where for greater clearness I have omitted the symbol CC' , which is to be understood as occupying each of the vacant squares.

The arrangement is the same as before; the right-hand margin shows the sextic cone; viz., for the node 4 this is made up of the singular planes 4, 12, 16 and of a cubic cone represented by (5, 6, 7) (as replacing the planes 5, 6, 7 in the case of the 16-nodal surface). Similarly for the node 5, the sextic cone is made up of the singular plane 4, the nodal cubic cone (2, 3; 5), and the quadric cone (9, 13) (the numbers in these last symbols indicating the planes in the case of the 16-nodal surface, which are here replaced by cones). So for the node 8, the sextic cone is made up of the singular planes 12, 16 and of the trinodal quartic cone (8; 1, 2, 3). As regards a nodal cubic cone, for example (2, 3; 5), the semicolon is used to indicate that the nodal line replaces the intersection of the planes 2, 3; the other intersections 2, 5 and 3, 5 having disappeared. And so for a trinodal quartic cone (8; 1, 2, 3), the semicolon is used to indicate that the nodal lines replace the intersection (1, 2), the intersection (1, 3), and the intersection (2, 3) respectively; the other intersections 1, 2; 2, 8 and 3, 8 having disappeared. Finally, in the body of the table, C is used to denote the cubic or the quartic cone (as the case may be); $\times$ to denote a nodal line of either of these cones; and C' the quadric cone; as already mentioned, the vacant places are considered to be CC' .

The reading of the table is then as follows; viz., for the node 4, the remaining twelve nodes lie on the nodal lines of the sextic cone 4, 12, 16, (5, 6, 7), as shown; viz., 5, 6, 7 are each of them on the intersection of the cubic cone with the plane 4; 8 is on the intersection of the planes 12 and 16; and so on.

I reserve for another Memoir the discussion of the 13 (β)-nodal surface, and the surfaces with less than 13 nodes.

Presents received:—

“*Monatsbericht*,” Nov., Dec., 1870; Jan., 1871.

“*Proceedings of the Royal Society*,” No. 126.

“*Journal of the London Institution*,” (4).

May 11th, 1871.

W. SPOTTISWOODE, Esq., F.R.S., President, in the Chair.

Mr. C. J. Monro, B.A., late Fellow of Trinity College, Cambridge, was elected a Member. Mr. J. Griffiths, M.A., Fellow of Jesus College, Oxford, was proposed for election; and the Hon. J. W. Strutt, M.A., Fellow of Trinity College, Cambridge, was admitted into the Society.

Prof. Henrici, V.P., indicated the method of treatment he had employed in his paper "On the Singularities of the Envelopes of a Non-Unicursal Series of Curves." Prof. Cayley made a few remarks upon the paper.

The Hon. J. W. Strutt then read the following paper:—

Note on the Explanation of Coronas, as given in Verdet's "Leçons d'Optique physique," and other Works.

Coronas are formed whenever, between the eye and the source of light, a large number of small opaque bodies, regular in size but irregular in distribution, are to be found. By a principle known as Babinet's, it is permitted to substitute for the layer of obstructing globules an opaque screen having apertures, which admit the light precisely where before it was stopped. From each aperture diverges a secondary wave, whose phase is arbitrarily related to those of the other waves on account of the irregularity in the distribution of the centres from which they emanate. In the theory of coronas, the intensities of the partial waves are supposed to be equal.

Any vibration may be represented by a straight line drawn from a given point, whose length is proportional to the amplitude, and angular position depends on the phase. When several vibrations affect the same point, the resultant may be found from the representative lines, according to the ordinary rules for compounding directed magnitudes. We have, therefore, to consider the resultant of a great number of lines radiating from an origin whose lengths are all equal and directions thoroughly irregular. From this statement, it appears at once that the question is perfectly indeterminate, and that no conclusion whatever can be drawn as to the probable value of the phase of the resultant, while all that could be inferred as to the amplitude relates merely to the order of its magnitude. Nevertheless, Verdet (vol. i., p. 297) comes to the conclusion that, the number of partial vibrations being n , the resultant is definitely $\sqrt{n}$ times greater than each component, provided, of course, that n is a large number. As to its phase, he is silent; and, if asked the question, would, I think, have found it difficult to explain how a resultant vibration could have a definite amplitude in conjunction with an indeterminate phase. I propose to point out what I conceive to be the error which vitiates his reasoning, and afterwards to show that, after all, his theory is substantially correct.

The component vibrations may be expressed by

$$\sin 2\pi \left(\frac{t}{\tau} \right), \sin 2\pi \left(\frac{t}{\tau} + \frac{\delta_1}{\lambda} \right), \sin 2\pi \left(\frac{t}{\tau} + \frac{\delta_2}{\lambda} \right), \dots\dots\dots$$

The intensity of the light corresponding to the sum of these is

$$\left(1 + \cos 2\pi \frac{\delta_1}{\lambda} + \cos 2\pi \frac{\delta_2}{\lambda} + \dots\right)^2 \\ + \left(\sin 2\pi \frac{\delta_1}{\lambda} + \sin 2\pi \frac{\delta_2}{\lambda} + \dots\right)^2,$$

or

$$n + 2 \cos 2\pi \frac{\delta_1}{\lambda} + 2 \cos 2\pi \frac{\delta_2}{\lambda} + \dots \\ \dots + 2 \cos 2\pi \frac{\delta_1 - \delta_2}{\lambda} + 2 \cos 2\pi \frac{\delta_1 - \delta_3}{\lambda} + \dots + 2 \cos 2\pi \frac{\delta_2 - \delta_3}{\lambda} + \dots$$

According to Verdet, this reduces to n when n is great enough, in consequence of the tendency of the series of cosines to zero. The inference is, I believe, unwarranted. All that we are entitled to say is, that when n is great, the sum of the cosines of the arbitrary angles tends to vanish, *in comparison with the number of terms of the series*, which is of the order n^2 . There is no reason for supposing that the series becomes small in comparison with n , as the argument requires.* The analysis, as it here stands, is taken from Billet ("Traité d'Optique physique," tom. i., p. 235). In Verdet's book there is, in addition to the above fallacy, a mistake in the work, which I need not further refer to.

I conclude, therefore, that the effect of the diffraction of a plane wave coming through a great number of small holes in an opaque screen is not correctly given. Yet the error disappears in the application to the theory of coronas as they occur in Nature. For Verdet's result comes only to this, that the sources of light are to be treated as

* It has been suggested to me that I have not sufficiently explained why the series of cosines does not tend to vanish when n is large enough. In order to examine this point more closely, let us, instead of comparing the phases of the second and succeeding vibrations with the first, refer them all to an external standard, so that the component vibrations are expressed by

$$\sin 2\pi \left(\frac{t}{\tau} + \frac{\delta_0}{\lambda}\right), \quad \sin 2\pi \left(\frac{t}{\tau} + \frac{\delta_1}{\lambda}\right), \quad \dots$$

The expression for the resulting illumination will be

$$\left(\cos 2\pi \frac{\delta_0}{\lambda} + \cos 2\pi \frac{\delta_1}{\lambda} + \dots\right)^2 + \left(\sin 2\pi \frac{\delta_0}{\lambda} + \sin 2\pi \frac{\delta_1}{\lambda} + \dots\right)^2.$$

The phases of the original vibrations being distributed with absolute irregularity, each series within brackets tends to zero; for the cosines or sines are, in the long run, as much positive as negative. The reason why the former series of cosines does not tend to vanish, but to the limit $-n$, is now evident. The reference of the phases of the succeeding components to the first as a standard has detracted something from the arbitrary character of the magnitudes which express them, so that the angles

$2\pi \frac{\delta_1 - \delta_0}{\lambda}, 2\pi \frac{\delta_2 - \delta_0}{\lambda}, \dots$ are no longer to be regarded as distributed absolutely

at random over the entire circumference. I hope that this explanation, which is somewhat difficult to put into words, though the idea is simple enough, will be considered sufficient. At the same time, I submit that the burden of proof lay with those against whom I am arguing, when once the absurdity of the conclusion was pointed out.

independent, and the illuminations due to them, and not the vibrations, are to be compounded. As it is commonly though not very correctly expressed, there is to be no interference. For this the sensible apparent magnitude of the sun, or other source of light, is an amply sufficient cause.

At page 106 of his excellent lectures, Verdet shows that the disturbance from the sun cannot be considered as a system of plane waves over a space greater than a circle of $\frac{1}{30}$ of a millimetre diameter. Between the vibrations at two points whose mutual distance is much greater than this, there is no permanent relation of phase. This shows that the vibrations corresponding to two holes in the imaginary screen cannot interfere regularly, but behave as if they were due to thoroughly independent sources of light.

Sir W. Thomson, Prof. Maxwell, and Mr. Strutt made some further remarks on the subject of the paper. Mr. Maxwell then gave a description of two singular solar halos which he had recently seen; and Prof. Adams, of King's College, gave some additional particulars in the case of one of the phenomena which had also been noticed by himself. Prof. Cayley, V.P., communicated an account of the following paper:—

On the Problem of finding the Circle which cuts three given Circles at three given Angles. By J. GRIFFITHS, M.A.

I shall first show that a circle which cuts three given circles S_1, S_2, S_3 at the given angles $\theta_1, \theta_2, \theta_3$, is always touched by a variable circle $\lambda S_1 + \mu S_2 + \nu S_3$, where λ, μ, ν are connected by a certain relation of the second degree.

To simplify our investigation, let us take the centre of the required circle, U say, as the origin of coordinates, so that, if R be its radius, we have for its equation $U = x^2 + y^2 - R^2 = 0$; and for those of the given circles S_1, S_2, S_3 , which are cut by U at angles $\theta_1, \theta_2, \theta_3$ respectively,

$$S_1 = x^2 + y^2 + 2g'x + 2f'y + R^2 - 2Rr_1 \cos \theta_1 = 0,$$

$$S_2 = x^2 + y^2 + 2g''x + 2f''y + R^2 - 2Rr_2 \cos \theta_2 = 0,$$

$$S_3 = x^2 + y^2 + 2g'''x + 2f'''y + R^2 - 2Rr_3 \cos \theta_3 = 0,$$

r_1, r_2, r_3 being the respective radii of S_1, S_2, S_3 .

Now let us consider the circle $\lambda S_1 + \mu S_2 + \nu S_3$. Its equation may be written in the form

$$(\lambda + \mu + \nu)(x^2 + y^2 + R^2) + 2\Sigma \lambda g'x + 2\Sigma \lambda f'y - 2R\Sigma (\lambda r_1 \cos \theta_1) = 0,$$

where $\Sigma \lambda g' = \lambda g' + \mu g'' + \nu g'''$, &c.;