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Alexander John Ellis ESQ., BA.,
F.R.S., F.S.A.

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JUNE 7, 1870.

R. H. M. BOSANQUET, Esq., M.A., F.R.A.S., F.C.S., FELLOW
OF ST. JOHN'S COLLEGE, OXON., IN THE CHAIR.

ILLUSTRATIONS OF JUST AND TEMPERED INTONATION.

By ALEXANDER JOHN ELLIS, Esq., B.A., F.R.S., F.S.A.

Mr. ELLIS commenced as follows:—‘By *intonation* is here meant a selection of musical sounds of certain degrees of pitch, made according to some fixed principle. It is called *just*, when that principle is to make the consonances as perfect as possible. It is called *tempered*, when the degrees of just intonation are slightly altered, with a view of materially facilitating performance on instruments having fixed tones, and, since the introduction of harmony, with the intention of injuring the consonances as little as may be compatible with that facilitation. That is to say, just intonation prefers harmony to manipulation—tempered intonation prefers manipulation to harmony. To understand this distinction thoroughly, and the various contrivances that have been proposed for serving at once the muse and the instrumentalist, it is necessary to know on what consonance and its deterioration depend. This requires a knowledge of the nature of musical sound itself, such as has been collected and clearly explained, within the last thirteen years, by Professor Helmholtz. But the results may be readily felt and appreciated by simply hearing a few experiments and harmonies. It seemed to me, therefore, that I should be rendering a service to the Musical Association if I showed a few of the elementary experiments in a manner which anyone could reproduce for himself, and also exhibited the effect of the principal tempered intonations, as contrasted with the just, in the chief forms of harmony. In doing so, I shall endeavour to reduce my observations to the merest thread sufficient to string my illustrations together, and refer those who wish to enter fully into the subject to my translation of Professor Helmholtz’s work on the *Sensations of Tone*.’

Mr. Ellis then proceeded to show simple tones, by the resonance of jars of air to tuning-forks; and illustrated the first great law affecting intonation, or *the law of beats*—namely, ‘that simple tones forming less than a certain interval with each other will beat, and that these beats will produce a maximum dissonance at from one-third to five-twelfths of that interval.’ According to Professor Mayer’s late complement to Professor Helmholtz’s law, the ‘beating distance’ is the number of equal semitones marked

B.D.,¹ and the interval of 'maximum dissonance' is the number of equal semitones marked M.D. for different pitches in the diagram on p. 161, where R.P. indicates the relative pitch of the notes, supposed to be simple tones, which, when multiplied by 64, gives the absolute numbers of vibrations in one second for the notes written over them, in just intonation. The second great law affecting intonation, or the law of differentials—namely, 'that two simple tones, sounded loudly enough to make the excursions of the particles of air bear a sensible ratio to the length of the wave, produce a third tone, having for its vibrational number the difference of the vibrational numbers of the generators'—he illustrated by two piccolo flageolets playing the G which is four octaves above the lowest note on the violin, and the F sharp next to it, and producing the low G (or more nearly the F below it, on account of the faulty intonation of these instruments) as the 'differential tone.' Such tones are also simple, and obey the first law.

The third great law affecting intonation, or the law of composition of musical tones, is that which forms the great feature of Professor Helmholtz's work—namely, 'that all musical tones are heard as if they were produced by a series of simple tones having the relative numbers of vibrations 1, 2, 3, &c., of very various degrees of loudness, sounded together.' In the diagram on p. 161 the columns show the relative pitches of the 'partial tones' which compose a 'compound musical tone,' for pitches of their 'lowest partials,' or 'primes,' corresponding to the notes at the head of the column. The loudness of the primes in each is generally much greater than that of any one of the partials. For the bass tones on harmoniums, Mr. Ellis showed that as many as sixteen at least of these partial tones are quite effective. By combining two musical tones together, such that the pitch of the prime of one corresponds to the pitch of a partial in the other (as any two or more in the diagram), the partials of each, and also their differential tones, will either fit in successively as partials of the lowest tone (in the first column), or coincide, and thus augment the power of certain partials, producing new 'artificial qualities of tone'—that is to say, 'chords.' If the partials strengthened lie within the 'beating distance' of one another, and especially lie near the interval of 'maximum dissonance,' the effects even in this case will be more or less rough, according to the pitch of those tones. The intervals 8 : 9, 9 : 10, 15 : 16, 24 : 25 become most important in producing these beats. Thus even Fifths, when taken low enough in the bass, beat strongly with 8 : 9 and 9 : 10. The tones combined in just intonation have always such relative pitches as those described. In

¹ These results were published by Professor Mayer in April 1874. In May 1876 he showed that very sensitive ears are capable of hearing beats at intervals nearly one semitone wider. But the intervals of maximum dissonance remain, as those marked M.D., and are even smaller intervals for many ears. The results vary much from individual to individual, but the diagram gives a good average.

tempered intonation the pitch of one or more notes is slightly altered (but by not exceeding the ratio of 80 to 81, or a comma), and the consequence is that the partial tones which do not beat, do not fit into the series of partial tones requisite to form a musical artificial quality of tone, and those partials which ought to coincide are always within the interval of maximum dissonance, and hence more or less rough and annoying.

R.P.	1	2	3	4	5	6	8	9	10	12	15	16	in $\frac{1}{64}$ sec.		
B.D.	$3\frac{1}{2}$	3	...	3	...	$2\frac{1}{2}$	...	...	$2\frac{1}{2}$	$2\frac{1}{8}$	...	$2\frac{1}{8}$	eq. sem.		
M.D.	$1\frac{1}{2}$	$1\frac{1}{2}$	...	$1\frac{1}{4}$	...	1	...	...	1	$\frac{9}{16}$	...	$\frac{5}{8}$	eq. sem.		
	1														
	2	2													
	3	...	3												
	4	4	...	4											
	5	...	...	...	5										
	6	6	6	...	...	6									
	7														
	8	8	...	8	...	...	8								
	9	...	9	...	...	...	...	9							
	10	10	...	...	10	...	...	...	10						
	11														
	12	12	12	12	...	12	...	...	...	12					
	13														
	14	14													
	15	...	15	...	15	...	...	...	...	...	15				
	16	16	...	16	...	...	16	...	...	...	...	16			
	17														
	18	18	18	...	...	18	...	18							
	19														
	20	20	...	20	20	...	...	...	20						
	21	...	21												
	22	22													
	23														
	24	24	24	24	...	24	24	...	...	24					
	25	...	...	...	25										

This law and its consequences were illustrated by simple tones struck with compound tones, by the compound tones of a common harmonium tuned with the relative pitches in the diagram, but with power to alter the pitch of one note by a comma (as described in Mr. Ellis's translation of Helmholtz, p. 681), and by four concertinas tuned in Greek (including Arabic) mean, equal, and just intonation. By means of these Mr. Ellis was able to compare the effect in these different intonations of Fifths, Fourths, Thirds, and Sixths, and also to play and compare

the six best forms of the major and three best forms of the minor triad of A according to Helmholtz, namely :—

Of these he dwelt especially on the fifth major form (E, C sharp, A), consisting of a major Sixth (E to C sharp), with a minor Sixth (C sharp to A) superimposed, and an Eleventh (E to A) between the extreme tones. He showed that this form was quite smooth in just intonation, was very little disturbed in mean and Arabic intonations, but was frightful in Greek and equal intonations. And he showed that this arose principally from the beats of the differential tone of the Eleventh (E to A), which beat with middle note (C sharp), and that of the major Sixth (E to C sharp), which beat at the same rate with the lowest note (E), and hence that it was independent of the quality of tone employed. Similar remarks applied to the first chord. The third and fourth chords also afforded examples of beats arising from differential tones. He also played a complete major scale, harmonised without discords, in all the different intonations, and a short chorale (also harmonised without discords, but introducing minor chords and slight modulations) in just, mean, and equal intonations, to show the difference of the effects, especially for the last three intonations, which are those with which music is at present principally concerned.

Mr. Ellis concluded as follows :—‘ Intonation has always been considered as of the utmost practical importance, because it affects the sound of every chord struck. Just intonation gives a gradation from perfect consonance to intense dissonance, unattainable strictly by any temperament, and especially unattained by equal temperament, which cannot produce a single triad without conspicuously dissonant beats, on any qualities of tone suitable for sustained musical expression, such as those of the voice, bowed or reed instruments.

‘ The history of temperament may be condensed into a few sentences. The Greek intonation was perfect for melody, and impossible for harmony, as even the Greeks themselves acknowledge that all its thirds were dissonant. Down to the eleventh century the Greek intonation influenced everything. Guido d’Arezzo then exercised his pupils by the monochord, presumably in just intonation, which offered no theoretical or mechanical difficulties as long as modulation was confined to passing from the key of C major to those of F and G major, or, provided the dominant chords were left minor, into A and E minor. It was not till the middle of the sixteenth century when either Salinas or Zarlino (for they dispute the honour) invented the mean-tone

temperament, that it became possible for the organ to modulate into B flat, and into G and A major, and into A minor, with partial effects of D and even G minor, all with major dominants. But the wheel of modulation once started, it could not stop here. C minor and F minor, and E major and E flat major soon became indispensable. The organist then struck G sharp for A flat, and so on, and produced those howlings which soon gained the name of "wolves," and wondrous efforts were made to avoid them. At last Sebastian Bach, in the middle of the eighteenth century, adopted an expedient previously suggested, and *sharpened his major thirds*. Finally, his son Emanuel, in 1753, announced his "*wohl temperirtes Clavier*," his "well-tempered clavichord," which became the parent of our present (intentionally) equally-tempered pianoforte, and it is to the peculiar adaptability of the tones of this instrument for disguising the defects of such a method of tuning that the general acceptance of that temperament is probably due. On the organ the shortcomings of this intonation were too manifest for them to be at once accepted as satisfactory, and the long struggle between the old organ-tuning and the new is not yet quite over. The great organ of St. George's Hall, Liverpool, built so lately as 1855, was tuned to the mean-tone temperament, and it was not till 1867 that it was re-tuned in equal temperament, for the purpose of playing with orchestras. (*Obit*, 27th February, 1875, p.131.) Equal temperament, or what tuners give us for it (a very different thing, generally), has indeed become a temporary necessity. If we will play no more than twelve sounds in an octave, we cannot avoid the fearful "wolves" in certain keys, now rather favoured, without introducing minor "wolves" in every key. But the discoveries of Helmholtz have sounded the knell of equal temperament, which must henceforth be recognised as a theoretical mistake and a practical makeshift—a good servant dismissed for becoming a bad master, and now merely retaining office till his successor is installed. The human voice, the violin, and the trombone do not require it, and will enable the composer to study the wondrous wealth of just intonation as a means of musical expression, when the laws of just modulation have been laid down. After Helmholtz's discoveries, they become comparatively easy to formulate, and in an Appendix to my translation of his work, I have endeavoured to follow out the hints he has given. There is really no theoretical difficulty left, either in notation, or the relations of tone, or the mechanical production of the notes required. For vocal execution just intonation is rendered easy by a constant reference of every sound to a known tonality, provided singers eschew the piano. For bowed instruments the same feeling cannot but lead to the same result, and similarly for trombones, because the ear so readily perceives and rejects the intrusion of inexact tones in harmony. Mr. Bosanquet's finger-board enables even organs and harmoniums to produce practically just intonation. I am unable to speak as to the wood and other brass of the orchestra. But they must certainly follow the lead, or

be left out of the race. At any rate just intonation, even upon a large scale, is immediately possible. And if I long for the time of its adoption, in the interests of the listener, still more do I long for it in the interests of the composer. What he has done of late years with the rough-and-ready tool of equal temperament is a glorious presage of what he will do in the future with the delicate instrument which acoustical science puts into his hands. The temporary necessity for equal temperament is passing away. Its defects have been proved to be ineradicable, because inherent in the nature of sound. An intonation possessing none of these defects has been scientifically demonstrated. It is feasible now on the three noblest sources of musical sound,—the quartet of voices, the quartet of bowed instruments, and the quartet of trombones. The issue is in the hands of the composer. Can any one doubt the result ?'

DISCUSSION.

Mr. AMBROSE SMITH asked if the second law enunciated by Mr. Ellis was not incorrectly stated, in representing that the resulting low sound produced by the concurrence of two high tones was that arising from the *difference* in the number of vibrations of the two upper tones? As he understood, it was represented by the *greatest common measure* of the two upper notes. Thus, if these tones were respectively produced by 280 and 860 vibrations, the subordinate sound would be that arising from 40 vibrations. This was laid down in a paper read before the Philosophical Society of Cambridge, some years ago, by Professor De Morgan.

Mr. ELLIS said the subject had been more fully investigated since then by Professor Helmholtz, on whose authority his remarks were based.

The CHAIRMAN said the question of Mr. Ambrose Smith was incontestably answered by the fact that the resultant low sound equally occurred when the number of vibrations of the upper tones were incommensurable. Professor De Morgan's paper was read before the investigations of Helmholtz had been made. With regard to experiments for the production of beats from high notes, he might mention that for some time past he had been in the habit of using an arrangement of small organ-pipes for that purpose. He did not know that he could perform the particular experiment mentioned by Mr. Ellis, but, as far as the production of difference-tones was concerned, they were readily produced by an arrangement of wide-stopped organ-pipes; and by the aid of a little tap in the foot of each pipe, the pitch of the tone could be regulated with the greatest nicety, and with care the difference-tone would almost resemble an ordinary bass-note. He had had some experience in mean-tone tuning, and it showed him that the mean-tone fifth was almost too bad for musical purposes in the treble, though as long as the bass was going it was bearable.

The quarter comma was very hard on the fifths, and if you struck a single triad high up in the scale, the beats were heard very sharply and unpleasantly, so that, although the third was perfect, the mean-tone had many disadvantages.

Mr. ELLIS, in conclusion, said his main object had been to show how experiments might be made, so as to verify the statements of Helmholtz. His harmonium was tuned by making seven major chords perfect, and by its help almost every experiment needed might be tried. Ordinary tuning-forks, and ordinary jars, tuned with a little water, served to produce the simple tones, and to hear their beats as distinguished from the others. Those who wished really to study Helmholtz's book would find it necessary to have something to guide their ear, for though it was said by Dr. Macfarren, that you could hear the right through the wrong, it was very difficult for the majority of people, who never happened to have heard the right.

A vote of thanks was then passed to Mr. Ellis, which concluded the proceedings.