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XLV. *On Direct Reading Resistance-Thermometers, with a Note on Composite Thermocouples.* By ALBERT CAMPBELL, B.A.*

THE measurement of high temperatures is a matter of growing importance both in scientific and technical work, and thanks mainly to Prof. Callendar's work on the subject, platinum resistance-thermometers at present hold the foremost place for accuracy in such measurements. In their most usual form, however, resistance-thermometers do not indicate the temperature directly, but it has to be deduced from the reading by means of a formula, table, or curve. In order to get rid of this trouble, I have recently invented two simple arrangements, by either of which the reading of a resistance-box, in connexion with a resistance-thermometer, gives directly the actual temperature to a good degree of accuracy.

I shall designate the two methods :—

- (A) The Rectifying Shunt Method,
and (B) The Rectifying Loop Method.

I have named them thus, because the result of using the device is to reduce to a straight line the curve connecting the box-reading with the temperature. I shall first describe (A), which is the simpler but the less accurate of the two.

(A) *The Rectifying Shunt Method.*

In this method the arrangement is extremely simple, and consists merely of the addition of an appropriate shunt to one of the arms of the Wheatstone's bridge used to measure the resistance. Before I show how this shunt is applied, let us consider the connexion between resistance and temperature. For platinum, the formulæ ordinarily used are equivalent to the following

$$\frac{R_t}{R_0} = 1 + \alpha t - \beta t^2,$$

where R_0 and R_t are the resistances at 0°C. and t respectively, and α and β are the constants. As to actual values, α and β

* Read March 10, 1905.

vary slightly in different specimens, α being of the order of 0.003, while β is near 0.0000005. This parabolic formula has been found to hold with considerable accuracy over a wide range of temperature (up to 1000° C.) *. For simplicity we may for the present make R_0 equal to 1 ohm.

Then

$$R_t = 1 + \alpha t - \beta t^2.$$

Fig. 1.

Now in fig. 1 let the Wheatstone's bridge shown have, at 0° C., four equal arms, of which BC is the platinum thermometer, and AB consists of a variable resistance x shunted by a fixed resistance s .

If the temperature of R_t is t , and x is altered until a balance is obtained, then

$$\frac{sx}{s+x} = R_t = 1 + \alpha t - \beta t^2.$$

Therefore

$$\begin{aligned} x &= \frac{sR_t}{s-R_t} = \frac{s(1+\alpha t-\beta t^2)}{s-1-\alpha t+\beta t^2} \\ &= \frac{s}{s-1} \left[(1+\alpha t-\beta t^2) / \left(1 - \frac{\alpha t}{s-1} + \frac{\beta t^2}{s-1} \right) \right]. \end{aligned}$$

* See, for example, J. A. Harker, Phil. Trans. vol. cciii. p. 343.

$$\text{Let } \frac{1}{s-1} = q.$$

$$\begin{aligned} \text{Then } x &= sq \cdot \frac{1 + \alpha t - \beta t^2}{1 - \alpha q t + \beta q t^2} \\ &= sq [1 + (1+q)\alpha t + (1+q)(q\alpha^2 - \beta)t^2 \\ &\quad + \alpha q(1+q)(\alpha^2 q - 2\beta)t^3 + \dots] \quad (1) \end{aligned}$$

Since α^2 and β are small let us neglect for the present the terms beyond t^2 .

If we can now make the term in t^2 vanish, x will reduce to the form

$$x_0(1 + mt).$$

The condition for this is

$$(1+q)(q\alpha^2 - \beta) = 0.$$

The factor $(1+q)$, being equal to $\frac{s}{s-1}$, cannot vanish since s is not zero, and thus the requisite condition is

$$q\alpha^2 - \beta = 0.$$

Hence

$$\frac{1}{s-1} = q = \frac{\beta}{\alpha^2};$$

therefore

$$s = 1 + \frac{\alpha^2}{\beta} \quad (2)$$

Since α and β are both positive for platinum, $\left(1 + \frac{\alpha^2}{\beta}\right)$ is positive and therefore s is a real resistance. When s is made to have this value, then

$$x = x_0(1 + mt)$$

where x_0 is the reading when $t = 0^\circ \text{C}$.

We have

$$x_0 = \frac{s}{s-1}, \quad (3)$$

$$m = \alpha(1+q) = \frac{\alpha s}{s-1}, \quad (4)$$

and

$$t = \frac{x - x_0}{m}.$$

Thus t is directly proportional to the increase of x above the zero-reading x_0 .

Clearly this proportionality still holds when the bridge-arms are not all equal, so long as x_0 and s are kept in the proper ratio. The compensating leads, being in the measuring arm, must have resistance in proportion to it. In practice, it is desirable to be able to use for the variable part ZB an ordinary resistance-box with sets of 10-ohm, 1-ohm, and 0.1-ohm coils (with $\frac{1}{100}$ ths also, if desired for the highest accuracy); and, in order to do this with a platinum thermometer of any resistance, the arm DC (fig. 1) is made to have the same resistance (at 0° C. say) as the thermometer, whilst the arm AD is made adjustable within a small range so that it can be made equal to AB at the zero-reading. Then if s , m , and x_0 be the values found from α and β as shown above, AZ is set to be $\frac{x_0}{10m}$, and the shunt is made $\frac{s}{10m}$. When these adjustments are right to start with, then each $\frac{1}{10}$ ohm between Z and B represents 1° C.

It will be remembered that in the above investigation we neglected the powers of t above t^2 [equation (1)]. The result of this is that the values of s , x_0 , and m calculated as above, give readings slightly in error, particularly at the higher temperatures. It is not difficult, however, by one or two trials to alter the value of s (and hence x_0 and m) so as to give readings whose accuracy is within two or three degrees, from 0° to 1000° C.

I have described this method mainly for two reasons: firstly, that it led up to the discovery of the more exact way (B), and secondly, because it is of some theoretical interest to observe that the resistance of a platinum wire at varying temperature can be very nearly represented by two resistances in parallel, one of them having a constant temperature coefficient and the other being invariable with temperature. Perhaps the first part represents pure metal, while the second (which is about $\frac{1}{30}$ of the first) may be an alloy produced by the impurities present.

If the resistance-thermometer be of iron (at moderate

temperatures) or any other metal for which

$$R_t = R_0(1 + \alpha t + \beta t^2),$$

the temperature-reading curve can usually be brought very nearly to a straight line by shunting the thermometer-arm itself by a suitable invariable resistance. Neglecting the terms above t^2 as in the preceding investigation, the requisite value of the shunt would be $R_0(\frac{\alpha^2}{\beta} - 1)$. This value is not necessarily always positive, and thus it may happen that the result is not attainable (—it is possible for *iron*). As in the case of metals like platinum, a better result can be got by trying various values of the shunt near to $R_t(\frac{\alpha^2}{\beta} - 1)$ in this case.

By introducing a little more complication, as in fig. 2, a nearer approximation to linear variation with temperature can be secured.

Here the main resistance R is shunted by $s_1 + s_2$, of which s_1 is invariable and kept cold, while a small part s_2 is of the

Fig. 2.

same metal as R and at the same temperature. By suitable choice of s_1 and s_2 the resistance from A to B can be made very nearly of the form $R_0(1 + mt)$.

To take an actual example of Method (A),—a certain platinum resistance-thermometer follows approximately the formula

$$\frac{R_t}{R_0} = 1 + 0.003946t - 0.0000005798t^2. \quad \dots (5)$$

Fig. 3 shows the values of the resistances necessary to give direct readings for this instrument.

It will be seen that the shunt is here equal to $25x_0$, while the approximate formula gives $27.86x_0$.

I need not discuss the first method further, but will pass now to the second, which is much more exact.

(B) *The Rectifying Loop Method.*

The arrangement of the measuring arm in this method is shown in fig. 4, in which a contact F can be moved along a loop of fixed resistance (n). (The resistance of the contact is assumed small and constant.)

As before, let us assume the parabolic formula

$$R_t = R_0(1 + \alpha t - \beta t^2),$$

and let us choose $R_0 = 1$,

making

$$R_t = 1 + \alpha t - \beta t^2.$$

In fig. 4 let $x + y = \text{constant} = n$, and let resistance AB be proportional to R_t when a balance (in the bridge) is obtained.

Then

$$kR_t = \frac{xy}{x+y} = \frac{x(n-x)}{n} \\ = x - \frac{x^2}{n}.$$

Now suppose that x is altered in proportion to the rise of temperature of R ,

$$\text{i. e.} \quad x = x_0(1 + mt).$$

Let $x_0 = 1$, and therefore $x = 1 + mt$.

Then

$$k(1 + \alpha t - \beta t^2) = kR_t = x - \frac{x^2}{n} = 1 + mt - \frac{(1 + mt)^2}{n};$$

$$\text{or} \quad k(1 + \alpha t - \beta t^2) = \left(1 - \frac{1}{n}\right) + m\left(1 - \frac{2}{n}\right)t - \frac{m^2}{n}t^2.$$

If this is true for all values of t , we have

$$k = 1 - \frac{1}{n} = \frac{n-1}{n}; \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\alpha = m\left(\frac{n-2}{n-1}\right); \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\beta = \frac{m^2}{n-1}. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Hence

$$\alpha = \frac{\beta(n-2)}{m}.$$

Therefore

$$n = \frac{\alpha m + 2\beta}{\beta} = \frac{\alpha m}{\beta} + 2.$$

Hence from (7) $m^2 - \alpha m - \beta = 0$.

Therefore

$$m = \frac{\alpha}{2} \left(1 \pm \sqrt{1 + \frac{4\beta}{\alpha^2}}\right). \quad . \quad . \quad . \quad . \quad (9)$$

Also

$$n = \frac{\alpha m}{\beta} + 2. \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and

$$k = \frac{\alpha m + \beta}{\alpha m + 2\beta}. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Since α and β are both positive (for metals like platinum), real values for m , n , and k can thus always be found by

equations (9), (10), and (11). As no terms in t^3 and higher powers have been neglected, *this arrangement gives an exact equivalent of the parabolic formula.* In other words, when m , n , and k have the values found from α and β by these equations, the alteration in x necessary to produce a balance for a given change of temperature will be proportional to that change of temperature.

From equation (9) it will be seen that two values of m satisfy the required conditions. The value got by taking the positive sign in (9) is the convenient one to use, as the other value gives very small change of reading for a given change of temperature.

In order to make the increase of x by 0.1 ohm correspond

Fig. 5.

to a rise of temperature of 1° C., as in Method (A) the resistances (fig. 5) in the loop are raised in suitable proportion*, and the arm AD is set to give a balance at some known temperature (say 0° C.), the compensating leads, increased in like ratio, being between B and the slider.

For example, for the thermometer already mentioned [equation (5)] we could make the total resistance of the loop 603.4 ohms, and the resistance x_0 equal to 24.46 ohms.

* A similar procedure is required to give equal bridge-arms.

Then each 0.1 ohm moved by the slider will accurately represent 1° C. Without altering these resistances, the combination could be used with fair accuracy with any other thermometer of nearly similar platinum. For the best work, however, the simplest way is to have two small resistance-coils adjusted to correspond to each thermometer, one representing x_0 , and the other $n - x_0 - 100$.

The arrangement shown in fig. 6 is convenient for the Rectifying Loop, the two adjusted coils being at P and Q.

Fig. 6.

Here A and B, the ends of the bridge-arm, are connected to turning-sliders on the ten 10-ohm and ten 1-ohm coils respectively, while the central turning head is a double one, altering the upper and lower resistances by equal and opposite steps of 0.1 ohm at a time. For clearness I have put this head (M) in the middle of the figure, but in the actual construction the dials should be in their proper order of magnitude, so as to give the temperature at a glance.

APPENDIX.

On Composite Thermocouples.

For the measurement of moderate or small differences of temperature, up to 150° C., sensitive thermo-junctions of several kinds have been successfully used. For example, in the measurement of the small temperature-differences due to the Peltier Effect, I found Iron-German silver thermopiles very convenient (Proc. Roy. Soc. Edin. 1882). The great

advantage of this combination lies in the fact that the thermoelectric power is large and almost constant over a good range of temperature, the lines of the iron and German-silver in the thermoelectric diagram being very nearly parallel: thus the voltages given are very nearly proportional to the temperature-differences. Fortunately the Nickel line is also very nearly parallel to that for Iron, and is lower than that for German-silver, making the combination Iron-Nickel considerably more powerful than that of Iron-German silver. Since the introduction of Constantan (or Eureka, a copper-nickel alloy), it has been largely used, particularly in Germany, in conjunction with iron, as a sensitive thermojunction. The thermoelectric power is likewise very nearly constant and it is considerably greater than either of the others already mentioned, being about 50 microvolts per degree C. The Iron-Nickel combination, however, has for some purposes the advantage over the other two, in that its resistance is much lower for the same size of wires. In all these combinations, the voltage obtained is only approximately proportional to the temperature difference. To make the proportionality much closer I would suggest the use of composite junctions, first employed by Tait in mapping out the thermoelectric diagram.

Fig. 7.

In fig. 7 let ABC and ADC be wires of two different metals joined in parallel, and let MP and NP be their respective lines on the thermoelectric diagram. Then the

line for the composite wire AC will pass through P (the Neutral Point of ABC and ADC) in some direction such as QP, and, by altering the ratio of the resistances of ABC and ADC, the direction of QP can be altered continuously so as to lie anywhere desired between MP and NP.

Thus by combining with the iron or the constantan (say) a "modifying" wire of another metal such as copper, the line of the composite wire can be turned into more exact parallelism to the line of the other metal used with it by merely altering the resistance of one of the branches of the composite wire. In the cases already mentioned the "modifying" wire (copper) will have a circuit of relatively high resistance.

In measuring high temperatures with platinum-platinum-rhodium (or PtIr) junctions, a "modifying" wire of less pure platinum, for example, would enable the thermoelectric voltage-curve to be modified at will, and perhaps "normalized" or even rectified.

Teddington, Feb. 23, 1905.

DISCUSSION.

Mr. W. DUDELL expressed his interest in the paper, and asked the Author if he had considered the question of putting a constant resistance in series with the rectifying-loop. By so doing, it would be possible to simplify the mathematical processes involved in the method. With reference to the Author's note on composite thermocouples, he asked if Mr. Campbell could give any information regarding the variation of the thermoelectric properties of constantan with the percentage of nickel and copper present in the alloy.

Prof. CALLENDAR said he admired the rectifying-loop method of reducing the direct readings to the Gas-Scale, and thought it would be useful in practical work, especially at high temperatures, but for the most accurate scientific work at moderate temperatures he thought the corrections would be too complicated, and that trouble would be saved by adopting the ordinary method. It was most important in practice to secure accurate compensation of the leads, which

could be insured in a satisfactory manner only by making the leads and the ratio arms equal. Instead of making the loop-resistance equal to kR as proposed in the paper, it would be necessary to make it equal to R , and to write $x=c+mt$ or its equivalent. The method suggested by Mr. Duddell of adding to the loop a resistance in series equal to R_0 , would simplify the algebraic relations of the coefficients, giving $m=aR_0$, and $n=a^2R_0/b$, but would prevent the thermometer reading on the loop below 0°C . For many purposes this would not matter seriously. Neither the loop method, nor the three-lead arrangement subsequently proposed by the Author, could be used with a bridge-wire, as both involved plug or sliding contacts of negligible resistance. The three-lead arrangement also precluded the possibility of making an insulation-test of the leads at any time.—With regard to composite thermocouples, he did not think the arrangement would be useful except for very moderate ranges of temperature on account of the irrationality of the curves, which could not be satisfactorily represented by parabolic formulæ. He had himself employed a similar arrangement with resistance-thermometers, combining wires of different materials in series, such as platinum and gold, or platinum and iron, so as to eliminate the b term. This was a simple and fairly reliable way of rectifying the curve, since the resistance-variation of most metals followed approximately parabolic formulæ, and the sign of b was positive for many metals but negative for platinum. It was difficult, however, to find a suitable metal capable of standing high temperatures without change.

Dr. CHREE pointed out that the resistance of the coils and shunt would vary somewhat with temperature unless made of some material with a negligible temperature coefficient. This would presumably necessitate some correction to the reading in the ordinary case. He asked Mr. Campbell whether a change in the total resistance of the wire due to temperature affected in any way the accuracy of the calculations.

Dr. R. T. GLAZEBROOK expressed his interest in the paper and in the usefulness of the methods described for ordinary

work, but pointed out that for the most accurate work it was advisable to work with the simplest possible form of apparatus and to apply the necessary corrections at the end. With regard to the question of compensation raised by Prof. Callendar, the mathematics given in the paper would embody his suggestion if α_0 were given a suitable value instead of being made equal to unity. The constantan wires referred to by Mr. Campbell had been tested over a large range of temperature, and it had been found that a constantan-iron couple gave a formula which was approximately linear.

Mr. CAMPBELL, in reply to Prof. Callendar, said it would be possible to arrange the Rectifying-Loop with plug-contacts instead of sliders; with 25 ohms in the measuring arm, sliders could not cause serious error. In reply to Mr. Duddell's second question, he did not know if any of the other copper-nickel alloys gave better thermoelectric results (against iron) than constantan. In both of the rectifying methods the temperature-coefficient of the measuring arm caused no more error than in the ordinary method. In further answer to Prof. Callendar's and Mr. Duddell's remarks, he mentioned that for both methods he had indicated clearly in the paper that when once the ratio of the resistances of the zero-reading and the total loop (or the shunt) was found from α and β , then the absolute values of these resistances may be altered in any ratio desired. The bridge can thus have equal or unequal arms at will.