless array of problems, each of which, if they are real problems and not mere examples, offers a fresh field for thought. Yet even here, how disheartening it is to see the attitude of the average pupil toward a problem that is not patterned after a predecessor.

This use of a teacher's opportunity to stimulate thought, rather than to impart information or ready-made thinking, is especially helpful in arousing interest in the subject. The matter of interest is of transcendent importance, as we all know. Now in many departments of school work interest is involved in the subject-matter. This is true, for example, in the case of the natural sciences, and in literature and history; but not so with the average boy or girl in pure mathematics. Our subject is so abstract, so essentially subjective, that its appeal is far from seductive. Any possible interest it may have for the pupil lies, not in its radicals and complex fractions, its determinants and goniometric formulas, but in the pleasure that every normal mind takes in its own exercise. As a boy once put it in my hearing, "It's fun to feel the wheels going 'round.'" This is above all true when he sets them in motion himself.

I am reminded of an experience that I had many years ago with

a member of one of my freshman mathematics sections. He was apparently a very dull fellow indeed, and his intellectual efforts were still further handicapped by his membership in the football team. Failure succeeded failure, and the case seemed hopeless, until I discovered, by an accident, that he was rarely successful at drawing. We were in solid geometry at the time, among parallelepipeds and pyramids. The famous "Devil's Coffin" figure loomed ahead, a terror not only to my students but to me as well—to this day I cannot draw the figure decently. At the end of the exercise of the day before the monster was to be upon us, I called the student of whom I have spoken to my desk, and told him that I should consider it a favor if he would put the drawing on the board in his best style. The next day we saw arise before our eyes a real work of art, the completion of which was greeted by a round of applause from the men, and by a warm word of appreciation from the desk. The event proved a turning point in the fellow's career. Finding that he could do one thing well, that he was indeed first with no second in one branch of class room activity, it occurred to him that other branches might be worth striving after. For the first time his interest was aroused, and the day of failure was behind him. He was never brilliant, but ever afterward (he elected all the mathematics which the college offered) he did his work substantially and well.

I have instanced this experience in order to emphasize the far-reaching importance of getting a boy interested. But let me digress a moment in order to draw a further inference from the story, one right at hand. It showed me the deep significance of personal contact with the pupil, the importance of individualizing the members of a class. The administration of a school or college should always make this possible, and the last economy should be that which overloads teachers with hours or with pupils.

I said at the outset that a teacher should have a broad, deep preparation, involving years of study and thought; that he should have an immediate preparation for the work of the day. To this may be added that, so far as possible, he should prepare himself for the pupil, guided in this work of preparation by his knowledge of idiosyncrasies, and helped to success by an abundant sympathy with boy nature. The cases are rare when this nature cannot be reached in some way; where there does not lie some avenue of approach to a source of interest in the pupil's mind. For many years now I have been closely identified with college administrative work, and have come into daily contact with many stu-

dents. My experience, though not, of course, without its shadowy side, has left me a confirmed optimist. The average boy intends to do the right thing, even in the performance of his daily scholastic tasks, and the average boy will respond to a frank, self-forgetful, enthusiastic appeal to his better side.

The individual interest of the student, and thus the chief business of the class room hour may be further promoted in the following more formal way. In proving a theorem, or giving the solution of a problem, the student should state his plan of campaign, carrying it clear to the end. Such a process should be like a lawyer's opening. Listen to the state's attorney: "We shall show, gentlemen of the jury, that A was foully murdered. We shall show, from physical signs attested by experts of the highest repute, that death must have occurred between the hours of 4:30 and 5:30 in the afternoon of a given day. We shall show, on the testimony of unimpeachable witnesses, first, that B was seen to enter and to leave the house within this time, and, on the evidence of the same witnesses, that no other persons within the same time either came or went. Finally, we shall show that in B's mind there existed hatred of long standing toward A, offering a sufficient motive to account for a deed of violence; and then we shall ask you, fair-minded gentlemen, in the light of such evidence, whether any other solution of the problem can possibly be found than that which involves the guilt of B."

Now let us take a mathematical case. "We wish to show that if the $f(x)$ represents a continuous function of x ; and if, further, a and b are two numbers such that the $f(a)$ is plus, and the $f(b)$ is minus: then between a and b there lies some other number, c , such that the $f(c)$ equals zero. We shall show first that by halving the interval from a to b , and then halving the subintervals corresponding always to values of the functions with opposite signs, we obtain two series of values of the given function, each of which approaches a limit. We shall then show that these limits are identical. From the continuity of the function it will follow that this common limit will actually be attained. We shall show finally that this limit cannot be negative and that it cannot be positive. We shall then ask the fair-minded gentlemen before us whether any other solution is possible than that this will be zero."

Corresponding to the "physical signs" of the lawyer's plea, we have mathematical facts; the expert is, in the last analysis, the ordinary mind, and our witnesses are previous propositions, quite

as unimpeachable in their evidence as A's neighbors across the way. The student will thus be more vividly impressed with the fact, rarely emphasized in our teaching, and generally unrecognized by the average boy, that mathematical reasoning is merely the reasoning of everyday life put into symbols.

What we owe our pupils first of all is not so much a knowledge of facts as discipline in method. "To achieve the mastery of one's mental processes is," to quote from President Garfield's inaugural, "the chief end of an education," and the day is far distant when mathematics can be spared from the list of those subjects which make directly for this mastery. Clear, clean-cut thought, the power of sustained reasoning, is an essential for any man, whatever his calling, and this our science if properly taught can peculiarly foster.

Here let me suggest something that is absolutely fundamental, if a teacher hopes to arouse interest. He must have a boundless enthusiasm for his subject and his work. Interest is contagious. Years ago a young college graduate came back to teach physics in his alma mater. He was fresh from three years of study in Germany, mainly under Helmholtz. Familiar with the subject in all its details, and having worked on its frontier, knowing its latest developments, he overestimated the capacity of his pupils—a common failure with young, thoroughly-trained scholars, and a serious fault, which in later years he corrected. But as one of his old pupils told me, at the very beginning the men hung on his words, strained to comprehend him, caught his enthusiasm, and left his lecture room with an ever-deepening interest in laws of physics.

A teacher must be unflagging to keep this enthusiasm on his own part alive. Not a little of his success will depend on his physical condition. There are many demands upon his time in the details of his daily work, but all this should be subordinated to freshness and elasticity of mind and body in the class room. And here let me say a word on the question of the correction of the written work. Such work is, on the part of the pupil, of high value and a certain amount of insistence upon it goes with successful teaching. But if at any time the burden of it is endangering the physical vigor which the hour of recitation has the right to demand, such work should be cut in two or stopped. I should emphasize this again and again, if I were in authority as the head of an institution.

A second and further help in developing interest on the part

of a teacher is an appreciation of the significance of his subject. Mathematics is the patriarch of sciences. It existed as a science when Abraham went down into Egypt, as we know from records recently discovered. Physics can hardly be said to date back farther than Plato; chemistry is a creature of the last two centuries, and biology was born less than a hundred years ago. Moreover, the history of mathematics has been distinguished by a steady advance. Other sciences have grown largely by a process of self-destruction: phoenix-like they have arisen from their own ashes. The corpuscular theory of light, championed by Newton and armed with all his prestige, had to give way before the undulatory theory of Young, and this in turn may have to yield before the modern theory of electrons. The old separate-creation theory in biology has been superseded by evolution. But the movement in mathematics has been like the flow of a great river, moving slowly at times, but ever steadily onward. There have been epochs in the history of science when by the discovery of some new method, some great tool, mathematicians have been frightened as to the foundations of some of their great theorems. But investigation in the light of all that is new, while it may have required a restatement, has left these theorems substantially undisturbed. But not merely in its history, but in the wonderful unity of its substance is our science unique and great. It was Kronecker, if I remember aright, who defined mathematics as the science of positive integers; he might well have said, signless integers. In a way, these are the only absolutely necessary data of mathematics. Taking them as given the mathematician begins his creation of artificial number; he inserts his fractions, extends his system by the creation of negatives, gives it continuity by the construction of irrationals; and then by number-pairs and number-triples creates his multiple algebras. But not satisfied with a unity that is contained in the realm of number, he reaches out into space, and establishes a one to one correspondence between the points of a line and the elements of his enlarged number system. In this way he harnesses geometry to analysis, and establishes an absolute unity in the whole realm of mathematical truth. As we go on with our teaching, we should have an ever-livelier sense of the dignity and essential greatness of the subject with which we are dealing.

Now at the close, some of you will pertinently inquire: Do you yourself follow out the suggestions which you have made? Frankly, no. It is notorious that pulpit orators do not practice

what they preach, or doctors take their own physic. Kuno Fischer used to narrate with great zest how Schopenhauer, the arch pessimist, who proved to his own satisfaction that this was about the worst world that could be conceived of—how this same Schopenhauer was an epicure of epicures, and lived a life of singular luxury. No, I don't do it; but I wish I did. I hold the ideals before me and strive toward them. Every now and then a teacher should examine himself, and see whither he is drifting; and then he should consult his charts. This means that he should have a chart, which is in fact a bundle of ideals, something at which to aim, a norm by which to regulate his life. What I have given you is my own chart. Each one of you has probably one of his own, one which shows rocks that do not appear on mine, and indicates favorable currents which I have not noted. But if I have succeeded in indicating any rocks or any currents, the knowledge of which will make your course more sure, I shall be satisfied.

TEXT-BOOK ENGLISH.

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A contributor to SCIENCE recently called attention to the philosophy in modern high school chemistries. Mine is the much less ambitious undertaking of showing some of their weak points. It has been my good fortune, or misfortune, to read a number of text-books and they left a bad taste in my mind. One pardons grammatical slips, minor misstatements, and errors of fact in papers and magazines as they must be hurriedly written, put into type and printed, and yet glaring errors are very rare there; but in text-books where no incorrect form should be seen; which are supposed to be written and rewritten; where the publisher takes his own time—and much of yours—to get them into type and off the press, they are as thick as the leaves in Vallombrosa. Some of them are due to careless proof-reading; some are errors which show distressing lack of training in the use of ordinary good English; while statements of fact—I should say *misstatements*—are too painfully evident. Surely teachers and those who prepare books for them should not be sinners in either respect, yet it is of these that I am to speak.