Therefore,

$$\begin{aligned}\text{arc LE} : \text{arc DE} &= \text{LF} : \text{AH}, \\ &= \text{sector LFE} : \text{sector DFE}.\end{aligned}$$

But,

$$\text{LF}^2 : \text{AH}^2 = \text{LF} : \text{X} \quad (\text{where X is defined by the proportion } \text{LF} : \text{AH} = \text{AH} : \text{X}),$$

and,

$$\text{LF}^2 : \text{AH}^2 = \text{sector LFE} : \text{sector AHB},$$

and,

$$\text{LF} : \text{AH} = \text{sector LFE} : \text{sector DFE}.$$

Therefore,

sector LFE : sector DFE = sector DFE : sector AHB, and on account of Lemma III,

$$\begin{aligned}\text{sector DFE} : \text{three-line DEK} &> \text{right angle} : \angle \text{DFE}, \text{ that is,} \\ &> \angle \text{LFE} : \angle \text{DFE}, \\ &> \text{sector LFE} : \text{sector DFE}.\end{aligned}$$

And

$$\text{sector LFE} : \text{sector DFE} = \text{sector DFE} : \text{sector AHB}.$$

Therefore,

$$\text{sector DFE} : \text{three-line DEK} > \text{sector DFE} : \text{sector AHB}.$$

Consequently,

$$\begin{aligned}\text{sector AHB} &> \text{three-line DEK}, \text{ and} \\ \text{semicircle ABC} &> \text{segment DEZ}.\end{aligned}$$

2. The proof for this part when DEZ is greater than a semicircle is exactly the same as the proof given above with the exception that Lemma IV is used instead of Lemma III. This proof will be left as an exercise for the reader.

DEFINITIONS IN GEOMETRY.

BY ROBERT R. GOFF,

Fall River, Mass.

A regular polyhedron has among other properties all its faces congruent polygons. Now a regular prism which is a polyhedron, is usually defined as a right prism whose base is a regular polygon, nothing being said about the lateral faces. This is inconsistent of course. A prism can have a regular base or regular faces, but in neither case is it necessarily a regular prism. The only regular prism possible is the cube.

The definition of a regular pyramid, which is a polyhedron, does not require the lateral faces to be congruent to the base. A pyramid can have a regular base or regular lateral faces but in

neither case is it necessarily a regular pyramid. The only regular pyramid possible is the tetrahedron.

The definition of circular cylinder makes no mention of the right section. As a circular stovepipe is one whose right section is a circle, it seems reasonable to expect a circular cylinder to be one whose right section is a circle. Also a circular cone should be one whose right section is a circle.

A segment of a sphere is that part of the sphere between two parallel planes. Then a segment of a circle might be that part of the circle between two parallel lines.

A median of a trapezoid is the straight line joining the middle points of the non-parallel sides. Then a median of a triangle might be the straight line joining the middle points of the non-parallel sides.

Now, I fully realize that it is unwise to bring in new terms unless a need for these terms is clearly shown. Nevertheless, I suggest the following definitions:

A so-called regular prism is a right prism with regular base.

A so-called regular pyramid is a right pyramid with regular base.

A so-called circular cylinder is a cylinder with circular base.

A so-called circular cone is a cone with circular base.

A circular cylinder is a cylinder whose right section is a circle.

A circular cone is a cone whose right section is a circle.

A segment of a circle is that part of the circle between two parallel lines.

The mid-line of a triangle or trapezoid is the straight line joining the middle points of two non-parallel sides.

Two lines that do not meet are not necessarily parallel; and a line and a circle are not necessarily tangent if they touch at one and only one point. The words "in the same plane" belong in the second definition just as much as in the first.

[At the suggestion of the author, this is published with the object of inviting criticism. In addition to the definitions noted, are there not others that need revision? During the years 1908 and 1909, in several articles published in this journal, G. W. Greenwood gave excellent discussions of the definitions of an angle, a limit, congruent figures, equal figures, and a prism. He showed that a solid can be constructed, satisfying the definition of a prism usually given in geometries, which is not a prism. Textbooks in geometry published recently contain this false definition of a prism and a false definition of a prismatoid.—MATHEMATICS EDITOR.]