The Efficiency of Generation of High-frequency Oscillations by Means of an Induction Coil and Ordinary Sparkgap. II. An Oscillographic Study of Some Induction Coil Phenomena

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XLIV. *I. The Efficiency of Generation of High-frequency Oscillations by Means of an Induction Coil and Ordinary Spark-gap. II. An Oscillographic Study of Some Induction Coil Phenomena. By Prof. G. W. O. HOWE and Mr. J. D. PEATTIE.*

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I. THE EFFICIENCY OF GENERATION OF HIGH-FREQUENCY OSCILLATIONS BY MEANS OF AN INDUCTION COIL AND ORDINARY SPARK-GAP.

The primary object of the experiments described in this communication was to determine the efficiency with which high-frequency currents are generated in small radio-telegraph stations, using an ordinary spark-gap and a coil supplied with continuous current through a mercury interrupter.

In order to explain the peculiarities observed it proved necessary to investigate the action of the spark coil itself, and an account of this investigation forms the second part of the Paper.

FIG. 1.

The apparatus employed and the connections are shown diagrammatically in Fig. 1. SC is a 10 in. Marconi spark coil supplied from accumulators through the motor-driven mercury break M. This break works in coal-gas; it is shunted with a condenser, K_0 . MCA and HWA are two ammeters connected in series, the former, of the moving coil type, reading the mean value of the intermittent current, the latter, of the hot-wire type, the root-mean-square value. K_1 represents three glass-plate condensers immersed in oil, the combined capacity at high frequencies being 0.00485 mfd. The coil

L_1 is 10 in. square, and consists of four turns of copper tape. The spark-gap G consists of massive brass electrodes of large radius (about 2 in.), with micrometer adjustment. The natural frequency of the primary oscillatory circuit corresponded to a wave-length of 360 metres.

The coil L_2 consists of 15 turns, 9 in. square, of 3 mm. copper wire; K_2 is an oil condenser of 0.0018 mfd. The circuit K_2L_2 takes the place of the aerial, and a resistance, R , is introduced to take the place of the total damping due to radiation and C^2R losses. The resistance R consists of fine Eureka wire having the same resistance to high-frequency and continuous currents. The energy dissipated in it could be determined from the temperature rise of the oil in which it is immersed, or more simply from the reading of the thermal ammeter TA . This energy constitutes the high-frequency output of the plant, and the overall efficiency is obtained by dividing it by the continuous-current input to the induction coil. The coupling between L_1 and L_2 can be varied by sliding L_2 along a graduated bench. Loosely coupled with the circuit K_2L_2 is the wavemeter K_3L_3 , by means of which the frequency and the nature of the oscillations can be determined.

Measurement of Input to the Induction Coil.—The unidirectional intermittent current was supplied from a battery of accumulators at some distance. If E be the constant E.M.F. of the battery and R the resistance of the battery and leads, the average power supplied to the coil will be $(E \cdot C_{av.}) - (C_{R.M.S.}^2 R)$, and this can be calculated from E and R and the two ammeter readings.

Determination of the Coupling.—If the coefficient of mutual induction between the coils L_1 and L_2 be M , then if $k = \frac{M}{\sqrt{L_1 L_2}}$, k is called the coefficient of coupling. This was determined for various positions of the secondary coil in the following manner. If the two coils are connected in series in such a way that their inductive effects are additive, the combined inductance $L' = L_1 + L_2 + 2M$, whereas, if the connections of one coil are reversed, the combined inductance $L'' = L_1 + L_2 - 2M$. Now L_1 , L_2 , L' and L'' can be determined by putting a spark-gap and a known capacity in the circuit and determining the frequency with a wave-meter. Since the coefficient k is merely a numerical ratio, it is not necessary to know the value of the capacity employed, if it be unchanged throughout. If

the coil of the wave-meter be also unchanged, the inductance under test is proportional to the capacity of the wave-meter condenser when adjusted to resonance. The leads used to connect L_1 and L_2 may have a considerable inductance, which we may call L_a , but their mutual inductance with the coils L_1 and L_2 can be made practically negligible, so that the combined inductance of L_1 and L_a , or of L_2 and L_a , is unaffected by reversing the connections to the coil. We have then

$$\begin{aligned} L_1 + L_2 + L_a + 2M &= \alpha K'_1 \\ \text{and} \quad L_1 + L_2 + L_a - 2M &= \alpha K'_2, \end{aligned}$$

where α is a constant and K'_1 and K'_2 are the capacities of the wave-meter condenser for resonance.

$$\text{Hence,} \quad M = \frac{\alpha(K'_1 - K'_2)}{4},$$

$$\text{and} \quad L_1 + L_2 + L_a = \frac{\alpha(K'_1 + K'_2)}{2}.$$

With coil L_1 alone we have $L_1 = \alpha K'_3$, and with coil L_2 alone, $L_2 = \alpha K'_4$, and

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{(K'_1 - K'_2)}{4\sqrt{K'_3 K'_4}}.$$

Hence, all that is required to determine the coupling is an accurately calibrated variable condenser. Some of the results of the calibration are given in the following table:—

$$K'_3 = 1.87 : K'_4 = 4.98 : \sqrt{K'_3 K'_4} = 3.05.$$

Reading on bench.	Approx. distance between mean planes of coils.	K'_1 .	K'_2 .	$\frac{K'_1 + K'_2}{2}$	$\frac{K'_1 - K'_2}{4}$	k .
70	4 cm.	10.24	4.40	7.32	1.46	0.48
65	9 "	9.48	5.21	7.34	1.07	0.35
60	14 "	8.58	6.06	7.32	0.63	0.21
50	24 "	7.81	6.90	7.36	0.23	0.075
40	34 "	7.51	7.17	7.34	0.085	0.028
30	44 "	7.41	7.21	7.31	0.05	0.016

By plotting these values of k , the coupling can be found for any position of the secondary coil.

The constants of the primary and secondary oscillatory circuits were as follows: $K_1 = 0.00485$ and $K_2 = 0.00183$ microfarads, $L_1 = 7.5$ and $L_2 = 20$ microhenries. $\lambda = 360$ metres.

Results of Tests.—On plotting either the aerial current or the current taken by the coil, against the spark-length,

peculiar results were obtained. After the electrodes had been polished readings were taken for different lengths of gap between 0.5 mm. and 5.0 mm., first gradually increasing the length and then gradually decreasing it. As shown in Fig. 2, the two sets of readings do not agree. This is due to the fact that with freshly polished electrodes the spark takes the shortest path, whereas when the central part becomes burnt and blackened the spark prefers the longer path between either the unburnt part of the electrodes or the edge of the burnt portion. This can be seen on observing the gap, and is also confirmed by the horizontal displacement of the return curve in Fig. 2. It will be noticed that with a gap of 1 mm. the average current taken by the coil gradually decreases, whereas with a gap of 1 mm. it gradually increases. The uncertainty thus introduced was avoided by polishing the electrodes before each experiment and then taking readings at

FIG. 2.

regular intervals after switching on. By plotting the results and producing the curves backward, the probable initial values were obtained. Except at the first moment, the effective length of the gap is greater than the measured length.

The first series of tests were made with a resistance of 6.4 ohms in the dummy aerial circuit, corresponding to a logarithmic decrement $R/2 \sim L$ of 0.19 per period. The induction coil was supplied from a battery of six accumulators, and the mercury break gave 23.8 interruptions per second. In all the experiments the spark-gap was subjected to a moderate air-blast. Six values of the coupling were tried, each with about 10 different lengths of gap. The results are shown in Figs. 3 and 6, where the high-frequency output and the over-

FIG. 3.

FIG. 4.

FIG. 5.

FIG. 6.

all efficiency are plotted against the spark-length, each curve referring to a given value of the coupling. With the same aerial resistance the tests were repeated with the coil supplied

FIG. 7.

FIG. 8.

from 8 and 10 cells respectively; the results when using 8 cells are plotted in Figs. 4 and 7. Using 6 cells the tests were repeated with "aerial" resistances of 21.5 and 41.2 ohms respectively, corresponding to logarithmic decrements of 0.65 and 1.24 per period (*see* Figs. 5 and 8).

In addition to the high-frequency output and the efficiency one must determine the nature of the oscillations produced. For this purpose resonance curves were taken by means of a wave-meter loosely coupled with the aerial circuit. These are reproduced in Figs. 12, 13 and 14 for the three different aerial resistances. In Fig. 12, which corresponds more nearly to the conditions found in practice, it is seen that the coupling cannot be increased above about 8 per cent. without decreasing the power transmitted from the aerial circuit to a slightly damped tuned circuit such as the wave-meter. If it were simply a matter of getting power into the aerial Fig. 3 shows that the coupling should be about 20 per cent. With this tight coupling, however, beats occur in the aerial oscillations as shown by the double humps in the resonance curves in Fig. 12. In practice, the decrement of the aerial is often less than the value, 0.19 per period, which the dummy aerial circuit had when the resonance curves of Fig. 12 were taken. This would necessitate a still looser coupling than 8 per cent. if beats are to be avoided in the aerial oscillation.

The coupling is sometimes calculated from the distance apart of the two humps of the resonance curve. If ω_1 and ω_2 be the frequencies corresponding to these humps, then assuming undamped oscillations, it can be shown that $k = \frac{\omega_1^2 - \omega_2^2}{\omega_1^2 + \omega_2^2}$. The following table shows, however, that the coupling, as determined in this way, may be far from correct:—

Fig. 4.	$\frac{\omega_1^2 - \omega_2^2}{\omega_1^2 + \omega_2^2}$	Actual value of k .
Curve D	0.059	0.125
Curve E	0.194	0.214
Curve F	0.348	0.363

The discrepancies are mainly due to the fact that the formula used assumes undamped oscillations whereas the dummy aerial circuit is heavily damped. That this is the correct explanation can be seen by comparing the curves marked F in Figs. 12, 13 and 14. The actual coupling was the same in each case—viz., 0.363—but the calculated coupling, as determined

FIG. 9.

FIG. 10.

FIG. 11.

Using 10 in. Spark Coil, with Mercury Break. Length of Spark Gap 1.2 mm. with Air-Blast.

A Coupling=2.9 %	D Coupling=12.55 %
B " " 4.75 %	E " " 21.4 %
C " " 7.8 %	F " " 36.3 %

FIG. 12.

FIG. 13.

FIG. 14.

by the position of the humps, decreases as the damping is increased, as shown in the following table :—

Curve F.	Resistance in aerial circuit.	$\frac{\gamma_1^2 - \gamma_2^2}{\gamma_1^2 + \gamma_2^2}$	Actual k .
Fig. 4.	6.4 ohms	0.348	0.363
Fig. 5.	21.5 „	0.298	0.363
Fig. 6.	41.2 „	0.157	0.363

If the figures in the third column are plotted against those in the second column, and the curve is produced backwards to find what the calculated value of k would be when the damping was zero, a result is obtained agreeing exactly with the actual value of k . The calculation of the coupling from the resonance curves by means of the above formula is, therefore, capable of giving very erroneous results if the damping is considerable. The position of the humps may also be affected to some extent by the presence of the middle hump, which is seen in Fig. 12, curve E, and which is due to premature quenching of the spark.

The absence of any pronounced double hump in Fig. 14 is due to the increased resistance in the aerial circuit, the dissipation of energy being too rapid to allow of its storage in the secondary circuit and subsequent return to the primary circuit.

Power taken by the Induction Coil.—These curves (Fig. 9) are of the same shape whether the supply be taken from 6, 8 or 10 cells, but the maxima and minima occur for longer spark-lengths as the supply P.D. is increased. The causes of the maxima and minima and the general shape of the curves are discussed later in connection with the oscillograph records. For short spark-lengths the increased input is shown to be due partly to sparking at make as well as at break, while the general shape of the curves is dependent on the time that elapses between the last spark and the re-making of the primary circuit.

High-frequency Output.—The curves in Figs. 3 and 4 follow, much the same lines as those for the input to the coil in Fig. 9, but, of course, vary in magnitude with the coupling. They show the futility of seeking to increase the output by using closer coupling, quite apart from questions of tuning, as very little is gained by increasing the coupling beyond a certain

point. The curves also show the tendency for the output to increase abnormally with short sparks and a coupling of about 20 per cent. This is undoubtedly due to premature quenching of the spark. In Fig. 5 the secondary circuit has a much greater decrement than an ordinary radio-telegraph aerial; there is here no sign of quenching and the output is largely increased by tightening the coupling up to about 50 per cent.

The Overall Efficiency.—From Figs. 6, 7 and 8 it will be seen that the efficiency is very low at short spark-lengths, due probably to the arcing that occurs between the electrodes. The efficiency increases with the spark-length up to a certain point, and then gradually decreases; it varies very little, however, over a wide range. The effect of premature quenching is seen in Figs. 6 and 7 for a spark-length of 0.5 mm. and 21 per cent. coupling. The effect of varying the coupling and the decrement of the aerial can best be seen from Figs. 10 and 11, which are plotted for fixed spark-lengths of 0.5 mm. and 2.0 mm. respectively. If the coupling be loose and the secondary damping large, we shall get a feeble forced oscillation, whereas if the secondary damping be small, a fairly large oscillation may be built up in the secondary before the gap in the primary opens, and the energy so stored will be dissipated entirely in the secondary circuit. With tight coupling, however, matters are very different. If the secondary damping is small, the energy surges backwards and forwards between the two circuits, and is largely lost in the spark-gap. As the secondary resistance is increased a greater proportion of the energy is dissipated in the secondary circuit. This explains the crossing of the curves in Figs. 10 and 11.

With an aerial resistance of 6.4 ohms and a coupling of 8 per cent. the overall efficiency has a maximum value of about 26 per cent. This represents approximately the conditions of radio-telegraphy when using sending apparatus similar to that employed in these experiments.

The logarithmic decrement due to the radiation from an earthed vertical wire can be shown from theoretical considerations to be about 0.2.*

The total decrement due to radiation and energy dissipation in the aerial and earth will depend largely on the nature of the earth connection. It may be much greater than that due to radiation alone.

* See M. Abraham, "Theorie der Elektrizität," Vol. II., p. 307.

T-shaped antennæ and other types having a large capacity relatively to the height have a smaller radiation decrement, and their total decrement may be much smaller than the smallest value employed in these experiments—viz., 0.19.

II. OSCILLOGRAPHIC STUDY OF SOME INDUCTION COIL PHENOMENA.

In order to explain the peculiarities of the curves showing the input to the coil for various spark-lengths, a number of oscillograms were taken. The oscillograph strip, suitably shunted, was placed in one of the two positions shown in Fig. 1. The frequency and regularity of the sparks was determined by means of a revolving mirror from which the sparks were reflected and focussed on a falling photographic plate.*

FIG. 15.—0.5 MM. PRIMARY. SPARKING ON MAKE.

FIG. 16.— „ „ NOT SPARKING ON MAKE.

FIG. 17.— „ SECONDARY. SPARKING ON MAKE.

With a gap of 0.5 mm. (Figs. 15, 16 and 17) the coil usually sparks on “make,” causing a rapid rise of the primary current. The fact that the core is already magnetised by the secondary current which has not yet died away will also cause the primary current to rise very suddenly, and Fig. 16, in which there

* See Dr. Fleming, “Journal” of the Inst. of Elec. Eng., Vol. XLIV., p. 363.

is no spark at make, shows that this is the principal cause of the sudden rise of the primary current. It appears from Fig. 17 that the "make" causes three or four sparks, whereas a torrent of sparks is maintained during the whole duration of the "break." Immediately after the break there are signs of one or two distinct sparks, followed by a period of great regularity in the current oscillations, from which it would appear that the phenomenon is similar to the musical arc, the frequency of the sparks being controlled by that of the

FIG. 18.

oscillations due to the inductance of the coil and the capacity of the condenser across the mercury break. After a time this regularity ceases, and the sparks occur at increasing intervals until finally the primary circuit is again closed. In such oscillograms one is always rather sceptical—and rightly so—as to the instrument being properly damped. This was always checked by making and breaking the current through a non-inductive resistance; a typical result is shown in Fig. 18.

FIG. 19.—1 MM. PRIMARY.

FIG. 20.—1 MM. SECONDARY.

Oscillograms are given of primary and secondary currents as the spark-gap is increased in length from 0.5 mm. to 6.0 mm. and it is interesting to trace the gradual changes. It is only with the shortest gap that sparking occurs at make. In all cases the exact number of sparks occurring at break can be counted in both primary and secondary oscillograms. This was confirmed by spark photographs. Fig. 24 was taken with a 2 mm. gap, and should be compared with Figs. 22 and 23.

FIG. 21.—1·5 MM. PRIMARY.

FIG. 22.—2·0 MM. PRIMARY.

FIG. 23.—2·0 MM. SECONDARY.

FIG. 24.—SPARK PHOTOGRAPH WITH 2 MM. GAP.

With a gap of 1.2 mm. the number of sparks varies between 8 and 10 per break, with 2.0 mm. between 4 and 5, and with a gap of 2.5 mm. between 2 and 3. It will be noticed that the successive sparks in each train die away in brilliancy, the first one being always especially bright, also the intervals between successive sparks gradually increase.

In all the oscillograms it will be noticed that there are oscillations of three distinct frequencies, occurring at various times. At make, the exponential rise of the primary current has superposed upon it an oscillating current which sometimes (Figs. 22 and 26) reaches a very large value. This oscillation ($\sim=266$) is equally prominent in the secondary circuit. It is due to the condenser K_1 , which is connected across the

FIG. 25.—2.5 MM. PRIMARY.

FIG. 26.—3.5 MM. PRIMARY.

FIG. 27.—3.5 MM. SECONDARY.

secondary of the coil. The inductance, which must be taken into account when calculating the frequency of this oscillation, is not the full inductance of the secondary winding, but only its leakage inductance, as the primary coil is practically short-circuited through the mercury jet and the battery.

There are other oscillations occurring only in the secondary circuit and of a considerably lower frequency—viz., 86 $\sim$ per second. They occur only when the primary circuit is open

and, therefore, when the effective inductance of the secondary winding is not diminished by the latter being tightly coupled with the closed primary winding. It is interesting to note the immediate effect on the frequency of this oscillation of closing the primary circuit in Figs. 23, 27 and 28. It is not quite correct, however, to say that the primary circuit is open during the intervals of break, for the mercury interrupter has a condenser of about 0.6 mfd. across its terminals. This condenser is connected, through the battery, across the primary winding. The natural frequency of this circuit is so high,

place. The primary current was previously as shown in Fig. 29; it is now modified as shown in Fig. 30, where the presence of the low-frequency oscillation is quite evident. We have simply decreased the stiffness of the primary circuit.

More striking in this last oscillogram is the change in the high-frequency oscillation to which we have not yet referred. It occurs on break and in Fig. 30 is only just damped out when the make occurs. This is the natural oscillation of the primary winding with the condenser across the interrupter. By increasing the capacity of the condenser to 16 times its former value we have lowered the frequency to a quarter and have greatly increased the amplitude. In all the preceding oscillograms the frequency of this oscillation has been so high that one might have thought it due to under-damping of the

FIG. 31.

oscillograph. The damping was, however, almost critical. In calculating the frequency of this comparatively rapid oscillation (under normal conditions $\omega = 4,250$) it must be remembered that the secondary winding is practically short-circuited by the glass-plate condenser K_1 . The inductance involved is only that due to the leakage flux of the primary coil. The free period of an oscillatory circuit is approximately $2\pi\sqrt{KL}$, but, if the coil be coupled with another short-circuited coil, it can be shown that the free period is reduced to $2\pi\sqrt{KL}\sqrt{1-k^2}$, where k is the coefficient of coupling $\sqrt{\frac{M^2}{L_1L_2}}$. The periods of the various oscillations as determined from the

oscillograms agree closely with the values calculated by means of this formula from the measured inductances and capacities.

An interesting point in connection with the oscillograms is the gradual change which occurs in the oscillations of the primary current at make, as the length of the spark-gap is increased. As there is no spark at make (except at 0.5 mm.) this effect is rather unexpected. At 1 mm. the primary current sets off in a negative direction. At 1.5 mm. the start is positive, and the magnitude of the oscillation gradually increases with the length of the gap as far as 3.5 mm. The reason for these differences can be seen from a consideration of the secondary current curves. The amount of energy left in the secondary circuit when the spark quenches depends on the length of the gap. This energy oscillates in the secondary circuit with a frequency of 86 cycles per second, the primary circuit being open, except for the condenser across the interrupter. When the primary circuit is closed again, the frequency is changed to 266 cycles per second, and an oscillation is set up in the primary circuit with this frequency. This oscillation is superposed upon the exponential growth of the primary current. The direction of the initial current at make may be negative, as in Fig. 19; this depends on the time which has elapsed since the last spark and on the energy left in the coil when the last spark was quenched.

An appendix has been added to the Paper in which this is shown mathematically. Curves have been plotted from the calculated results to the same scale of current as applies to the oscillograms, and the results will be seen to agree very well. In Figs. 19 and 20 the make occurs at the moment when the secondary current is zero; in this case the primary oscillations are superposed on an exponential curve rising from the base line.

In Fig. 23 the secondary current at the moment of make is a maximum; as shown in Fig. 22, the primary current under these conditions consists of oscillations superposed upon an exponential curve, which starts from a point above the base line. In Figs. 28 and 29 the secondary current at the moment of make is approaching a maximum, but in the opposite direction, and the primary current curve is here displaced downwards. The result of this is that the final value reached by the primary current at the moment of break, and still more so the average value of the current, is much smaller in Fig. 29 than in Fig. 22. This explains the shape of the curves in Fig. 9, where the input to the coil is plotted for various spark-lengths; in

the oscillograph experiments six cells were used, and under these conditions the input is a maximum for a spark-length of 2 mm. The position of the maximum in Fig. 9 depends, therefore, on the time which elapses between the last spark and the subsequent make ; it will obviously vary with the number of cells employed, for with more cells sparking will continue longer, unless the spark-gap is lengthened. This explains the curves for 8 and 10 cells in Fig. 9 being displaced to the right.

It is obvious that the spark-length for which the input is a maximum will vary with the design of the mercury interrupter and with the speed at which it is run.

What has been said applies also to the high-frequency output curves, which follow generally the same shape as the input curves.

The effect of the oscillations in the primary current was evident in the relative values of the readings of the moving coil and hot-wire ammeters. With a gap of 0.5 mm. the ratio was 1.33, while with a gap of 3.3 mm. it had increased to 1.64.

Although in many cases the initial oscillation of the primary current exceeds in amplitude the final value of the current, it would be useless breaking the circuit at this moment, for the large primary current is almost neutralised by the secondary ampere-turns and the resultant magnetic flux is small.

APPENDIX.

It has been seen that the growth of the current in the primary circuit of the induction coil when the mercury interrupter closes the circuit depends on the magnitude and phase of the remanent oscillation in the secondary circuit of the coil at that moment.

Let L_1 and L_2 be the inductances of the primary and secondary windings of the coil and M their mutual inductance. Let E be the E.M.F. of the battery supplying the coil. The secondary circuit has an oscillatory current with an amplitude I , and the primary circuit is suddenly closed. Since we are mainly concerned with what happens at the moment of closing the switch we can assume that both R_1 and R_2 are zero. The equations for the two circuits may be written :—

$$L_2 \frac{d^2 q_2}{dt^2} + M \frac{d^2 q_1}{dt^2} + \frac{q_2}{K} = 0, \quad \dots \dots (1)$$

$$L_1 \frac{d^2 q_1}{dt^2} + M \frac{d^2 q_2}{dt^2} = E. \quad \dots \dots (2)$$

Eliminating q_1 we have

$$\frac{d^2 q_2}{dt^2} \left(1 - \frac{M^2}{L_1 L_2} \right) + \frac{q_2}{KL_2} + \frac{ME}{L_1 L_2} = 0.$$

Putting $\frac{M^2}{L_1 L_2} = k^2$, $\frac{1}{KL_2(1-k^2)} = \omega^2$ and $\frac{ME}{L_1 L_2(1-k^2)} = \alpha$,

we have
$$\frac{d^2 q_2}{dt^2} + \omega^2 q_2 + \alpha = 0$$

and
$$q_2 = A \sin(\omega t + \varphi) - \frac{\alpha}{\omega^2},$$

$$i_2 = \frac{dq_2}{dt} = A\omega \cos(\omega t + \varphi). \quad (3)$$

From (2)
$$\frac{d^2 q_1}{dt^2} = \frac{E - M \frac{d^2 q_2}{dt^2}}{L_1}$$

$$= \frac{E + MA\omega^2 \sin(\omega t + \varphi)}{L_1}.$$

$$\therefore i_1 = \frac{dq_1}{dt} = \frac{E}{L_1} t - \frac{\omega AM}{L_1} \cos(\omega t + \varphi) + B, \quad (4)$$

where A and B are constants to be determined from the initial conditions.

If, in the first place, the "make" be assumed to occur at the moment of maximum current I in the oscillating secondary circuit, then we have for $t=0$, $q_2=0$ and $i_2=I$.

$$\therefore A \sin \varphi = \frac{\alpha}{\omega^2} \quad \text{and} \quad A\omega \cos \varphi = I.$$

$$\therefore A = \frac{\sqrt{I^2 + \frac{\alpha^2}{\omega^2}}}{\omega} \quad \text{and} \quad \tan \varphi = \frac{\alpha}{\omega I}.$$

$$\therefore i_2 = \sqrt{I^2 + \frac{\alpha^2}{\omega^2}} \cos \left(\omega t + \tan^{-1} \frac{\alpha}{\omega I} \right). \quad (5)$$

Hence, the oscillatory current is increased in amplitude

whichever way the battery is switched in, but the change of phase depends on whether E is positive or negative.

$$i_1 = \frac{E}{L_1}t - \frac{M}{L_1}i_2 + B,$$

where B is a constant, such that when $t=0$, $i_1=0$.

$$\therefore i_1 = \frac{E}{L_1}t - \frac{M}{L_1}i_2 + \frac{M}{L_1}I. \quad (6)$$

Hence, the primary current consists of three components—the first, directly proportional to the time (neglecting resistance); the second, a sine wave; and the third, a constant current. The first term may be positive or negative, and on this depends the phase displacement of the second term.

If, in the second place, the primary circuit be closed when the secondary current is passing through zero, the constants in (3) and (4) are such that for $t=0$, $i_2=0$; $q_2=\frac{I}{\omega}$; $i_1=0$.

$$\therefore A = \frac{I}{\omega} + \frac{\alpha}{\omega^2}, \quad \varphi = \frac{\pi}{2} \text{ and } B=0.$$

$$\therefore i_2 = - \left(I + \frac{\alpha}{\omega} \right) \sin \omega t \quad (7)$$

and

$$i_1 = \frac{E}{L_1}t - \frac{M}{L_1}i_2. \quad (8)$$

In this case the oscillatory current is unchanged in phase, but is increased or decreased in amplitude, depending on the direction of E as compared with the direction of the initial oscillation induced in the primary circuit. The amplitude changes from I to $I + \frac{\alpha}{\omega}$, where α can be $+$ or $-$, and

$$\frac{\alpha}{\omega} = E \sqrt{\frac{K}{L_1}} \sqrt{\frac{k^2}{1-k^2}}.$$

Hence if the coefficient of coupling k be nearly unity, as it is in the case of the induction coil, the increase of amplitude of the secondary current and the oscillation set up in the primary circuit may be very considerable, as is shown by the oscillograms.

In Fig. 32 the four cases have been plotted. The values used in the calculation are as follows :—

$L_1=0.02$; $L_2=570$; $M=3.2$ henries; $K=0.005$ mfd.; $E=12$ volts; $I=0.025$ ampere; $k=0.946$.

Although the only difference in the assumptions in the four cases is in regard to the phase of the secondary current at the

FIG. 32.

moment of make, the differences in the nature of the oscillations produced are very great. In the actual case the oscillations are gradually damped out by the resistance and the term

$\frac{E}{L_1}t$ becomes an exponential term, rising finally to $\frac{E}{R_1}$.

ABSTRACT.

The apparatus used was similar to that employed in small radio-telegraph stations. A 10 in. induction coil, operated from cells through a mercury interrupter, supplied power to an oscillatory

circuit containing a spark-gap between spherical electrodes. Coupled to this circuit was another oscillatory circuit representing the aerial, and containing a variable resistance which constituted the high-frequency load. The input, output and efficiency were determined for various degrees of coupling, various aerial decrements, different lengths of spark-gap and with various primary voltages, the object being to determine the effect of these various factors on the working of a small radio-telegraph station.

A second part of the Paper dealt with some oscillographic records of the primary and secondary currents of the induction coil. These showed many points of interest, and explained several peculiarities in the working of the coil, especially the variation of input with spark length. This was shown to be due to the particular phase at the moment of "make" of the remnant current in the primary induced in it by the oscillations in the secondary produced by the last spark; this depended on the time which had elapsed since the last spark.