Philosophical Magazine Series 6

ISSN: 1941-5982 (Print) 1941-5990 (Online) Journal homepage: http://www.tandfonline.com/loi/tphm17

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To cite this article: G.T. Bennett (1905) LXXVII. The parallel motion of Sarrut and some allied mechanisms , Philosophical Magazine Series 6, 9:54, 803-810, DOI: 10.1080/14786440509463333

To link to this article: http://dx.doi.org/10.1080/14786440509463333

Published online: 15 Apr 2009.

LXXVII. *The Parallel Motion of* Sarrut *and some Allied Mechanisms.* *By* G. T. Bennett, *Emmanuel College, Cambridge**.

§ 1. ANY student of mechanism who has gained his knowledge of the subject from the current text-books might be pardoned for supposing that the cardinal problem of obtaining exact rectilinear motion by the use of linkwork was first solved in the year 1864 by Peaucellier. Most of the works which deal with the subject, after describing the approximate "parallel motions" of Watt and others, place first in their account of the exact rectilinear motions now known a discussion of the inverting mechanism of Peaucellier. The *Encyclopædia Britannica* (vol. xxii. p. 512), more explicit, and expressing certainly the common view, states† that "it was for long believed that the production of an exact straight-line motion was impossible until the problem was solved by the invention of the Peaucellier cell." When this discovery had become known, the subject of linkworks attracted much fruitful attention; and other solutions of the problem, some requiring a smaller number of pieces, were soon found. These mechanisms, however, gave rectilinear motion only to a single point: (or, more strictly, to all points of a line, in one of the pieces, parallel to the hinge-lines; or else to all points of a hinge-line connecting two pieces). There remained yet to be solved the more complete problem, embracing and including the other, of giving rectilinear translational motion to a whole piece. This also has since been contrived in several ways; all, however, more or less complicated. It seems therefore very desirable to point out what appears to have escaped notice or to have been entirely forgotten: namely, that this last problem itself was solved exceedingly simply by Sarrut in the year 1853, eleven years before Peaucellier's announcement of his inverting cell.

§ 2. Sarrut's description of his mechanism appears in the *Comptes Rendus* of the Académie des Sciences for 1853 (vol. xxxvi. p. 1036); and later (p. 1125) is given the report of Poncelet upon Sarrut's mechanism. The Academy agreed "de lui accorder son entière approbation et de decider que le court mémoire qui en contient l'explication sera inséré dans le Recueil des Savants étrangers, avec la description du modèle que l'auteur y a joint." But it seems never to have appeared.

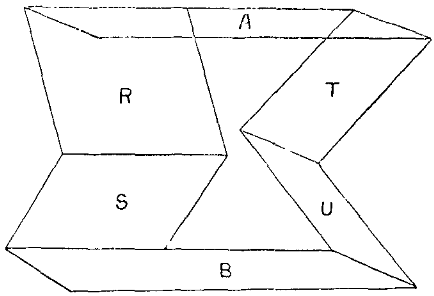
The mechanism may be thus briefly described:—A moving

* Communicated by the Author.

† Quoting Kempe's "How to draw a straight line."

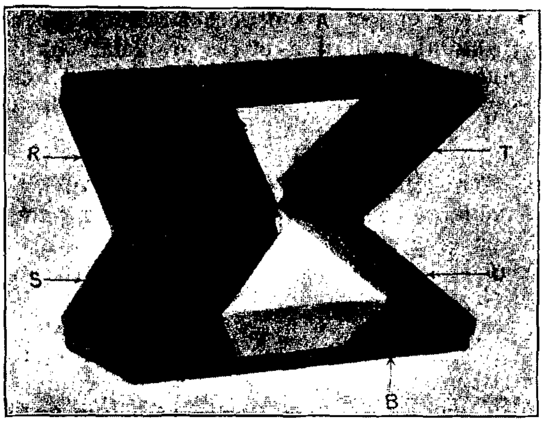
piece A is to have rectilinear motion, say vertically up and down, relatively to a frame or base B (figs. 1, 2, 3). To

Fig. 1.



A perspective outline of fig. 2.

Fig. 2.



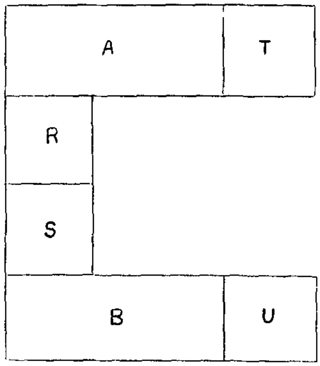
A model of one form of the Sarrut parallel motion.

effect this, four connecting pieces RSTU are used. The pieces ARSB are consecutively hinged by three parallel horizontal hinges*; and, again, the pieces ATUB are consecutively hinged by three parallel horizontal hinges; the two directions being different. Connected thus with B the piece A has a movement which is rectilinear and vertical.

* By the term "hinge," here and throughout, is meant, quite generally, any form of connexion which ensures a permanent axis of pure rotation for the relative movement of two bodies. "Axis," "turning pair," "cylinder pair," "pivot," "pin-joint," are other terms to be found in use with the same theoretic meaning.

The whole forms a closed kinematic chain of six pieces. Seven being the normal number of pieces in a closed kinematic chain necessary to ensure freedom of one degree, it

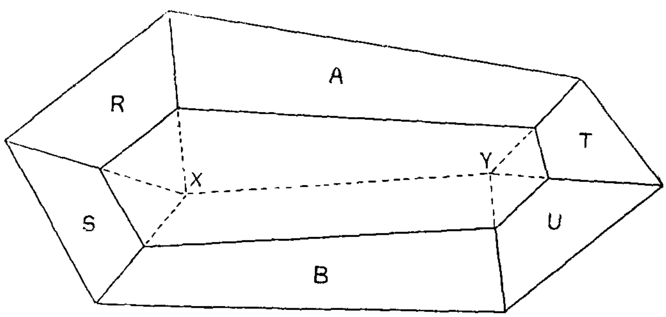
Fig. 3.



The hinge TU of fig. 1 being disconnected, the mechanism flattens out as shown here in plan.

follows that the mechanism of Sarrut belongs to a special class of mechanisms, with only six pieces, possessing singularly and exceptionally one degree of freedom.

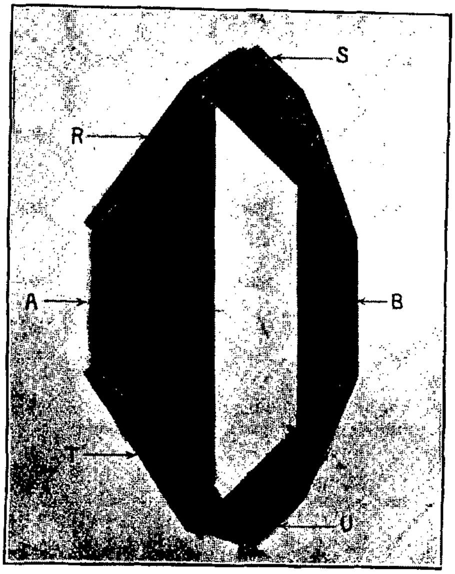
Fig. 4.



The general mechanism of class (a). The hinges of A are non-intersecting, as are also the hinges of B.

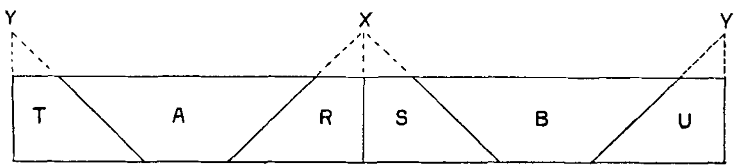
§ 3. It is instructive to compare the mechanism of Sarrut with a certain generalized form to which it stands in the relation of a degenerate case. Retaining the same number of pieces and hinges, let it be supposed (figs. 4, 5, 6) that the hinges consecutively connecting ARSB are concurrent in a finite point X instead of being parallel; and, further,

Fig. 5.



A model, of a special form, of the mechanism (*a*). The two vertical plates have relative rotation about a virtual hinge.

Fig. 6.



The hinge TU of the model of fig. 5 being disconnected, the mechanism flattens out as shown here in plan.

that the hinges consecutively connecting ATUB are concurrent in a finite point Y. The piece A has then, relatively to B, a motion of pure rotation about the line XY. Like the Sarrut mechanism, this is a closed kinematic chain of six pieces singular in possessing one degree of freedom.

Another view of this last mechanism is useful. It may be regarded as composed of two spherical* mechanisms, each of four pieces, placed in tandem with different centres. One consists of a closed chain of four pieces ARSB consecutively hinged along four lines concurrent in X; the other similarly of four pieces ATUB with hinges concurrent in Y; the pieces AB and also their hinge-line XY being common to the two mechanisms. But the singular six-piece mechanism described above arises only after the removal or omission of the hinge XY, which is superfluous or redundant. The line XY is then no longer an actual and mechanical hinge, but yet remains kinematically as a *virtual* hinge-line, in respect of the relative movement of A and B.

This type of mechanism shall be referred to as (*a*).

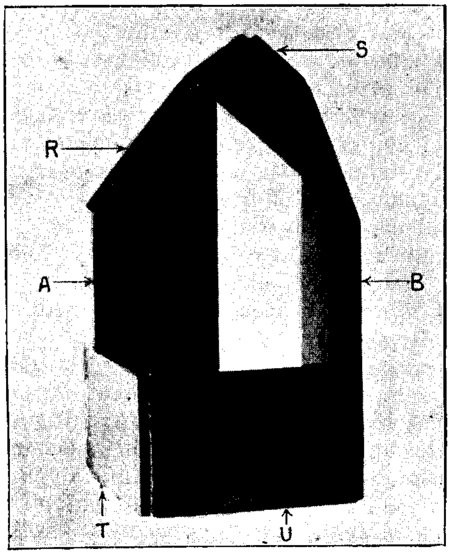
§ 4. From the type (*a*) two special forms are derivable by taking one or both of the points XY at infinity. If one only, say Y, is at infinity, a case (*b*) is obtained. The three hinges connecting ARSB meet still in a finite point X; but the three hinges connecting ATUB are parallel; and the motion of A relative to B is a movement of pure rotation about a virtual hinge-line parallel to the hinges last named and passing through X (figs. 7, 8). This mechanism may be regarded as composed of a plane mechanism and a spherical mechanism in tandem, with the common hinge of the two common pieces omitted.

If both X and Y are at infinity a case (*c*) arises. Each set of three hinges forms a parallel system; the movement of A relative to B is a rotation about the line at infinity which meets all the hinges, a rectilinear motion therefore in the direction perpendicular to all the hinges. It is the type of mechanism described by Sarrut. It may be regarded as composed of two plane mechanisms in tandem; these mechanisms having parallel sets of hinges in two different directions and having the virtual common hinge of the two common pieces at infinity. In technical terms the two mechanisms would be called crossed slider-crank chains,

* The term is perhaps not well enough established to pass without explanation. Mechanisms composed of pieces connected by hinges which are all parallel, being sufficiently represented by any plane section perpendicular to the hinges, are loosely and commonly spoken of as "plane" mechanisms. And by analogy mechanisms of which all the hinge-lines are concurrent in one point, being sufficiently represented by the section in which they are cut by any sphere centred at that point, may be called "spherical" mechanisms. Custom, however, is variable. Reuleaux uses the terms "cylindric" and "conic"; the relative motion of any two pieces being representable by the rolling of cylinder on cylinder in the one case and cone upon cone in the other.

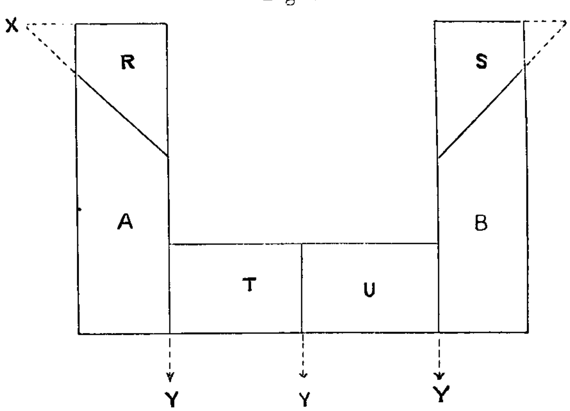
having the two sliding pieces in common and the slide-connexion itself entirely omitted.

Fig. 7.



A model, of a special form, of the mechanism (*b*). The four upper pieces all move as in fig. 5.

Fig. 8.



The hinge RS of the model of fig. 7 being disconnected, the mechanism flattens out as shown here in plan.

§ 5. After deriving the mechanisms (*b*) and (*c*) from the class (*a*), I was interested to try to discover any references to them. A brief account of what I have been able to find out may be set down here; and may perhaps be the means of eliciting further information.

Among the collection of models left by Professor Robert Willis occurs a specimen of class (*a*). It departs slightly from generality (see fig. 4) in that the two hinges connecting the pieces here called RAT intersect each other: a non-essential peculiarity. There is also a specimen which would belong to class (*b*) if the hinge XY were made virtual instead of actual. Of class (*c*) there appears to be no specimen. It was in searching*, however, through the voluminous MS. lecture-notes of the Professor, that, though finding no mention of either (*a*) or (*b*), I first came upon a reference to the article of Sarrut; together with references to a quotation of his result in two French text-books † of slightly later date, and a rough sketch, marked with dimensions, of a simple form of the mechanism. It seems quite likely that a model was made, but that it was subsequently dislocated and the parts adapted to other uses; a frequent custom (so Mr. J. Willis Clark tells me) with the Professor. In any case the parallel motion failed to become known; and twice at least (not to count the present article) it has been reinvented and republished apparently with but little more success. In 1880 a patent (Specification 5492, Dec. 30) was taken out by H. M. Brunel; and in 1891 it was again invented by Professor Archibald Barr (Proc. Phil. Soc. Glasgow, March 18, 1891)‡. Yet to this day it appears to remain practically unknown to mechanicians. Its obvious merits and its long neglect of fifty years seem therefore to be worth insisting upon with some emphasis. Compared indeed with any of its later rivals in the presentment of rectilinear motion, it has obviously two points of great superiority. In the first place, there is an entire absence of any special restrictive metrical relations to be satisfied by the dimensions of the parts as a condition of the movement. In the second place, the connecting pieces are but four in number, with six hinges; whereas rectilinear motion of a piece seems not to have been obtained otherwise §

* By kind permission obtained at the Engineering Laboratory, Cambridge.

† Laboulaye, *Traité de Cinématique*, 1854, p. 634; Girault, *Transformation du Mouvement*, 1858, p. 267.

‡ I have to thank Professor F. G. Baily for referring me to Professor Barr; the latter, in his turn, had heard of Brunel's patent from Sir Frederick Bramwell, to whom Brunel had once shown it.

§ By Kempe, Hart, and Darboux.

with less than eight pieces of connexion and eight or more hinges (some of them multiple) ; or six pieces of connexion and ten hinges. The simplest rectilinear point-movement uses five moving pieces and seven hinges.

§ 6. It may be not amiss to mention certain very familiar objects in which the mechanisms here discussed are foreshadowed and, in a sense, are even potentially contained. The bellows used in blowing organs * have, since about the year 1419 †, been made with each of the four collapsible sides or walls composed of two wooden plates (ribs so-called), hinged together along the horizontal median line, as an improvement on mere folds of leather. A horizontal lid and base, hinged to the ribs, with suitable gussets of leather at the eight corners, complete the apparatus. The step, which took so long in making, from this mechanism to the parallel motion of Sarrut, consisted in removing two of the adjacent walls, and realizing that the rectilinear up-and-down motion of the lid was then not only still a possible movement but the only movement possible. (Removal of *opposite* walls would of course at once leave the mechanism with *two* degrees of freedom.) Again, too, in the folding flaps which connect the edges of the two boards of some portfolios we may see, in an approximate form, the details of the mechanism (*a*). Some purses, pocket-books, and card-cases show the same arrangement. In all such instances, however, the very crucial and distinctive omission of the actual and principal hinge remains to be made before the singular mechanism (*a*) is realized.

§ 7. Some of the foregoing explanations may appear needlessly explicit ; but the difficulties of picturing and describing adequately mechanisms of a specifically three-dimensional character (a large class too little studied so far) may be sufficient excuse. Mistakes and misconceptions arise only too easily in this region. Sarrut himself, for example, appears to have thought that any closed chain of six pieces would be normally free ; and Brunel, desiring to convert his parallel piece-motion into a rectilinear point-motion, figures and describes a spurious mechanism which is nothing but a stiff framework.

* Willis has the suggestion "organ-bellows" written against his sketch.

† Grove's Dictionary of Music : Article "Bellows."