DISCUSSION ON "ELECTRICAL MEASUREMENTS ON CIRCUITS
REQUIRING CURRENT AND POTENTIAL TRANSFORMERS."
FRONTENAC, N. Y., JUNE 30, 1909

C. H. Sharp: This important question of the measurement of ratio and phase-angle of transformers is beginning to receive the attention it deserves. I should like to call attention to similar work done at the Electrical Testing Laboratories under my direction by Wm. W. Crawford.

By the two-ammeter method of measuring the ratio of current transformers, it is possible to get quite accurate results, but their accuracy is limited by the steadiness of the available alternating-current supply. It is also desirable that the two instruments should have as nearly as possible the same natural frequency of oscillation.

Being dissatisfied with the results obtained by the two-ammeter method, we sought a method which would be free from

its disadvantages, and two years ago worked out the two-shunt method outlined by Mr. Robinson. This is essentially a bridge or zero method in which the fluctuations of the supply do not go into the result directly, and which has also the great advantage of giving the result in terms of the ratio of two resistances. We have developed also the shortcomings of this method, which are described below, and have devised means whereby they might be overcome and also a scheme for measuring the phase-angle of the transformer at the same time that its ratio of transformation is obtained.

The simple connection shown in Fig. 4 of Mr. Robinson's paper does not give a wholly exact value for the ratio of the transformers, for the reason that the primary and secondary currents are not in phase with each other and their phase-angle increases with decreased load. Referring to Fig. 1, we have e_p the fall of potential on the primary shunt, e_s that on the secondary shunt, and e_x the reactive potential difference which is present

even when $e_p = e_s$ numerically. Now, if the rotating commutator is adjusted so that its axis of commutation bisects the angle between e_p and e_s , the galvanometer measures the difference between the projection of e_p on the axis of commutation, and of e_s on the same axis. When the deflection of the galvanometer is reduced to zero by adjusting the secondary shunt, the ratio of shunt resistances gives very accurately the ratio of the transformer. The difficulty is to set the axis of commutation to approximately the right position. If this axis of commutation falls some distance outside the angle made by e_p and e_s , the ratio measured will be incorrect.

To obviate this difficulty, the reactive potential difference e_x must be nullified. The first method used for this purpose is illustrated in Fig. 2. In this method a fixed non-inductive resistance R'' is introduced into the secondary circuit, and this is shunted by a potentiometer arrangement R_s of 100 ohms,

in series with which is a variable standard of self-induction L by means of which the current through R_s may be brought into phase with that through R' .

The procedure then is as follows: the axis of commutation is brought as nearly as possible into the general direction of the vectors of the fall of potential of primary and secondary shunts, by noticing the position of the commutator which gives the maximum sensibility to the arrangement; that is, the position in which a given change in the secondary shunt throws the apparatus farthest out of balance. Having determined this position, and having adjusted R_s so that the galvanometer deflection is zero, the line of commutation is rotated through an angle of 90 deg. At this point the sensitiveness of the arrangement in measuring the inductive difference e_x is the maximum, and the deflection here observed is brought to zero by adjusting the value of the self-induction L . This operates to bring the

potential differences on the shunts substantially into phase with each other.

When this has been accomplished, the brush holder is rotated back into its original position and the adjustment of R_s is again made. The second value of R_s so found must be almost exactly correct since it is made when e_p and e_s are in phase with each other, and consequently the ratio of their projections upon any line gives the true ratio of the potential differences. The ratio of the transformer is then given by the expression:

$$\text{Ratio of transformation} = \frac{r}{R'} \cdot \frac{R''}{R'' + R_s} \cdot \frac{1}{\sqrt{1 + \tan^2 \alpha}}$$

where α , the phase-angle of the transformer is given by the relation,

$$\tan \alpha = \frac{2 \pi f L}{R'' + R_s}$$

The principal disadvantage of this method is that the ordinary Ayrton-Perry form of standard of self-induction is such that it is subject to influence by stray alternating fields, and a very feeble stray field would in the case of these measurements be sufficient to destroy the accuracy of the measurements. It is evident that in the foregoing work, not only has the ratio of the transformer been determined, but at the same time the phase-angle between primary and secondary currents has been measured.

Following this, a second method, shown in Fig. 3, was developed in which a variable standard mutual-induction was substituted for the variable standard of self-induction, and since a standard of mutual-induction can be made astatic so as not to be influenced by stray fields, this difficulty of the first method was overcome. The relations involved in applying this second method are rather complicated, and as a result the method has been abandoned for one illustrated in Fig. 4, in which the arrangement of cir-

cuits was suggested by Mr. F. K. Vreeland. As will be seen, the primary of the mutual-inductance is in series with the secondary of the transformer, while the secondary of this mutual-inductance is connected in series with the galvanometer. The operations are exactly as described in the first method but the formula is even more simple than in that method, while the trouble from stray field is avoided by using an astatic mutual-inductance.

The methods here given have the advantage over Mr. Robinson's method of not requiring two wattmeters which are accurate on low power-factors, together with a voltmeter and ammeter, three observers, and a steady current. By this method not only is the true ratio of the transformer obtained as a ratio of resistances, but the phase-angle is measured by actually nullifying the reactive potential difference which corresponds to it. With the exception of an ammeter to show approximately what the load is, no direct-reading instruments are required.

Moreover, the arrangement is independent of sensitive suspended instruments. It is quite feasible to use a resistance of very small fall of potential for the primary and secondary shunts and still have sensitiveness enough even when a pivoted direct-current galvanometer is used. On account of the character of the apparatus employed, it is possible by one of these methods to measure the ratio of a transformer installed on a switchboard and to do it with a high degree of accuracy. This at once extends the range of possibilities of this sort of testing beyond the walls of the laboratory—a very important consideration in many instances.

We have found that the rectifying commutator in the form in which Mr. Robinson describes it is not a success. As soon as the current becomes very small the contact resistance of brush with commutator is very uncertain, sometimes reaching a high value. We have obviated this difficulty by substituting a vibrating rectifying device operated by a cam on the shaft of

the motor. Straight pressure contact instead of rubbing contact is thereby obtained, and operates satisfactorily. The primary non-inductive shunts for this sort of work present very serious problems of construction as soon as heavy currents have to be dealt with, and they must be arranged to be cooled by some liquid.

A similar method can be used for the measurement of the ratio of potential transformers. As a current detector an alternating-current—direct-current voltmeter can be used, provided its connections are changed so that its field coils can be separately excited. For most purposes the sensitiveness of such an instrument is sufficient. The methods and apparatus for carrying them out as described in the foregoing have been developed in the Electrical Testing Laboratories quite independently of any similar work done elsewhere.

M. G. Lloyd: Mr. Robinson's table and diagrams will no doubt be useful as showing the general way in which those quantities change with the conditions of load.

I think it may be of interest to speak of some methods that have been developed at the Bureau of Standards and are now being used in this kind of work. Referring first to the current transformer and the diagrams in Figs. 2 and 3: we tried out the first method and found it objectionable, at least not quite as serviceable as the method indicated in Fig. 4, and similar to the one described by Dr. Sharp, in which only two shunts are necessary in the circuit of the transformer. In addition to the points raised by Dr. Sharp, one other is that while there is no trouble in getting accurate measurements with the arrangements shown in Fig. 2, since a dynamometer must be in the secondary circuit, it limits the load conditions which must be used; that is, the resistance and reactance cannot be cut down below the values necessary for the watt-dynamometer itself. Consequently, the secondary circuit cannot be made as small as it might be in practice, and therefore as small as it would be desirable to have the conditions in calibrating.

The alternative diagram in the lower left-hand corner of Fig. 4, is a method that we have found of more service. At the terminals 2, we connected one coil of the dynamometer; the other coil had current supplied to it in phase with the primary of the transformer. The result is that the potentials across the two resistances, R_p and R_s are opposite and not exactly in phase, their resultant being a small voltage in quadrature, and the dynamometer is made to read zero. In the case of any good transformer the ratio of resistances gives the ratio of the currents since the cosine of the small angle is a negligible factor.

We have found it simpler to obtain the phase-angle measurement almost simultaneously by using a somewhat different scheme. Instead of using a three-phase supply we have tried using a two-phase source of supply, and we have found that preferable. Using a two-phase supply we have a second dynamometer whose field is supplied with current in quadrature; we then apply the same electromotive force, which represents

the difference in the drop in the two shunts, R_p and R_s , to the other coil of the dynamometer having its field in quadrature. The deflection, then, measures a small quantity which, in ratio to the primary potential drop, gives the phase-angle of the transformer; so that by a simple throwing of the switch the second measurement is obtained.

The method with the mutual inductance, mentioned by Dr. Sharp, I believe is the same as that used at the National Physical Laboratory, in England, and described in their last annual report.

In regard to potential transformers, our method is very similar to that used by Mr. Robinson; that is, we balance a fractional part of the potential of the primary against the potential of the secondary. I need not speak of that in detail; suffice it to say that here again in obtaining the phase-angle we use the quadrature current as the auxiliary instead of the phase-shifting transformer, with the result that by simply throwing the same difference of potential on the second dynamometer, that quantity is measured directly and gives the phase-angle. These methods which we have finally found the most preferable were given several months ago in a paper read before another society.*

As Mr. Robinson has pointed out, the transformer is an instrument of precision when used under proper conditions. There is one way, however, in which an error may be introduced even when the transformer has been calibrated and when it is not subject to such errors as aging. That is due to the fact that iron requires a different magnetizing current in some cases from what it needs in others. Such a case can be brought about by magnetizing highly in one direction and then subsequently breaking the magnetizing current. It will be then in a condition that is not neutral. If that operation is performed with the current transformer, and its ratio then measured, it will be found that the ratio will not be of the same value as when the iron is in a neutral condition.† That does not apply to a potential transformer, because there the magnetizing current comes in only as a second-order effect. In the current transformer, where the ratio is thrown off from the nominal value entirely by the exciting current, this effect comes in very strongly and may produce an appreciable error.

Hence when calibrating the current transformer and using it afterwards, one should be sure that a definite condition of the iron exists. That can be arranged by putting the core through a demagnetizing process before using it. Such an abnormal condition may easily result, because if the secondary happens to be open-circuited we have the condition spoken of. The magnetism of the core jumps to a high point and if the flux is suddenly cut down by closing the secondary, it is brought down without the demagnetizing process.

* Agnew and Fitch, *Phys. Rev.* 28, p. 473; June, 1909.

† Attention was first called to this by Agnew and Fitch loc. cit.

In regard to which angle should be called positive and which negative, I wish to take issue with Mr. Robinson. The point mentioned earlier, depending upon whether we shall call a lagging angle positive or negative, I will not go into because that is a point which depends largely upon the way in which one has been trained. But coming to the specific case of designating these angles, it seems to me that the method used in this paper is inconsistent and undesirable. It will be noticed in the case of the current transformer that when the secondary leads, the angle is called positive, and in the case of the potential transformer it is called negative. There is an inconsistency between the two cases. The reason for doing this is to make it possible to take the sum of the two angles and determine the apparent power-factor. It seems to me that is confusing. Anyone understands that if the current and the potential are thrown off simultaneously in the same direction; that is, if each one is caused to lag by the same number of degrees, the reading of the wattmeter will not be altered. The first thing that naturally occurs to one is that the difference of the two phase-angles must be taken. So it would be more logical and less confusing to call the phase-angle positive in one definite direction, preferably when the secondary leads.

In the formula on the fourth page, I would suggest that having indicated which angles are positive and which negative, it would be better to omit the combined sign plus and minus and let it stand as plus.

In an early part of the paper, under the things to be investigated, Mr. Robinson has mentioned the effect of wave-form. I think he takes up all the other points, but he says nothing further about this. Over a year ago I published an article* giving some experiments on that point. The results showed that with any distortion that would be likely to be present on an ordinary circuit, the ratio is not affected. There is a small effect with very large distortion. In a book by Arnold,† this section of which was written by LaCour, there is a theoretical discussion on this subject, based on the assumption that harmonics can affect the result only by their value and regardless of their phase. In other words, if the same harmonics are in such phase as to produce a flat wave and a peak wave, the result should be the same. This is not in agreement with the experimental facts.

L. W. Chubb: I would like to ask if the transformers giving the curves shown in Figs. 7 and 8 were of special design for the test, or whether they were commercial transformers, and what were the load conditions 1, 2, 3, 4, and 5?

I would also like to ask Dr. Sharp if the extra circuit containing the resistance and inductance as shown in his diagram would take enough current to prohibit getting any desired conditions of instrument load during a test?

* *Electrical World* 52, p. 845, 1908. *Bull. Bur. Stds.* 6, No. 1, 1909.

† *Wechselstromtechnik*, Vol. 1, p. 254.

Albert F. Ganz: I wish to ask Mr. Robinson whether he has any method to suggest for measuring the phase-angle α in the wattmeter itself, also whether in a properly designed transformer, β is not always positive and γ negative? Is there anything about the tests that will enable one to tell whether these angles are positive or negative?

C. H. Sharp: Answering Mr. Chubb's question: in this method of testing current transformers the high relative sensibility of direct-current galvanometers can be utilized. Consequently it is not necessary that the fall of potential in these shunts should be large. As a practical matter we work with one-quarter of an ohm in the secondary circuit of the transformer, but we could just as well use two-tenths or one-tenth. The inductance, is entirely negligible.

L. T. Robinson: In reply to Dr. Sharp, I would say that it is not necessary to take into account the small error due to the fact that the primary and secondary currents are not exactly in phase. I think that the somewhat elaborate arrangements which he has shown are an entirely unnecessary refinement. By reference to a table of cosines it will be seen that if the transformer is reasonably good; that is, if the angles are nearly as they should be, it will be entirely beyond the accuracy wanted in a commercial test. If the angle β is as much as 2 deg. the necessary correction ($\sec \beta - 1$) amounts to only about 0.05 per cent. If desired, correction can easily be made for this small angle, but if a transformer were brought to me for test, and I had to take this correction into account, I would not make a complete test. The transformer would evidently not be a good one.

I think that the method suggested for measuring the phase-angle is very ingenious. I have not had the time to try it but my opinion is that it would not allow a determination to be made with the same degree of precision that could be obtained by using the method given in the paper.

The question was raised as to the number of observers and the time required to complete the work. I would say frankly that I expected better results with the resistance methods that have been described, and the only reason I can give for not having adopted long ago for commercial work what will no doubt prove to be the final method is, that where one has so much to do it is difficult always to bring into daily use methods which promise to be better. I am quite willing to say, however, that from my point of view the length of time for these tests is quite satisfactory.

The commutator I myself do not like. We did try it and it will work, but I think I would prefer some other method.

Dr. Lloyd referred to Figs. 2 and 4. I prefer Fig. 4 unless some arrangement like that shown in Fig. 6 can be used. I believe this will be the ultimate outcome.

It is true that the secondary load conditions are limited by the dynamometers. From my own point of view this is no ob-

jection, because we can go to the limits of the smallest connected load that it would ever be necessary to use, and there is no use in going further. For precision work we would not use a transformer with less on it than one portable ammeter and one portable wattmeter, and the combined load of these two is just about equal to what we can easily get by the methods we have used.

As to the effect of the altered magnetizing current due to the previous state of the core affecting the ratio, I would say that this fact has been appreciated for some time. I thought it was so well known that I did not need to refer to it. All users of current transformers have for some time been cautioned against leaving the secondary circuit open.

M. G. Lloyd: The same effect is brought about if the dynamometer is calibrated with the secondary or primary in circuit with direct current.

L. T. Robinson: I know that, but we do not do it.

As to the angles and sines, I appreciate this fact, that it may not be logical and I do not offer it as such; but after one has carefully selected the best method and no one else is doing the same thing at that time, the only way is to proceed and afterward tell how it was done and what one's opinion of it is. I am perfectly willing to make a change if we can come to any satisfactory understanding about what it is best to do.

In reply to Mr. Chubb's question as to the loads used: I described their properties though perhaps not in sufficient detail. The loads are given in electrical terms and below is given a brief description of the actual loads used.

The tests of transformers were all made on commercial specimens. Professor Ganz asked how to determine the angle α . I would say that one cannot tell everything in a single paper. I shall be glad to tell him privately what we do.

J. Dalemont (by letter): I wish to call attention to the influence of the core-loss and magnetic leakage upon current transformers. I have recently made some measurements on two current transformers having iron cores of the same cross-section and nearly the same length, but of different form: one as in Fig. 1, the other as indicated in Fig. 2.

On one of the legs of the prismatic core, the following sets of coils were mounted in succession:

1 { Primary; 5 turns.	2 { Primary; 41 turns.
{ Secondary; 460 turns.	{ Secondary; 460 turns.

The transformer was supplied with an alternating current of almost sine shape and a frequency of 36 cycles. On the torus core, a secondary winding of 460 turns was wound uniformly. The primary wire carrying the current was placed in the direction of the axis of the torus, perpendicular to the plane of the paper and was supplied with the same current as before. The secondary circuits were closed on standard ammeters. Table I gives the values of the primary and secondary currents in each case.

It will be seen that in the first case (ratio of 5 to 460, prismatic core), the average of the primary and the secondary currents is 83.4, while the ratio of the number of turns is 92; with the other coils (ratio 41 to 460) on the same core, the average ratio of the currents is 12.2, while the ratio of the turns is only 11.2.

FIG. 1

Taking the relation between the ampere-turns, we have:

$$\frac{n_2 I_2}{n_1 I_1} = \frac{4 \pi n_2 p}{\sqrt{R^2 r_2^2 + (4 \pi n_2^2 + l_2 R)^2 p^2}}$$

where p is the frequency,

FIG. 2

R = the magnetic reluctance of the core.

r and $p l$, the resistance and the reactance of the secondary circuit.

It will be seen that this ratio is equal to 1 when the losses are negligible.

TABLE I

Ratio $\frac{5}{460}$		Ratio $\frac{41}{460}$		Ratio $\frac{1}{460}$	
Core 1		Core 1		Core 2	
I_1 ampere	I_2 ampere	I_1 ampere	I_2 ampere	I_1 ampere	I_2 ampere
93	1.09	15.5	1.23	567	1.15
116	1.37	19.8	1.58	755	1.58
135.2	1.61	23.4	1.89	936	2
150	1.79	27.7	2.23	1021	2.18
170	2.03	32.3	2.62	1098	2.32
184	2.19	35.8	2.91	1175	2.52
205	2.45	39.2	3.17	1265	2.68
233.3	2.77			1355	2.87
263	3.13			1495	3.17
275	3.25			1670	3.54
299	3.53			1772	3.77

TABLE II

Curve 1		Curve 2		Curve 3	
$n_2 I_2$	$\frac{n_1 I_1}{n_2 I_2} - 1$	$n_2 I_2$	$\frac{n_1 I_1}{n_2 I_2} - 1$	$n_2 I_2$	$\frac{n_1 I_1}{n_2 I_2} - 1$
502	-.073	565	.122	529	.048
632	-.082	727	.115	727	.036
742	-.088	870	.102	920	.020
825	-.092	1025	.098	1000	.024
936	-.093	1205	.100	1068	.032
1008	-.088	1339	.095	1160	.030
1130	-.092	1458	.095	1232	.028
1275	-.087			1320	.026
1440	-.086			1958	.025
1498	-.082			1628	.025
1625	-.080			1732	.022

The curves in Fig. 3 show the variation of the ratio

$$\frac{I_1 n_1 - I_2 n_2}{I_2 n_2}$$

plotted to $I_2 n_2$ as abscissas. The advantage of the torus form for the magnetic core is thus clearly demonstrated by these results. On the other hand, a rectangular section for the magnetic core is preferable to a square section, but with the low values of induction used in such apparatus (2750 c. g. s.), the difference is very small. For a comparison of the two cores from this point of view, the numerical values of the above expression are given in Table II.

FIG. 3

Albert F. Ganz (by letter): Mr. Robinson shows that where a wattmeter is used with current and potential transformers, the wattmeter reading must be multiplied by the factor

$$\frac{\cos \theta}{\cos [\theta - (\alpha + \beta + \gamma)]}$$

in order to correct for the error due to phase displacement between current and voltage caused by the wattmeter itself, and by the transformers where

θ = phase angle between current and voltage in the main circuit assumed *positive* when the current is *lagging*.

α = equivalent angle of phase displacement between current in the potential coil and voltage across the potential terminals of the wattmeter, assumed *positive* when the current *lags* behind this voltage.

β = angle of phase difference between the primary and the secondary current in the current transformer, assumed *positive* when the secondary current *leads* the primary current reversed.

γ = angle of phase difference between the primary and the secondary voltage in the potential transformer, assumed *positive*, when the secondary voltage *lags* behind the primary voltage reversed.

Mr. Robinson has also given convenient tables of correction-factors for various power-factors of the load, and for phase displacements up to 5 degrees. The correction-factor as given by the above formula, and the tables as given by Mr. Robinson, are directly applicable to a single-phase circuit, in which case θ of the formula is the angle whose cosine is the power-factor of the load.

Where three-phase power is measured by two wattmeters, the indications of these wattmeters are proportional, not to $\cos \theta$, but to $\cos (30^\circ + \theta)$ and $\cos (30^\circ - \theta)$, respectively. Suppose the three-phase power to be delivered over three line wires, 1, 2 and 3, and let the two wattmeters be connected with their current coils in lines 1 and 3 and their potential coils across 1 and 2, and 3 and 2, respectively. Assuming the load to be balanced, and taking θ the angle whose cosine is the power-factor of the load, then the wattmeters will give indications equal to

$$P_1 = E I \cos (30^\circ - \theta),$$

and

$$P_3 = E I \cos (30^\circ + \theta).$$

Where current and potential transformers are used with two wattmeters, the displacement of phase in the transformers affects the angle $(30^\circ - \theta)$ in wattmeter #1, and the angle $(30^\circ + \theta)$ in wattmeter #3. With a lagging current in the load—that is, with θ positive— P_3 will be the smaller wattmeter reading and P_1 the larger wattmeter reading.

To correct for phase displacement, the reading of wattmeter #1 must be multiplied by

$$\frac{\cos (30^\circ - \theta)}{\cos [30^\circ - \theta + (\alpha + \beta + \gamma)]},$$

and that of wattmeter #3 must be multiplied by

$$\frac{\cos (30^\circ + \theta)}{\cos [30^\circ + \theta - (\alpha + \beta + \gamma)]}.$$

In applying Mr. Robinson's tables to three-phase power measured by two wattmeters, the θ used in the tables must be taken equal to 30 degrees minus the angle whose cosine is the power-factor of the load, for wattmeter reading #1; and equal to 30 degrees plus the angle whose cosine is the power-factor of the load, for wattmeter reading #3.