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AN EXPERIMENT IN LEARNING AN ABSTRACT SUBJECT¹

EDWARD L. THORNDIKE AND C. B. UPTON

Institute of Educational Research, Teachers College, Columbia University

The purpose of the experiment was to observe the process of learning an abstract subject by adult learners in a case where the task is for them of somewhat the same novelty and difficulty as the learning of algebra is for the first-year high school pupil or as the learning of physics is for the third year pupil. We were also concerned with finding out how profitable it seemed to be for teachers to study themselves as learners in the case of such abstract material.

The subjects were a score of college graduates—teachers of mathematics. The task was to acquire understanding of certain elementary laws of electricity and magnetism. It was defined by the following instructions:

EXPERIMENT, APRIL, 1922

LEARNING AN ORGANIZED ABSTRACT SUBJECT

Study pages 1 to 22 (up to Section 15) of FRANKLIN and ESTY, "Elements of Electrical Engineering; Direct Currents" for 6 hours. Keep notes of difficulties and of how you met them.

There are six or seven copies on reference in the library.

¹ The investigation reported in this article was aided by a grant from the Commonwealth Fund.

Begin on page 1; refer back to pages XI, XII and XIII as is needed. Refer to attached notes on physics as is needed.¹ Refer to physics text-book if you find the attached notes insufficient. There are several copies on reference in the library.

Then work problems 4 to 18 inclusive on pages 448 to 453, so far as you have time in 4 hours more. Refer back to pages 1 to 22 as you need to. Keep notes as before.

The answers which the text-book provides were erased, since it seemed more instructive to make the first experiment without aid

¹ The attached notes were as follows:

C.G.S. UNITS, WHICH MAY BE NEEDED, IN SOLVING THE EXAMPLES ASSIGNED

Dyne	= force which acting on 1 gr. for 1 second increases its velocity 1 cm.
The number of dynes	= weight in grams times acceleration of gravity in centimeters per second per second.
Erg	= unit of work or energy = 1 dyne acting through a distance of 1 cm.
Joule	= 10^7 ergs.

ENGLISH UNITS

Foot-pound	= force required to move 1 lb. a distance of 1 ft.
Pound-inch	= force required to move 1 lb. 1 in.

DEFINITIONS

Torque equals the force times the distance through which the power works.

Center of gravity is the point on which any object will balance.

Armature, a core of metal surrounded by a coil of wire, rotating near the poles of magnets in a dynamo.

Dynamo, a machine to convert mechanical energy expended upon it into electrical energy.

Equivalent units of measure that may be needed in solving the examples assigned:

1 horse power	= 33,000 ft.-lbs. per minute
1 horse power	= 746 watts
1 watt	= 0.00134 horse power
1 joule	= 0.74 ft.-lb.
1 watt	= 44.3 ft.-lb. per minute
1 cm.	= 0.3937 in.
1 ft.	= 30.48 cm.
1 lb.	= 453.6 gr.
1 gram	= 0.0353 oz. avoirdupois
1 kilogram	= 2.2046 lb. avoirdupois

from them. We hope to repeat the experiment with a similar group using the answers.

It was explained to most of the subjects that the 6 hours of work on the text, the 4 hours of work on the text, and the problems could be divided up and alternated in any way that the subject might choose. All work was signed with a number or pseudonym. The subjects were asked to hand in all their work and to put down in writing any matters concerning the learning process, such as difficulties and the means taken to overcome them which seemed instructive, taking out time therefor.

This last injunction resulted in only rather meager notes, probably for two reasons. First, the teacher does not, unless he has an exceptional interest in the psychology of learning, watch himself learn when he learns. On the contrary he becomes absorbed in the content which he is trying to master. In the second place, there is a natural tendency to avoid the labor of making such notes.

The 10 hours were spent at times of the subject's choice during a week. At the close of the week certain questions about their experiences were asked of all the subjects and a 12-minute test was given. What follows in this article is the writer's opinion based on the answers to these questions, the notes on learning, the written work, and the test.

The experiment seems a useful one as a means of increasing the subject's appreciation of pupils' difficulties and sympathy with their efforts, and as a means of increasing the experimenter's knowledge of learning. The task assigned is somewhere nearly as hard for college graduates who know physics only from an elementary course, as beginning algebra and physics and economics are for their students. This is evidenced by the records with the problems. The highest score was $12\frac{1}{2}$ correct out of 15; the lowest was 0; the median was 5; half were from 3 to 7 inclusive. The errors gave by their nature, and especially their variety, much the same impression of inadequacy, confusion and carelessness that one gets from the results of a hard assignment to a high school class in an abstract subject.

It is not grossly inaccurate to say to a group of teachers who have done this experiment: "This is to you approximately what a hard series of lessons is to a high school class. The difficulties and discouragements and confusion which you have felt are what they feel. Although you did your best, your work looks stupid and careless, much as theirs does. It looks in places as if you 'did not try' or

'would not make yourself think,' or 'did not keep your attention on what you were doing.''' The experiment should be a warning and protection against underestimating pupils' difficulties and imputing to perverseness or lack of effort results which are really due to the general laws of mental action. It should enforce the general lesson of psychology that commands and exhortations and rebukes are only a very small part of teaching, the great part being to discern the forces of the pupils' own minds and hearts and manoeuvre them to desirable ends.

So much for the benefits to be derived from the experiment in making teachers more humane and reasonable. With respect to our knowledge of the learning process itself, its chief result is its evidence that gifted, highly trained adults are much more like ordinary untrained children in methods and procedure in learning than it has been the fashion to suppose. It has been customary to contrast the inferior young, untrained child sharply with the superior, trained adult in that the former is impulsive, leaping before he looks; is uncritical, accepting any idea that happens to strike him regardless of its appropriateness; and is confused and dreamy and irrelevant, lacking clear-cut ideas of what he is to think and why he is to think it. This contrast is treated as general, pervading all the thinking of the two types of mind, and constitutional, depending on their distinctive natures, not on the tasks in which they engage. The statement that superior trained adults seem more rational, critical and clear-headed in large measure because they do only what is easy for them, would be regarded by all educational psychologists as a paradox and by most of them as a very silly one. Yet our inspection of the work of this score of superior trained adult thinkers strongly suggests that the paradox has a large element of truth, that the difference in thinking is not all-pervasive and constitutional, but is, in part, a consequence of specialized habits.

Consider, for example, the answers given by these gifted trained adults to the question "Why are magnets which are used to pick up pieces of iron bent into U form?" which was asked at the end of the 10 hours of study of the elementary principles of magnetism. Many of them show an impulsive, uncritical, and confused thinking to a notable degree—to the person who is really master of the topic.

(4) The intensity of magnetic field is increased by bringing poles near each other. Attraction between two poles is product of their intensities divided by distance squared, and as distance is decreased the intensity increases inversely as its square.

(B) Magnetic force tends to concentrate at one spot or another. Therefore, if the iron is bent horseshoe fashion and magnetized, there is more strength.

(C) To get both poles on a level to attract either or both poles of the article you want to pick up.

(D) In order that the object may lie along the lines of intensest force, thus the strongest pull being conducted through the material picked up.

(E) Magnets are bent into U form in order to create a magnetic field of

(F) Magnets are bent into U form to increase intensity of field. The two poles being

(G) So that the poles are more convenient for use. More handy for use than a long magnet.

(H) The magnet is in U form to bring poles nearer together.

(I) To strengthen field; $F = \frac{m'm''}{r^2}$ as distance r decreases, F increases.

(J) Pull is both ways and stronger north and south poles give equal pull.

(K) Bringing both poles together condenses the external magnetic field, thus making it stronger. Both poles are brought to bear on an object, thus

(L) To attract both north and south pole. The flux is from north to south pole; by having U-shaped magnet the two poles are near enough to make the circuit.

(M) So that both poles may operate at once; putting the poles together concentrates the field, i.e., shortens and pushes closer the lines of force.

(N) Because the closer the poles are together the more intense is the magnetic flux.

(O) Magnets are bent into U form to bring the poles nearer together.

(P) Bent to give two poles so that the current is attracted to the second pole, otherwise there would be no flow of current.

(Q) U form is due to the fact that like poles repel and unlike poles attract. If the poles are close together the force of attraction is greater and exerts greater force on external objects.

(R) The poles are opposite in attraction and have power that is concentrated.

(S) So that the entire force of the magnet is at one point and so that both directions of force may be exerted at once.

(T) Double the strength, or attraction on the iron.

Impulsiveness and uncriticalness are witnessed further by the large number of wrong answers in comparison with the number of omissions or confessions of inability. In the book problems there were (counting each task requiring a separate answer as one problem) 341 answers given of which only 53 per cent were right. In the case

of few of these was there any apology or other expression of doubt about the answer. In the test problems a similar state of affairs obtains. The extreme of uncriticalness appears where the thinker merely "fishes about" for any promising formula and puts the number of the problem into that formula, almost haphazard. Consider these answers all given to the same problem and all wrong.

15,503 lines
15.5035
93,020 lines
348,000 cos 63
174,000 cos 63
4.739 gauss
78,994 dynes
7.8996

A general mental confusion is evidenced by much of the work, especially perhaps by the very frequent failure to define the answer numbers as dynes, gaussses, maxwells, lines, centimeters, or whatever they should be, and by the frequent attachment of erroneous names to the answer numbers.

Thus in the test the question: "What is the intensity of the magnetic field due to a unit pole at a distance of .001 cm.?" received a correct number answer six times but the name was correct only once.

The "childishness" of these adults appears also in their eagerness to learn from some authority whether the answers which they obtain are right, rather than trust to their own proofs and checks.

We are not, of course, saying that the work of first-year high school pupils *at this task* would not be easily distinguished from these adults' work at it. Nor do we say that these adults are as childish at it as first-year pupils are at their tasks in mathematics, grammar, or science. What is claimed here is not that the difference between the thinking of superior trained adults and that of children is zero, but that it is less than the orthodox educational psychology and child psychology of the last two decades has taught. In proportion as a task for thought is novel and hard, the superior trained adult tends to jump at conclusions instead of planfully mastering each necessary step, to let results stand without surety that they are true and useful, and to become confused, making computations and statements without any clear realization of what he is doing and why he does it.¹

¹ I may be permitted to note that I observed these tendencies in my own work at this task.

ON THE PSYCHOLOGY OF TEACHING IN GENERAL

The experiment provides evidence corroborating certain pedagogical doctrines which are already accepted or on the road to acceptance. There was general agreement among the subjects that each item of fact or principle should be applied as soon as learned, that miscellaneous problems requiring the selection of appropriate principles should be given later, and that many short problems with a minimum of computation and interpretation of complex situations should be given first. There was general agreement that, even with the very scanty problem list of the book used, the problems explained the text as truly as the text explained the problems. There was substantial agreement that the learner should be enabled to ascertain whether his work was correct very soon after he finished any part of it and very often along its course. Sample problems requiring the use of formulas should be solved in the text as illustrations of the formulas.

The irritation caused by features of the task that were, or at least seemed, irrelevant, such as laborious computations, a problem about the torque on a pulley, and the recalculation of an answer in another system of units, was notable. Irritation at being thwarted by a difficulty that was, or at least seemed, not provided for by the text was also great. The text left, or seemed to leave, the meaning of gauss and the relation of a gauss to the force exerted by a unit pole at a distance of 1 cm. to be inferred. We estimate that if it had stated it clearly and emphatically there would have been a saving per person of 30 minutes time, much irritation and at least ten per cent in errors.¹

Finally the facts of the experiment suggested to the writer as highly probable an important amendment to the common view of the difference between the "easy" subjects like English, History, French, or book-keeping and the "hard" subjects like mathematics, Latin, or physics. The common view is that the latter are hard because they are more abstract, more organized, more rigorous and more precise. They require, to a greater degree, analysis and reasoning in place of memory, systematized related knowledge in place of an accumulation of details, an absolute adherence to certain rules of the game rather than a general following of their spirit, and 100 per cent precision in the operation of the mental bonds instead of a certain free play for

¹ Neither this paragraph, nor anything else in this report, is intended or should be interpreted as an adverse criticism of the text-book pages used. Our experiment put them to a use very different from that for which they were designed.

errors of moderate amount. This is all true; but it may also be true that part of the greater difficulty is due to the sheer unfamiliarity of the date, the greater amount to be learned per unit of time, the more subtle symbolism, and the larger number of missing links—facts that have to be read between the lines, connections that have to be worked out by the pupil by inference. If high school pupils studied the history of philosophical systems or of credit facilities, history would be made harder by the unfamiliar data. If, when they had studied a certain phenomena, say the causes of revolt in Greece and Rome, they were expected to apply that knowledge to all cases of revolt anywhere, distinguishing when it was and when it was not applicable, history would be harder because learning the causes of revolt in Greece and Rome would then be learning a great deal more than a chronicle plus certain suggestions. If, in the text-book on history, *e* stood for envy, *r* for race antagonism, *p* for George Washington, the area of a square for the number of people concerned, *l*₁, *l*₂, *l*₃ etc., for certain typical cases previously taught, and the like, history would be harder. If any facts that a competent thinker could infer were omitted from statement in history (as if we read "Charles" instead of "King Charles," "Gregory" instead of "Pope Gregory," "He infringed upon the six most important civil rights of the population," instead of a long paragraph stating in detail what he did, history would be harder.

In the experiment all these four factors appeared: (1) Strength of pole, magnetic field, intensity of magnetic field, and magnetic flux are hard partly because they are strange to us. (2) Formulas like $F = mH$ or $\Phi = sH$ look like small bits of learning, but each is really, if understood, a large body of fact and a still larger possibility of handling other facts. Thirteen such formulas in 6 hours means a very large amount of learning per hour. (3) The symbolism which will in the end economize thought is in the beginning a burden. (4) Consider the number of inferences that a student must make to obtain a true and adequate learning of this single half page.

"Magnetic Flux.—Consider a plane surface, *s* square centimeters in area, stretched across and at right angles to a uniform magnetic field of intensity *H*. The product *sH* is called the *magnetic flux* across the surface. That is:

$$\phi = sH \quad (4a)$$

in which Φ is the magnetic flux across a plane surface of area *s* square centimeters at right angles to a uniform magnetic

field of intensity H . When the plane surface is not at right angles to the uniform magnetic field then:

$$\phi = sH \cos \theta \quad (4b)$$

in which θ is the angle between H and the normal to the surface. When the field is not uniform or when the surface is curved then:

$$\Delta\phi = H \cos \theta \Delta s \quad (4c)$$

in which $\Delta\phi$ is the flux across an element of the surface of which the area is Δs , H is the intensity of the field at the element of surface, and θ is the angle between H and the normal to the element of surface. In this case the total flux across a finite surface is found by integrating equation (4c) over the finite surface."

So, we repeat, the experiment suggests that these other characteristics deserve consideration along with abstractness, organization, rigor and precision. Precision and rigor may be rated as indispensable features of secondary education. Abstractness, organization, and subtle symbolisms are very desirable features of it for those whose intellects can handle them. What I have called *reading between the lines*, supplying necessary interpretations and connections, is desirable in moderation. The mere difference in amount of learning whereby an assignment in English or history seems more than it is and an assignment in mathematics or physics is much more than it seems, is not of any intrinsic educational value. The difference in strangeness is perhaps of negative value educationally, it being unwise to try to understand, analyze, compare and reason about facts until we have a certain familiarity with them as facts. In so far as Latin is hard because of the sheer strangeness of datives and ablatives, its hardness is chiefly an unfortunate interference to be overcome. In so far as algebra is hard because the use of letters to represent numbers is strange, the lesson to its teachers is to remove that much of its hardness as quickly as they can by suitable experiences with formulas and their evaluation.