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66. Note on the Parabola through Four Concyelic Points

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66. *Note on the parabola through four concyclic points.*

Let A, B, C, D be given fixed concyclic points upon a given parabola. Produce AB, CD , which are equally inclined to the axis, to meet externally at E . At any point P on the curve draw the tangent PQR and the diameter PUV , meeting AB, CD in Q, R and U, V respectively.

Then, by a known property of the parabola,

$$QU^2 = QA \cdot QB, \text{ and } RV^2 = RC \cdot RD$$

(*Milne & Davis*, p. 26). Hence QR is the radical axis of the circle through A, B, C, D , and a circle touching EAB, ECD in U, V .

Conversely, the envelope of the radical axis of a given circle and a variable circle touching two given fixed straight lines is one of the two parabolas through the four points in which the straight lines intersect the fixed circle. [One parabola corresponds to the series of variable circles inscribed within the acute angle between the straight lines, and another to those within the obtuse angle.]

It will be seen that the above property furnishes a method of drawing the tangent in any proposed direction to the parabola through four given concyclic points; whence the tangent at the vertex and directrix can be determined.

(*Cf. Educational Times*, May, 1898, Question 13840.)

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67. *Circles are drawn through two fixed points. Trace the path of a point which cuts them all at the same angle.* (Question 172, p. 89.)

Let O, A be the two fixed points.

Invert the system of circles, taking O as centre and OA ($=a$) as radius of inversion. The inverted system is a system of straight lines through A .

If A is taken as pole, the spiral curve which cuts these straight lines at a given angle α has the equation

$$r_1 = be^{\mu\theta_1}, \dots\dots\dots(1)$$

where $\mu = \cot \alpha$, and b is any constant length. The inverse of this spiral with regard to O is the curve required.

Let P be a point on the equiangular spiral, and let Q be its inverse point.

$$\begin{aligned} \text{Let } OQ &= \rho, & AQ &= \rho', \\ QOA &= \theta, & QAO' &= \theta'. \end{aligned}$$

Then, in the similar triangles OAP, OQA ,

$$\frac{r_1}{a} = \frac{\rho'}{\rho},$$

also

$$\theta' - \theta = \pi - \theta_1;$$

therefore from (1)

$$\alpha\rho' = b\rho \cdot e^{\mu(\pi + \theta - \theta')} = c \cdot \rho e^{\mu(\theta - \theta')}, \text{ if } c = be^{\mu\pi}, \text{ i.e. } \alpha\rho'e^{\mu\theta'} = c\rho e^{\mu\theta}.$$

This is the equation of the required curve in bipolar coordinates. Its equation in rectangular or polar coordinates could easily be deduced from this,