XVII. On the cooling of cylinders in a stream of air

J. Alfred Hughes B.Sc. E Coy R.A.M.C.

To cite this article: J. Alfred Hughes B.Sc. E Coy R.A.M.C. (1916) XVII. On the cooling of cylinders in a stream of air , Philosophical Magazine Series 6, 31:182, 118-130, DOI: [10.1080/14786440208635482](https://doi.org/10.1080/14786440208635482)

To link to this article: <http://dx.doi.org/10.1080/14786440208635482>

Published online: 08 Apr 2009.

Submit your article to this journal [↗](#)

Article views: 8

View related articles [↗](#)

Citing articles: 13 View citing articles [↗](#)

On using the formula of Gauss

$$\psi(z) = \int_0^\infty \left\{ \frac{e^{-t}}{t} - \frac{e^{-zt}}{1-e^{-t}} \right\} dt,$$

we get, without much difficulty,

$$\begin{aligned} \pi T_n = & 2n \left[\log \left\{ \lambda n \cot \left(\frac{1}{2} \lambda \pi \right) \right\} - \frac{1}{2} \psi(1 - \lambda n) - \frac{1}{2} \psi(1 + \lambda n) \right] \\ & + \sum_{m=1}^n \frac{(-)^m B_m}{(2m)! (2n)^{2m-1}} \left\{ \psi^{(2m-1)} \left(\frac{1}{2} - \frac{1}{2} \lambda \right) + \psi^{(2m-1)} \left(\frac{1}{2} + \frac{1}{2} \lambda \right) \right. \\ & \left. - \psi^{(2m-1)} \left(1 - \frac{1}{2} \lambda \right) - \psi^{(2m-1)} \left(1 + \frac{1}{2} \lambda \right) \right\}, \end{aligned}$$

the remainder after r terms having the same sign and being numerically less than the $(r+1)$ th term; where λ is equal to $\beta/(2\pi)$.

The corresponding series occurring in the transverse component of force is

$$\begin{aligned} U_n = & \sum_{m=1}^{n-1} \cos \left(\frac{m\pi}{n} - \frac{1}{2} \beta \right) \operatorname{cosec}^2 \left(\frac{m\pi}{n} - \frac{1}{2} \beta \right) \\ = & 2 \frac{\partial T_n}{\partial \beta} = \frac{1}{\pi} \frac{\partial T_n}{\partial \lambda}; \end{aligned}$$

this may be determined from the asymptotic expansion for T_n , which may legitimately be differentiated term by term with respect to λ .

XVII. *On the Cooling of Cylinders in a Stream of Air.* By Pte. J. ALFRED HUGHES, B.Sc., E Coy, R.A.M.C., formerly Research Student, University College of North Wales, Bangor*.

THE interchange of heat between a solid and a moving stream of gas is a subject of considerable technical importance, and also not devoid of scientific interest. A considerable amount of work has been done on the cooling of thin wires† in a current of air, mainly with the object of constructing instruments for the measurements of air-velocity. These experiments have shown that the heat lost by the wire is proportional to the difference of temperature and to the square-root of the velocity. There are, however, no data available relating to the convection of heat from bodies of large diameters; and the following experiments were undertaken with a view of throwing some light on the problem of convection in these cases.

* Communicated by Prof. E. Taylor Jones, D.Sc.

† King, Phil. Trans, 1914, p. 373. Morris, 'Electrician,' Oct. 4, 1912.

Apparatus.

The measurements were carried out on a series of copper tubes all of the same length, but with external diameters ranging from 4.33 cm. to 5.06 cm. These tubes, of about a metre in length, were placed vertically in the wind-channel, and were heated by steam from a boiler placed on top of the channel; after a steady temperature was attained, the weight of the water which condensed in the cylinder in a given time measured the heat which it lost through convection and radiation.

The wind-channel, fig. 2, consisted of a wooden tunnel placed horizontally, which was of square section, of side 3 feet, and was 10 feet long. This terminated in a rectangular box of greater section in which the fan rotated.

The fan was made by attaching four narrow steel blades to the axle of a three-phase motor, the speed of which could be regulated (*a*) by a rotor rheostat, and (*b*) by varying the voltage of the alternator supplying the current. A good range of wind-velocity could thus be obtained. The velocity of the air currents produced by the fan was measured by the ordinary Pitot-tube and gauge. This consisted of two glass tubes of $\frac{3}{16}$ inch bore, the dynamic tube facing the wind, and the static tube being placed with the plane of its orifice parallel to the direction of the flow. These tubes were placed close to one another in the centre of the channel, immediately in front of the cooling cylinder. By means of rubber tubing they were connected to a water-gauge, the difference of level in which measured the difference of pressure, which was proportional to the square of the velocity as given by

$$v = \sqrt{2gh},$$

h being the pressure difference in metres of air, and *v* the velocity in metres per second. The difference between the two sides of the U-tube was measured by a cathetometer microscope reading to $\frac{1}{100}$ of a millim. The gauge-tubes were 2 cm. wide so as to avoid surface-tension errors. Large oscillations were produced in the liquid in the gauge on account of variation in the wind-pressure, and this caused a difficulty in measuring the wind-velocity. The oscillations were damped, and the amplitude reduced by loosely stopping the bottom of the U-tube with cotton-wool. In some of the later experiments, thick paraffin was adopted as the liquid in the gauge. With this, the difference of level was increased on account of the smaller density of the liquid, while the oscillations were reduced by reason of its greater viscosity.

The Pitot-tube determined the wind-velocity at the middle of the channel; and since an average value of the velocity over the cylinder was required, a determination was made of how the wind-velocity varied along a vertical line through the position of the Pitot-tube. A reading was taken with the Pitot-tube in its ordinary position: then readings were taken for the same wind velocity at different points across the channel. Thus an average value was calculated for the channel, and this was done for different velocities of the wind. A graph was plotted (fig. 1) showing the relation between the Pitot-tube reading at the centre of the channel and the average reading for the section. From this an average value can be read for any value of the axial wind-velocity.

Fig. 1.

The heat lost was determined by finding the weight of water condensed. The arrangement of the steam generator is shown in fig. 2. The boiler A is placed on the top of the channel, and the steam passes into the top of the tube through the water-trap B placed directly over the tube. The whole was shielded from air-currents by an asbestos cover. With cylinders of the largest diameters and with the stream-line tubes, a metallic steam-trap covered with cotton-wool was used, and the tubes were soldered to the bottom of the can as shown in the diagram.

The condensed water was collected in a beaker placed underneath the tubes, a glass funnel being attached to the bottom end of the copper tube to ensure that all the condensed water should be caught in the beaker; the excess of

Fig. 2.

Fig. 3.

steam which came out with the condensed water escaped to the air past the sides of the tube. The end of the copper tube and the funnel were covered with cotton-wool. A wooden box was built round this receiving apparatus beneath the wind-channel, to shield it from air currents produced by the fan.

The experiment was carried on in the following manner. The water was boiled and the steam allowed to pass through the copper tube for about five minutes, to ensure that the steady temperature had been attained.

The first reading of the Pitot-tube was taken, the motor started, and after a short interval, a weighed beaker was placed underneath the end of the cylindrical tube, and the stop-watch started. Readings of the water "head" in the Pitot-tube gauge were registered, and after 10 minutes the beaker was taken away, and the motor stopped. The second zero of the gauge was taken, and the beaker and water weighed. The temperature of the air and the atmospheric pressure were also taken.

In the case of the large cylinder, 20 cm. length, 15.5 cm. diameter, the arrangement of the apparatus is shown in fig. 3. The condensed water was allowed to collect in the cylinder C itself, the whole being suspended from a balance B placed on the top of the channel, the steam connexion being made by rubber-tubing R. When the balance-beam is down, the cylinder rests on a stand S in the channel; the excess of steam escapes through the exit E.

The water condensed, which measures the heat lost, is estimated by taking the weight of the cylinder at the beginning and end of the interval of time, viz. 10 minutes. Steam is passed through for about 5 minutes, and after the steady temperature has been attained, the weight is taken and the motor started. After 10 minutes the motor is stopped and the weight again taken. The weight of the water condensed will be the sum of three parts: (1) that condensed by convection and radiation in still air in the interval from the first weighing till the motor was started; (2) the condensation during the wind-current; (3) the condensation in still air during the interval between the stopping of the motor and the time when the final weight is taken. The "water equivalent" of the excess of time above 10 minutes is found, and this subtracted from the total weight gives the weight of water condensed by the wind alone. Still-air condensation is small, as can be seen from Table II., and therefore no serious error is introduced by this method.

Corrections.

Most of the heat is lost from the cooling cylinders by convection, but some also by radiation. A correction can be applied for the heat lost by radiation, and this subtracted from the total loss of heat gives the heat lost by convection alone. The radiation correction was calculated from the equation

$$R = k(\theta^4 - \theta_0^4),$$

where R is the heat lost by radiation;

k the coefficient of emissivity in gram-calories per sec. per sq. cm.;

$\theta - \theta_0$ the excess of temperature of the hot cooling body above the temperature of the surroundings.

k^* was calculated from Bottomley's results to be $\cdot 63 \times 10^{-12}$ gm.-cals. per sec. per sq. cm. for tarnished copper;

* Phil. Trans. vol. clxxxiv. p. 591 (1893).

and therefore R per cm. length

$$= k \frac{A}{l} t (\theta^4 - \theta_0^4),$$

A = area in sq. cms.

l = length in cms.

t = time in secs.

The values obtained are given in Table I.

TABLE I.

R (cals. per cm. length).	Diameter of Cylinder.
4 calories.	·43 cm.
12·4 „	·81 „
28·7 „	1·93 „
75 „	5·06 „
234·3 „	15·5 „
113 „	Stream-line Section.

It has been assumed that the temperature of the outside surface of the cooling cylinder was the same as the temperature of the steam. This is practically true, as can be shown by calculating the value of $\theta_2 - \theta_1$ from

$$(\theta_2 - \theta_1) = \frac{Hd}{2\pi rkT},$$

$\theta_2 - \theta_1$ being the drop of temperature in the tube,

H the largest value of the heat which passes across the tube per cm. length in time T,

r the mean radius = 15·4 cm.,

d the thickness = ·2 cm.,

k the conductivity of copper.

The value of $\theta_2 - \theta_1$ is ·03° C., so that the temperature of the outside of cylinder can be taken to be 100° C.

The quantity of water condensed depends on the density and the temperature of the air, so to obtain uniform results a correction was applied, and the quantity of water was reduced to a standard temperature and density.

Results.

Experiments were performed with five copper cylinders of diameters ·43, ·81, 1·93, 5·06, 15·5 cm. respectively, and the results obtained for the loss of heat in calories per cm. length (H) and the velocity (V) in metres per sec. are given in Table II.

TABLE II.

Loss of Heat (H) and Velocity (V) results for Cylinders.

H = Cals. per cm. length. R = Radiation in cals. per cm. length.
V = Velocity in metres per sec.

Cylinder (1). $R = 4$. Diam. = 43 cm.		Cylinder (2). $R = 12.4$. Diam. = 81 cm.		Cylinder (3). $R = 28.7$. Diam. = 1.93 cm.		Cylinder (4). $R = 75$. Diam. = 5.06 cm.		Cylinder (5). $R = 243$. Diam. = 15.5 cm.	
H.	V.	H.	V.	H.	V.	H.	V.	H.	V.
68.7	0	82.9	0	87.4	0	120.8	0	652	0
164.1	3.311	206.4	3.651	249.2	2.338	379.3	2.177	1752	3.666
166.1	4.252	279.8	7.034	292.3	3.036	461.4	3.333	2289	4.908
191.8	5.484	312.4	7.964	304.7	3.549	501.9	3.751	245.1	5.396
186.6	6.148	313.6	8.248	337	4.1	602	4.776	2620	5.908
209.9	7.452	316.9	8.396	335.7	4.144	633.7	4.944	2507	5.936
216.6	7.720	315.1	8.456	364.6	4.404	686.5	5.540	2813	7.132
220.2	8.136	335.9	9.108	406.9	5.496	687.8	5.744	3081	8.284
239.8	8.668	335.5	9.444	453	6.648	736	6.380	3165	8.756
237.9	8.708	350	10.132	462.7	7.364	753.8	6.6	3498	9.804
		351.8	10.740	509.7	8.036	812.6	7.284		

Cooling of Cylinders in a Stream of Air.

125

253	9.60	357.2	10.484	498.4	8.272	835.5	7.384	3405	9.824
248.4	9.88	356	10.872	500.1	8.336	860.8	7.832	3659	10.108
266.4	11.252	371	10.772	528.9	9.084	931	8.668	3633	10.16
270.3	11.376	364	11.284	553.7	9.38	904	8.752	3766	10.70
288.5	12.276	374.8	11.48	563.9	9.488	935	9.212	3766	10.94
292.3	12.492	376.6	11.412	554.5	9.528	973	9.576	3974	11.028
291.6	13.024	374.7	11.484	556.2	9.528	992	9.672	4072	11.852
		392.2	11.716	571.1	10.304	1012	9.824	4453	12.492
		390.9	11.916	589.2	10.492	1033	10.132	4544	12.748
		405	12.34	623.5	11.044	1066	11.048	4974	13.888
		400.3	12.836	624.1	11.064	1096	11.364	5275	14.504
		404.9	13.012	612	11.12	1090	11.032		
		413	13.224	638.6	11.604	1124	11.56		
		434.8	13.912	642.7	12.056	1164	12.036		
		427	14.088	661.7	12.3	1192	12.536		
		435.8	14.008	670	12.596	1180	12.78		
				672.6	12.412	1219	12.936		
				685	13.008	1232	13.204		
				684	13.112	1259	13.64		
				692.4	13.272	1298	14.312		
				694.9	13.052				
				731	13.812				
				705	13.896				
				732.8	14.396				

These are also shown in curves (fig. 4), the loss of heat having been plotted against the velocity.

Fig. 4.

These curves show that the heat-loss is a function of the velocity which may approximately be represented by:

$$H = kv^n.$$

In order to find the index n , $\log H$ is plotted against $\log V$, and a straight line drawn through the points. The inclination of this straight-line graph gives n .

The values of n obtained for the various cylinders are exhibited in Table III.

TABLE III.

n .	Diameter.
.55	.43 cm.
.6	.81 "
.65	1.93 "
.7	5.06 "
.98, .6 at low velocities.	15.5 "

The above table shows that n increases from .5 for cylinders of very small diameter to 1 for cylinders of large

diameter, thus bringing into agreement the two formulæ which had been obtained by different experimenters, $H = ktv$ given by Reynolds, Nicholson, Williams for large bodies, while Boussinesq, Morris, King arrived at the formula $H = kt \sqrt{v}$ for wires. In the case of those cylinders, with which a number of readings at fairly low wind-velocities have been obtained, there are indications that the value of n is much smaller. With the cylinder of diameter 1.93 cm., n at very low velocities is .57, while at higher velocities n is .65; and with the large cylinder of diameter 15.5 cm. $n = .6$ at low wind-velocities, whereas at high velocities it is .98.

Variation of Heat-Loss with Diameter.

The way in which the heat lost varies with the diameter can be obtained by comparing the heat-loss from the different tubes at the same velocity. These values are plotted in fig. 5, and it will be seen that the heat lost is proportional to $d^{.57}$ at all velocities.

Fig. 5.

Stream-Line Section.

In several technical applications it is important that the cooling system should have as small an air resistance as possible, and accordingly a few experiments were made on a tube of stream-line section with the object of finding its efficiency for cooling purposes. The perimeter of the cross section of the tube was 23.1 cm., its maximum thickness being 2 cm.

Experiments were performed with the tube placed thus

and a third set of readings was obtained with the tube in the second position, with a metallic strip, $\frac{1}{2}$ inch in width, placed in front. This was caused to oscillate by the wind and produced irregular convection. The results are given in Table IV., and they are also shown in curves in fig. 6.

TABLE IV.

Loss of Heat (H) and Velocity (V) results for Stream-Line Tube.

H = Calories per cm. length. V = Velocity in metres per sec.

R = Radiation corr. in cal. per cm. length = 113.

(1) Tube placed Direction of wind.		(2) Tube placed Direction of wind.		(3) Tube placed with oscillating strip in front.
H.	V.	H.	V.	H.	V.
137.3	0	136.2	0	136.2	0
136.2	0	137.3	0	137.3	0
482.5	3.2512	553.1	1.912	686.7	2.285
520.6	3.6304	628.9	2.341	759	2.821
591.9	4.304	628.9	2.555	918	3.502
635.7	4.528	675.4	2.704	984	3.908
645.4	4.752	720.3	2.890	1032	4.612
680.9	5.560	795.0	3.254	1099	5.184
732.3	5.956	930	4.228		
797.3	6.576	1080	5.560	1180	6.352
793.3	6.652	1140	6.228		
823.5	7.060	1187	6.352	1317	7.856
860.1	7.292	1190	6.652	1354	7.980
917	7.780	1267	6.920	1417	8.656
903	8.016	1405	8.300	1490	9.004
889	8.160	1486	9.076	1627	10.692
898	8.420	1594	10.784	1682	11.196
966	9.216	1614	11.116	1778	12.132
973	9.360	1614	11.468	1851	13.212
1071	10.080	1610	10.928	1905	14.872
1084	10.464	1682	12.172		
1060	10.280				
1116	11.304				
1139	11.376				
1135	11.456				
1255	13.332				
1231	13.504				

Fig. 6.

The values of n obtained were :—

- (1) $n = .67$,
- (2) $n = .62$,
- (3) $n = .61$.

These results show that the heat lost from the stream-line tube is slightly greater than that lost from a circular tube of same total area, but much greater than from a circular tube of the same width or same air resistance. This will be more clearly shown by the following comparative values.

- (1) Heat-loss from stream-line tube at velocity 13 metres per sec. = 1735 cal. per cm. length.
- (2) Heat-loss from cylinder of same diameter as the maximum thickness of stream-line tube at velocity 13 metres per sec. = 695 cal. per cm. length.
- (3) Heat-loss from a cylinder of the same total area (7.3 cm. diam. approx.) = 1476 cal. per cm. length at velocity 13 metres per sec.
- (4) Heat-loss from cylinder of the same air resistance as stream-line tube ($\frac{1}{4}$ diam. = .3 cm.) at 13 metres per sec. = 233 cal. per cm. length.

The curves in fig. 6 also show that the cooling is much greater when the stream-line tube is placed in position (2)

than it is when placed in position (1), and with the oscillating strip the heat-loss is increased at low and very high velocities.

It is evident, then, that as a cooling apparatus a stream-line tube is in every respect more efficient than a circular one.

The above experiments were carried out in the Physical Laboratory of the University College of North Wales, and my best thanks are due to Professor E. Taylor Jones for placing the necessary apparatus at my disposal, and to Mr. W. E. Williams, B.Sc., A.F.Ae.S., for suggesting the subject and for giving most valuable help and advice.

XVIII. *An application of Nomography to a case of Discontinuous Motion of a Liquid.* By W. B. MORTON, M.A., and ELEANOR J. HARVEY, M.A., *Queen's University, Belfast**.

THE problem of the impact, in two dimensions, of a jet of frictionless liquid on a plane strip, when both jet and strip are of finite width, can be solved by the method of conformal representation. The solution was given by Prof. Love†. The equations which embody the result of the analysis are too complicated to admit of arithmetical calculation of special cases. It is the purpose of this note to point out how numerical results can be obtained by means of the graphical methods developed by M. d'Ocagne.

Fig. 1.

Taking the width of the undisturbed stream before incidence as unity (fig. 1), the data of the question are the

* Communicated by the Authors.

† Proc. Camb. Phil. Soc. vii. p. 175 (1891).