CURVES OF GROWTH OF INTELLIGENCE

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Up to the present time most investigations in the use of mental tests have been directed toward the practical value of such tests from the point of view of prognosticating the future behavior of individuals. Because of the comparative recency of the work in mental testing, there has been little opportunity to make systematic measurements of the intelligence of a number of individuals over a period of years. This type of research will undoubtedly give us the best knowledge of the growth of intelligence. In the meantime, however, we may endeavor to gain some knowledge of the growth curve from an examination of group tests given to a large number of children of various ages.

PART I

The purpose of the results here presented is to suggest a few possibilities of individual prognosis to be expected as soon as group tests can be refined to the point of being as accurate as the individual examinations used in the past. Simplification of methods of administering, scoring and evaluating tests to the point where any school teacher of average intelligence can accurately give, score, and statistically evaluate mental and educational group tests, will be a big advance in the direction of securing the masses of data so necessary for determining group norms.

Our results are based on the median mental ages of 1,987 children, tested on a series of six group tests.¹ Chronological-age-mental-age distributions for all school ages from six to

¹ Pintner, R., 'The Mental Survey,' New York, 1917, 116 pp.

thirteen were obtained. To secure additional accuracy, the mental ages were grouped in classes of one half year mental age. This distribution table is too extensive to print here and is also unnecessary for the discussion. The percentages

FIG. 1. Distribution of Mental Ages, by Chronological Age and by Per Cent.

derived from the table were turned into a cumulative distribution by chronological age and by mental age with the result as shown in Fig. 1. Very small irregularities have been ironed out of these curves by mechanically smoothing the curves in plotting. Each curve of Fig. 1 represents a

TABLE I

CHRONOLOGICAL AGE DISTRIBUTIONS OF THE MENTAL AGE LIMIT REACHED BY THE DESIGNATED PERCENTILE PERSON OF EACH CHRONOLOGICAL AGE GROUP

(Disregarding the sampling involved, and assuming constancy of percentile rank of the individual, the figures in the body of the table are ordinates for individual curves of growth of intelligence, as indicated.)

Percentile Person	Mental Age at the Chronological Age:									
	6	7	8	9	10	11	12	13	14	15 ¹
Lowest...	—	4.8	5.5	6.0	6.45	6.8	7.5	8.0	8.65	8.95
5.....	—	5.45	6.05	6.9	7.3	8.0	8.55	9.70	10.15	10.45
10.....	—	5.85	6.45	7.35	7.8	8.55	9.2	10.45	10.95	11.15
15.....	—	6.1	6.75	7.75	8.25	8.95	9.8	10.95	11.5	11.8
20.....	—	6.25	7.0	8.05	8.55	9.35	10.25	11.35	11.85	12.3
25.....	—	6.4	7.25	8.3	8.85	9.7	10.6	11.7	12.25	12.8
30.....	—	6.5	7.4	8.45	9.1	10.0	10.9	12.0	12.6	13.25
35.....	—	6.6	7.55	8.65	9.3	10.25	11.2	12.3	12.9	13.7
40.....	—	6.7	7.7	8.7	9.55	10.5	11.45	12.55	13.25	14.1
45.....	—	6.85	7.9	8.85	9.75	10.8	11.75	12.8	13.6	14.55
50.....	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0
55.....	6.15	7.1	8.15	9.15	10.2	11.25	12.25	13.3	14.3	15.4
60.....	6.25	7.25	8.35	9.3	10.35	11.45	12.5	13.6	14.6	15.8
65.....	6.45	7.4	8.4	9.45	10.55	11.7	12.75	14.0	15.05	—
70.....	6.55	7.55	8.55	9.6	10.8	11.9	13.05	14.4	15.5	—
75.....	6.7	7.75	8.65	9.8	11.1	12.15	13.45	14.9	16.0	—
80.....	6.9	7.95	8.80	10.55	11.4	12.45	14.0	15.45	—	—
85.....	7.1	8.15	8.95	10.3	11.85	12.9	14.55	—	—	—
90.....	7.3	8.4	9.20	10.7	12.35	13.4	15.3	—	—	—
95.....	7.6	8.8	9.65	11.4	13.15	14.2	—	—	—	—
Highest...	8.0	9.6	10.9	13.0	14.6	15.7	—	—	—	—

cumulative percentage distribution (ogive) of the mental age of all the cases of the chronological age designated on each curve. This chart was drawn to a very large scale, enabling accurate graphical interpolations to be made. The points of intersection of each chronological age curve and the respective percentage ordinates of Fig. 1 were thus graphically determined. Thus from the figure we can find the mental age equivalent of any percentile of any of the chronological ages. For example, the forty percentile eight year old has a mental age of 7.7, and so forth. The points of intersection, graphically determined, for every fifth percentile of all chronological ages are shown in Table I.

From Table I, we may plot directly percentile curves of growth of intelligence, chronological age being used as abscis-

¹ Age 16 not included. At most percentiles their M.A.'s are not as high as the 15-year-olds, due to mental retardation. They are also one to two years or more retarded in school.

sae, and mental age as ordinates. The result of such plotting is shown in Fig. 2, the circles representing plotting points. Only each ten-percentile curve is here shown. The plotting points are given for only five of the curves to prevent confusion.

FIG. 2. Plotted Least Square Equations of Percentile Curves of Growth.

$$\text{Now the equation } I.Q. = \frac{\text{Mental Age}}{\text{Chronological Age}} \quad (1)$$

may be written

$$M = I.C., \quad (2)$$

where M = mental age; I = I.Q.; and C = chronological age. This equation is a straight line. By the method of *least squares* the straight line equations of best fit of the actual percentile distributions have been calculated for each 10-

percentile and have been plotted on Fig. 2. The equations of these lines are shown in Table II. With the tests used, and with the distributions shown in Fig. 2, greater constancy of I.Q. on the whole would result from using the formula,

$$I = \frac{M - 3}{C - 3} \quad (3)$$

since the straight lines are seen to intersect at approximately $C = 3$, $M = 3$.

TABLE II

EQUATIONS OF LEAST SQUARE STRAIGHT LINES OF BEST FIT OF DATA OF TABLE I
($M = I \cdot C + k$)

Percentile Curve	Straight Line Equation
Highest.....	$M = 1.589C - 1.536$
90.....	$M = 1.327C - .991$
80.....	$M = 1.213C - .587$
70.....	$M = 1.128C - .397$
60.....	$M = 1.057C - .150$
50.....	$M = 1.000C + .000$
40.....	$M = .931C + .261$
30.....	$M = .858C + .581$
20.....	$M = .784C + .813$
10.....	$M = .705C + .884$
Lowest.....	$M = .518C + 1.260$

Figure 2, although statistically correct, gives an entirely erroneous view of the probable actual differences in intelligence. None of these curves have any decided tendency towards becoming horizontal in the later chronological ages, indicating decrease in the rate of growth, as is known to be the case. We need a diagram, therefore, that will show the gradual decrease in the growth of intelligence with increasing chronological age. To obtain such curves we must obviously calculate the difference in amount of intelligence from one chronological age to the other. These amounts will determine the gradually decreasing distances for our mental age ordinates.

In Table III. are shown the standard deviation intervals between the median mental ages (half-year mental age classes) by chronological age. This table was constructed from the original frequency distribution table of mental ages by chrono-

TABLE III

STANDARD DEVIATION INTERVALS OF THE SEVERAL MENTAL AGES AS MEASURED BY THE SEVERAL CHRONOLOGICAL AGE DISTRIBUTIONS

Chronological Age	Standard								Intervals								(Sigma)							
	6.0-6.4	6.5-6.9	7.0-7.4	7.5-7.9	8.0-8.4	8.5-8.9	9.0-9.4	9.5-9.9	10.0-10.4	10.5-10.9	11.0-11.4	11.5-11.9	12.0-12.4	12.5-12.9	13.0-13.4	13.5-13.9	14.0-14.4	14.5-14.9	15.0-15.4	15.5-15.9	16.0-16.4	16.5-16.9	17.0-17.4	17.5-17.9
6.....	.54	.44	.59																					
7.....	.97	.64	.38	.36	.46	.48																		
8.....			.47	.48	.44	.64	.49	.35																
9.....			.53	.48	.36	.61	.34	.29	.45	.40														
10.....					.21	.40	.29	.22	.38	.26	.20	.24												
11.....							.25	.24	.33	.25	.26	.24	.35	.18										
12.....									.24	.26	.25	.21	.32	.19	.20	.15								
13.....											.16	.23	.33	.22	.25	.15	.20	.18						
14.....													.20	.27	.13	.19	.19	.16						
15.....															.25	.13	.30	.20	.12	.21				
16.....																	.22	.24	.16	.06				
Weighted Sigma Interval.....	.75	.54	.48	.43	.37	.55	.34	.27	.35	.29	.22	.23	.31	.21	.20	.16	.23	.19	.14	.13				
Cumulative Sigma Interval..	.75	1.29	1.77	2.20	2.57	3.12	3.46	3.73	4.08	4.37	4.59	4.82	5.13	5.34	5.54	5.70	5.93	6.12	6.26	6.39				

logical ages. For each chronological age cumulative percentage frequencies for each mental age step in the table were calculated. The points represented by these cumulative percentages were then turned into sigma values by means of the usual table of values of the probability integral (Thorn-dike, 'Mental and Social Measurements,' Table 44). We thus have sigma values for each point on the table and the differences between these sigma values give us the sigma intervals shown in Table III. To make the matter still more concrete, we find from our frequency table that at chronological age six there are 36.8 per cent. who test at a mental age of 5.99 or below; 57.9 per cent. test at a mental age of 6.49 or below; 73.7 per cent. test at a mental age of 6.99 or below; 89.0 per cent. test at a mental age of 7.49 or below. These percentages are now turned by means of the integral table into the sigma values (*i.e.*, deviations from the median divided by sigma) — .34, + .20, + .64, + 1.23. For example the 73.7 percentile is 23.7 per cent. above the median and we find from the table that a range from the median to + .64 sigma includes about 23.7 per cent. of the cases, and so with the others. Subtracting these values (*i.e.*, — .34 from + .20, + .20 from + .64, etc.) we get .54, .44, and .59 as standard deviation intervals for the three mental age intervals as shown in Table III. The figures give, for a normal frequency distribution of mental ages of children six years old chronologically, the distance or range on the base line (in terms of sigma) between the points 6.0 and 6.4 mental age, between 6.5 and 6.9 mental age, and so on. In a similar manner the standard deviation intervals for the other chronological ages have been computed. The table shows the general tendency of these standard deviation intervals, representing amounts of intelligence growth, to decrease gradually as we go from the lower to the higher ages.

In the table those standard deviations included within the blocked areas are direct measures of the interval, that is to say they are the chronological age distributions within one year of the mental age considered. Each standard deviation within the blocked areas was arbitrarily given a weight

of 3 and those figures of the table outside of the blocked areas were given a weight of 2 each and the average weighted intervals between the mental ages were thereby determined.

FIG. 3. Method of Determining M.A.-limits in Terms of Amount.

These weighted intervals are shown at the bottom of the table in the row called "Weighted Sigma Intervals." It will be noticed that there is a gradual decrease in the interval with increase in age but that these decreases are not perfectly

regular. To eliminate this feature, and secure a probably more accurate determination of the interval as well as having a high utilitarian value in plotting the curves, the mental age

FIG. 4. Curves of Growth of Intelligence. Percentile Rates of Growth.

intervals were cumulated, as shown in the last row of Table II., and plotted as the curve of Fig. 3.

In this curve, very small irregularities were mechanically ironed out in drawing the curve. The intersections of each

mental age ordinate with the cumulative curve are projected to the right and form our foreshortened ordinates of Fig. 4.

Using these foreshortened mental age ordinates and the usual equal-interval chronological age abscissae, the data of Table I. were plotted with these coördinates. Due to the method of measurement of mentality in terms of mental age, mentality units above age sixteen are not available. The percentile curves have, however, been prolonged graphically until they come to a point where the rate of growth no longer increases. If we do this, we find that the median of the fifty percentile curve seems to stop at age eighteen. This is probably too high and due to the fact that the sampling of children in the higher ages was very likely more selected than those in the lower ages. As we know from other data, it is more likely that average mental growth ceases about age fourteen or fifteen. There may be some increase after these ages, but such increase is probably so slight as not to be measurable with the rough scales of intelligence that we possess at the present time. Our curves, therefore, based as they are upon the data derived from a specific set of tests, probably show too much increase in the higher ages, but they will serve, nevertheless, to indicate the general trend of mental development for all degrees of intelligence at the various chronological ages.

PART II

We must keep in mind the theoretical assumptions underlying the procedure in Part I. The assumption is made first of all, that intelligence tests of the type used actually do measure intelligence. No intelligence test has ever yet been devised which will give a perfect correlation with any independent social criterion of intelligence. Partial correlation may be used in combining mental tests with the proper theoretical weighting of each test to give the maximum correlation with any given social criterion. It is to be noted, however, that even the social criterion itself will never give a perfect correlation with the presumably true intelligence of the individuals involved.

The second assumption, which has often been used, but which has as yet not been put to experiment, is that the r th percentile child at any given chronological age will be the r th percentile child at any succeeding chronological age. If this be assumed to be true, then the use of the procedure outlined above is justifiable. If there are any "spurts" in intelligence, these would quite necessarily be covered up by the procedure used. Future measurements of the mentality of large groups of children over a period of years is very desirable in this connection. The assumption here made is purely arbitrary but consistent with the assumption ordinarily made by most experimenters in group testing.

The third assumption that the average weighted standard deviation is the best measure of mental age intervals is also purely arbitrary. A fourth assumption is that rates of growth of intelligence computed from mental tests of *school children* give the desired curves of growth. The statement often made to the effect that a distribution of *school children* is a representative sampling of the population as a whole is manifestly false; very apparently so, when one considers the very high amount of elimination from school in the higher grades. Such elimination in the seventh and eighth grades is largely of two kinds:

- (a) Elimination of the mentally inferior, and
- (b) Elimination by graduation of mentally advanced pupils who have also been advanced in school.

In point of numbers, (a) and (b) most certainly do not counterbalance each other. If we resort to the high school in order to secure a large number of persons of sixteen years or over, we again have a more decidedly selected group of people and not a *random selection* of test subjects at such ages. The difficulty of obtaining a *random selection* of test subjects at any age necessitates resorting to the schools for test subjects and such has been true in all investigations, at least so far as published. This will probably always continue to be the case. Curves of growth of intelligence based upon school children are decidedly not the best for purposes of prognosis.

Thorndike has clearly shown this fact in his investigation of school elimination.¹ Such curves are based upon the 'survivors' and the 'obtainable' and not on a random selection of the whole population of the chronological age limits considered.

¹ Thorndike, E. L., 'The Elimination of Pupils from School,' U. S. Bureau of Education Bulletin, No. 4, 1917, pp. 45-55.