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IV. *The Comparison of Electric Fields by means of an Oscillating Electric Needle.* By DAVID OWEN, B.A.(Camb.), B.Sc.(Lond.), Lecturer in Physics, Birkbeck College, London*.

EXPERIMENTS on fields of force due to electric charges suffer as compared with the corresponding measurements in magnetic fields by reason of the impracticability of obtaining an "electric needle" corresponding to the oscillating magnetic needle. But while it is true that a permanently electrified needle cannot be obtained, it is possible to have one that has equal charges of opposite sign induced by the field. This paper contains an account of experiments on the use of such a needle for the measurement of electric fields, steady or alternating, and for the experimental illustration of some of the laws of electrostatics.

Theory.

Consider a cylindrical needle supported at its centre by a fibre whose torsional control is negligible, placed with its axis horizontal in a uniform horizontal field of strength F . The needle will move so as to set itself parallel to the field. If disturbed from that position a restoring couple will come into play. This couple will be proportional, both to the induced charge, *i. e.* to the "pole-strength," and to the field. As the former is proportional to the latter, it follows that the couple will vary as the square of the field-strength. It will diminish as the angle between the axis of the needle and the direction of the field increases, and will clearly be zero when the two are at right angles. In the case of an ellipsoid the couple is proportional to the sine of twice the angle θ between the long axis and the field. Assuming that law to hold for a cylindrical needle, we may write the couple equal to $\alpha F^2 \sin 2\theta$, where α is some constant. If θ is small we have

$$\frac{\text{Couple}}{\text{Angular displacement}} = \frac{\alpha F^2 \sin 2\theta}{\theta} = 2\alpha F^2 \text{ (constant),}$$

* Read June 30, 1905.

in which case the vibrations of the needle will be isochronous, the time being given by

$$T = 2\pi\sqrt{\frac{I}{2aF^2}} \dots \dots \dots (I)$$

where I = moment of inertia of the needle.

Writing $T = \frac{1}{N}$, we may denote the frequency of vibration by

$$N = bF, \dots \dots \dots (II)$$

where b is a constant depending on the form, size, and mass of the needle. Thus it appears that the strengths of uniform fields are directly in proportion to the frequencies of such a needle oscillating in them.

Of course if the controlling couple due to the suspending fibre is sufficient to take into account, this may be done by observing the frequency N_0 when the electric field is nil. Denoting the frequency actually observed in field by N' , we have

$$N = \sqrt{N'^2 - N_0^2} = bF. \dots \dots \dots (II)'$$

This law was tested by establishing an electric field between a pair of parallel circular plates, each 12 cms. in diameter, kept at a constant distance apart. The difference of potential between them was obtained by means of a Kelvin voltaic pile, whereby any voltage from 400 volts downwards could be applied. The volts were measured by a Kelvin multicellular electrostatic voltmeter reading to 400 volts. The vibrations were observed by the help of a plane mirror placed below the needle, as shown in fig. 1, which represents the apparatus used. The image of the needle in the mirror was viewed through a telescope, in the focal plane of which an image of the needle appears vertical in the position of rest. The vertical cross-wire is set to bisect it.

The time of a counted number of oscillations was obtained by means of a stop-watch reading to fifths of seconds, sufficient passages being allowed to admit of the whole period of time measured being from one to two minutes.

The apparatus was enclosed in a balance-case in order to avoid draughts. Observations were restricted within an

amplitude of oscillation of about 5° , in which case the error due to finite amplitude is negligible.

Fig. 1.

The needle used was of copper wire, 1.4 cms. long and 2 mms. in diameter, suspended by a quartz fibre about one-tenth of a mm. in diameter and 8 centimetres long. The plates were at a distance of 3 cms. apart. The results gave as expected a straight-line law connecting V and N .

Disturbance of the Field due to the presence of the Needle.

The effect of the presence of such a needle in an electric field is in every case to produce a concentration of the field near the needle. Further, if the charged conductor forming one of the boundaries of the field is sufficiently near the needle, the distribution of the charge on that boundary will be altered. This of course prevents the application of the electric needle for the accurate comparison of the electric forces *near* the surfaces of conductors of different forms and curvatures. It was considered desirable before proceeding

further to investigate the nature of the disturbance caused, and to determine its bearing on the applicability of the needle for exact quantitative measurements of field.

A case of importance is that of the field between two parallel metal plates. The question may be put thus: if an electric needle be placed in an infinite uniform field, at what distance does the disturbing influence of the needle cease to be appreciable? I know of no mathematical treatment for the case of a short cylindrical needle such as was used above. The following experiment was therefore performed. The parallel plates already described were used, and the arrangement of apparatus was as in fig. 1. The plates could be moved perpendicular to themselves so as to alter their distance d apart. A constant potential-difference of 400 volts was maintained between them. The needle used was of aluminium, of the following dimensions:—

$$\begin{aligned} \text{length} &= 1.526 \text{ cms.}, \\ \text{diameter} &= .101 \text{ ,,} \\ \text{and mass} &= .0329 \text{ grams.} \end{aligned}$$

The distance d was varied between the limits of 6.04 cms. and 1.92 cms., and values of N for these and several intermediate values of d were determined. The needle, suspended by a quartz fibre, was set centrally between the plates in each case. The results may be tabulated as follows:—

$$T_0 = 18.80 \text{ secs.}$$

No. of exp.	d in cms.	T in secs.	N' per sec.	$\frac{N}{\sqrt{N'^2 - N_0^2}}$	$N \times d.$
1	6.04	7.20	.1392	.1282	.775
2	5.33	6.40	.1562	.1468	.784
3	4.82	5.82	.172	.163	.785
4	4.40	5.34	.1866	.178	.783
5	4.03	5.00	.200	.193	.778
6	3.00	3.69	.271	.2656	.797
7	2.53	3.04	.329	.325	.822
8	2.325	2.72	.368	.364	.844
9	2.07	2.32	.433	.430	.885
10	1.92	2.04	.490	.487	.916

These numbers are represented graphically in fig. 2, where products of frequency $\times$ distance apart of plates ($N \times d$) are

plotted against distance apart of plates (d). From this curve it may be seen that beyond $d=4.5$ cms., or say

Fig. 2.

Fig. 3.

exceeding three times the length of the needle, the disturbing effect of the needle is inappreciable, and the product $N \times d$

remains constant. This may be seen further by the aid of a diagram (fig. 3) representing the lines of force and the equipotentials near the needle when the axis of the latter is along the direction of the undisturbed field.

The lines of force are shown as full lines, the equipotentials as broken lines. The equipotentials near the needle are much curved, but the effect diminishes rapidly on going away from the needle, so that the equipotentials PP and $P'P'$ are practically straight. Beyond these the equipotential surfaces are a series of parallel planes, and the field is uniform and of the same value as if the needle were not present.

The result suggests the possibility of determining by the use of an electric needle the specific inductive capacity K of a dielectric. For if a pair of parallel plates kept at fixed potential-difference be used to establish a uniform field, their distance apart well exceeding the limiting distance at which the disturbing effect of the needle is appreciable; and if between one of the plates and the critical plane a parallel slab of the dielectric be introduced, the effect is to increase the field in the air space by an amount involving K in a simple and well-known way. Of this nothing further will be said in this paper, as careful experiments are being undertaken dealing with the determination of specific inductive capacity by such a method.

In regard to the disturbance introduced in other cases, it is important to notice that for a given arrangement of conductors and given position of the needle, variations of field due to alteration of potential will be measured by the frequency of the needle, the ratio of frequencies being unaffected by the distortion of the field. For example, in the case of the field between a pair of parallel plates at a distance apart only slightly exceeding the length of the needle; though the frequency of the needle is greater, as shown above, than in an infinite field of the same strength, yet it increases exactly in proportion to the difference of potential between the bounding plates.

The choice of Needles.

It is desirable that the needle should be as small as possible, to reduce the disturbing effect; and sensitive, for the measurement of as small fields as possible.

To investigate this latter condition six cylindrical needles were made of aluminium, with dimensions as follows:—

Three needles of diam. of 1 mm.; lengths $1\frac{1}{2}$ cms., 1 cm., .5 cm.
 „ „ „ .5 mm.; „ $1\frac{1}{2}$ „ 1 „ .5 „

A similar set was afterwards made in brass.

These needles were drilled centrally at right-angles to their length with a hole of about .3 mm. in diameter, into which was fitted a suspending wire of the same diameter, and length varying from 5 to 10 mms. To this wire was cemented a quartz fibre about $\frac{1}{15}$ mm. in diameter, and about 8 cms. long, the upper end of the fibre being cemented to a supporting piece attached to one of the plates. These needles were supported in succession between the plates, which were kept at the same distance apart and at the same difference of potential throughout the experiment. The time of vibration of each was thus determined:—The dimensions and mass of the needles being known, the moments of inertia could be calculated, and by equation (I) the couple on each when in a specified field could be determined.

No high degree of accuracy was to be expected where the moment of inertia of such small needles was involved. The results given below, however, show sufficiently the effect of alteration of length and of diameter. The frequency came out less for the brass than for the aluminium needles, just as calculated from their relative density. The values of the frequency in column 4 are for needles of aluminium.

Dependence of Frequency on Dimensions of Needle.

Electric field = 100 volts per cm.

Length (cms.).	Diameter (cms.)	$1000 \times \text{Couple}$ (dyne-cms.)	Frequency (per sec.) N.	Relative couples.	Relative frequencies.
1.526	.101	9.82	.197	100	100
1.103	.1025	4.42	.233	45	118
.510	.1025	.643	.275	6.54	140
1.500	.050	.693	.337	70.5	171
1.000	.050	.253	.371	25.7	178

These numbers show that although the couple on the needle increases rapidly with length and slowly with diameter, yet the effect on frequency is more than counterbalanced by the increase in the moment of inertia. Thus whilst, as the first three rows of numbers above show, the length of needle decreased in the proportion of about 3·1, the couple decreased in the proportion of about 15 : 1, whilst the frequency went up in the proportion of only 1 : 1·4.

Hence, while as the needle gets smaller the disturbing effect on the field rapidly diminishes, its frequency of vibration increases, though slowly. This is a favourable circumstance to be borne in mind in experiments such as that suggested above for the determination of specific inductive capacity.

Where the disturbance is not of importance, and it is desired to show the vibrations, either directly or by projection in a lantern, the needle may with advantage be large. Aluminium needles of $1\frac{1}{2}$ cms. length and 1 mm. diameter have been found very satisfactory; brass needles should be of smaller diameter. For strong fields the needles may be still heavier, and the suspension may be of silk threads consisting of several single-fibres. There is thus little danger of breakage of the suspension, where, as for purposes of lecture demonstration, the needle has to be moved about and may be more or less roughly used.

Applications.

The electric needle may be regarded as a measurer of *electric force*, and be applied to test the field-strength due to any arrangement of conductors, effect of alteration of their relative position, or of the potential, &c. In some cases exact results will not be obtained owing to the disturbance introduced by the needle itself. In others the disturbing effect of the needle does not enter. Of the first class the following example may be cited:—

Comparison of the electric field at different points on the surface of any conductor.

The case of a pear-shaped conductor (fig. 4) was examined.

Four positions on it were chosen, namely, at the pointed end

and blunt end, at the equator, and opposite the middle of the flat side (where the curvature is single). The needle

Fig. 4.

used was $1\frac{1}{2}$ cms. long. Its nearer end was set in each case at a distance from the conductor equal to half the length of needle. The results were as follows:—

Position.	Time of oscillation T (secs.).	Relative frequency.
Pointed end	2.55	1
Blunt end	3.20	.8
Equator.....	3.25	.78
Flat side	3.85	.66

For a couple of spherical conductors at a distance from other conductors the following numbers were obtained:—

Diameter of sphere.	T.	Relative frequencies.	Electric force (theoretical).
12 cms.	2.40 secs.	1	1
3 „	1.23 „	1.96	4

These comparative numbers indicate the increase of electric force F with curvature. By means of a proof-plane the increase of surface-density σ of charge with curvature may be shown. In both cases the numbers suffer in accuracy owing to the disturbance introduced by test needle and proof-plane respectively.

Such experiments as the above are best performed by attaching the conductor by a fine wire to the inner coating of a charged Leyden jar. The potential then changes only very slowly with time.

It may be interesting to add that the potential of the pear-shaped conductor, as determined by an electric needle, was 6200 volts in the case to which the numbers above refer.

In the following two cases the disturbing effect of the needle is of no significance :—

(1) Wherever the variation of electric force with alteration of potential is to be found, no motion of the bounding conductors or of position of needle taking place. The electric force as measured by the frequency of the needle is proportional to the difference of the potential.

(2) To prove that inside a charged conductor (not enclosing an insulated charged conductor) the electric force is zero. Though this theorem is one of fundamental importance, it is not easy to demonstrate it experimentally. In the electric needle we have a means of doing this of a simplicity corresponding to that which the proof-plane affords for showing that the charge inside the conductor is zero.

As the needle is lowered into the conductor, *e.g.* a metal can or a cylinder of wire netting, the frequency of the needle falls until when well inside its value is the same as when the conductor is discharged.

On the Shielding Effect of Dielectrics.

Having mounted a needle within a glass specimen-tube in order to do away with the effects of air-currents, it was observed that even when in the strongest electric fields the needle was quite unaffected. The tube acted as if it were a conductor. As this effect might be due simply to the unclean condition of the surfaces, the experiment was repeated with a carefully cleansed glass lamp-chimney, with the same result. It appears then, that owing to the conductivity of the glass, slight as it is, a closed glass vessel screens electric force from its interior, just as if it were a conductor.

The same result was found for a sheet of mica of thickness $\frac{1}{12}$ mm. bent into a cylinder. A mica cylinder in ordinary air screens the interior completely (when placed in a steady field, as in fig. 5). If heated over a hot plate and quickly

transferred to the field so as to surround a needle swinging in it, there is seen to be electric force at the needle ; but it quickly dies down, and in 20 or 30 seconds the needle once more vibrates in its own natural period, depending on the torsional couple of the quartz suspension.

This conductivity of the mica appears to be a surface effect, probably due to the deposition of moisture from the atmosphere. For if the mica sheet be immersed for some time in melted paraffin-wax at a temperature close on 200°C ., then taken out and allowed to cool, the sheet (made into a cylinder as before) is found to be capable of transmitting the field for some time. But even in this case the field within the cylinder falls off with time. Thus in one case a needle within a paraffined mica cylinder gave 12 vibrations in 10 seconds at first, but four minutes later only 8 vibrations were counted in the same time, the fall of frequency being according to the exponential law. Electrical separation has slowly taken place in the mica. If now the plates between which the field was produced be discharged, the cylinder should be left charged, and an electric field should exist in the interior. This is found to be the case. The field dies out with time, as in the experiments in the constant field. Another way of showing this effect of minute conductivity is to introduce the cylinder into a steady field, leave it there for some time, and then turn it about its axis through 180° . The field inside the cylinder is seen to be for the moment greater than that of the original field, for on the latter is superposed the field due to the separated charges on the walls of the cylinder. With cylinders of thicker sheets of mica similarly treated with paraffin-wax, the rate of falling off of the interior field is found to be greater. These experiments indicate an appreciable though exceedingly small conductivity in the mica itself.

Experiments on the shielding effect of ordinary white paper were made. Cylinders of paper screen the interior completely from the surrounding field. But on heating for a short time over a hot plate the shielding effect vanishes ; in about one minute, however, the frequency of the needle within is found to have fallen considerably, and a minute later is the same practically as in zero field. Dry paper then acts as a perfect insulator. But by the absorption of

moisture when exposed to the open air it quickly gains appreciable conductivity.

With paraffined paper the shielding power was nil, and remained nil for days in the open air of the room. Of all the materials tried this alone permitted for any length of time the undiminished transmission of electric force. Thus by applying a thin coating of paraffin wax to a sheet of dielectric, and then observing its shielding effect on an electric needle, we have a convenient means of testing the conductivity of the dielectric, which removes all doubts as to surface-action. This effect of the high insulating, non-hygroscopic, quality of paraffin-wax is made use of in the experiments on mica above described.

Experiments with alternating fields.—As with steady fields glass and mica proved sufficiently conducting to screen off electric effects, experiments were made with these materials, using alternating electric fields. The alternating voltage was obtained by the use of the Musical Arc. Mr. Duddell, I may say here, very kindly advised me as to the conditions for obtaining high voltages by this means. The connexions were as shown in fig. 5.

Fig. 5.

The alternating voltage on the self-induction *P* in the shunt circuit to the arc was found to be about 200 volts. This was transformed up by means of a second coil *S*, to the terminals of which the parallel plates were connected. In this way any voltage up to 1000 volts could be maintained between the plates. The frequency of alternation of this voltage was found, by means of a sonometer and standard fork, to be 1700 per second. The needle was allowed to

vibrate in the field with

- (a) glass cylinder surrounding it ;
- (b) cylinder taken away.

The time of vibration was found to be the same in each case, namely 1.83 secs. The glass cylinder had a diameter of 5 cms., the thickness of wall being 1.4 mm. With such dimensions the weakening of the field inside the cylinder due purely to the specific inductive capacity effect should only be small. With a cylinder of thicker glass, but diameter only slightly exceeding the length of the needle (1 cm.), the frequency of vibration was distinctly less in case (a) than in case (b). With mica cylinders the same result was obtained.

These experiments emphasise the necessity of using alternating fields in all cases where from the measurement of the force experienced by a mass of a dielectric its specific inductive capacity is to be determined. In steady fields, unless the specific conductivity of the material be quite nil, the force will ultimately be the same as for a conductor. This consideration will account for the high values of k obtained in some of the experiments of Boltzmann (see Gray's *Abs. Measmts.* vol. i. p. 465).

Use of Electric Needle to measure Volts.

A pair of insulated parallel metal plates with a needle suspended centrally between them forms a simple means of measuring high voltages. For we have

$$\text{volts across plates} = \text{frequency of needle} \times \text{a constant.}$$

The constant may be determined at a low voltage by any voltmeter available. The highest voltage measurable is limited by the difficulty of counting the frequency of the needle if this exceeds 4 or 5 per second. But the range may be extended by opening out the plates ; for as already shown the frequency of the needle for constant difference of potential is inversely as the distance apart of the plates, provided that the minimum distance used exceeds three times the length of the needle.

The constant k is most simply defined as the volts across the plate required to produce a frequency of oscillation of 1 per sec. for a distance apart of the plates equal to 1 cm.

Then if

d = distance apart of plates,

N = frequency,

V = voltage between plates,

we have

$$V = kd \times N. \quad \dots \dots \dots (III)$$

An instance of this application has been given already. Another may be added :—

The difference of potential yielded by a Wimshurst machine was measured by charging a Leyden jar, the outer coating being connected to one pole, the inner to the other pole, the poles being kept well apart. The jar was connected with the pair of parallel plates, which were at a distance of 10·5 cms. apart. The frequency of the needle in the field between them was 2·04 per sec. The needle used had a constant (reduced to $d=1$ cm.) of 1300 volts. Applying formula III, we have

$$\begin{aligned} V &= 1300 \times 10\cdot5 \times 2\cdot04 \\ &= 27800 \text{ volts.} \end{aligned}$$

The machine, it may be added, was working badly, giving a spark between balls of 2 cms. and 5 cms. diameter respectively, Leyden jars being in, of something less than 1 cm. length.

These experiments were made in the laboratories of the Birkbeck College. I would record my thanks to the Principal for facilities for carrying them out.

V. *Note on Constant-Deviation Prisms.*

By T. H. BLAKESLEY.*

It appears that any prism of three faces can be made to give a spectrum in which the light, that occupies the centre of the field of view of the telescope at any moment, has undergone passage through the two refracting surfaces of the prism in such a way that its original angle of incidence is equal to its final angle of emergence. This condition, which in the ordinary employment of the prism is associated with minimum deviation, must be described as isogonal passage, the property which has the minimum value being not the

* Read November 10, 1905.