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LVI. *Some Spectroscopic Notes.* By ROBERT A. HOUSTOUN, M.A., B.Sc., *Research Student in the University of Glasgow**.

§ 1. **I**N a communication made to the Royal Society ("On the Radiation of Helium and Mercury in a Magnetic Field," by Prof. A. Gray, F.R.S., and Walter Stewart, D.Sc., with Robert A. Houston and D. B. Macquistan, Proc. Roy. Soc. vol. lxxii. No. 477) certain measurements were given of the wave-lengths of the satellites of the green line in Hg. These measurements disagree with those of Fabry and Perot; they have since been verified, and there seems evidence to show that the satellite system of the green line varies considerably under different conditions.

According to Michelson (Phil. Mag. (5) xxxiv. p. 280, 1882) the green line of Hg is fourfold. It consists of a narrow bright line in the middle of a broader weaker one. Then follow three weaker lines towards the red (fig. 1), the values of $d\lambda$ being about +0.075, +0.100, and +0.125 Ångström units.

Fabry and Perot (*C. R.* 1898, p. 409) represent the line as having two attendant lines +0.008 and +0.092 (fig. 2).

Later they make it fivefold (*C. R.* 1899, cxxviii. p. 1156), one very weak component on the violet side, -0.044, and three on the red side, 0.009, 0.082, and 0.136 (fig. 3).

Finally, they make it sevenfold (P. Zeeman, *Astroph.*

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Journal, xv. p. 218), the differences being -0.224, -0.076, -0.052, 0.008, 0.082, and 0.136 (fig. 4).

* Communicated by Prof. A. Gray, F.R.S.

Our own measurements represent the line as sixfold when most highly developed, the distances of the satellites being -0.208 , -0.096 , -0.059 , $+0.032$, and $+0.067$, with

the condition that these figures are right only to a multiple of $.464$ the distance between two successive orders. When a faint line is seen in the field—the echelon spectro-scope was the instrument with which our observations were

made—the different orders of the central line D are very close together, and it is impossible to tell to what order the faint line belongs. Considerations of symmetry lead us to suppose that B C E and F are rightly placed. It is quite as probable that A is on the other side of D at a distance of $+0.256$.

These measurements were made on an ordinary Geissler tube with aluminium electrodes, and were verified on a tube the electrodes of which were little mercury cups. The tubes were fed from an induction-coil that gave a 10-inch spark. The E.M.F. in the primary was usually 6 volts.

The system did not always appear as above. B and C appeared at times as one, as so also did E and F. In that case the distance of (B C) was $.084$, and of (E F) $.069$. Then B C might be a doublet and E F a single line, or the system might reduce to two satellites corresponding to A and the unresolved doublet B C. It was usually thus when a less powerful induction-coil was used.

Although Zeeman did not measure the position of the satellites with his echelon (which is slightly less powerful than ours), he apparently obtained the same arrangement, for he says, in the article quoted above: "Using the echelon in a position in which two strong lines of equal intensity corresponding to successive orders of the radiation were visible, I could distinguish also five faint, very narrow lines between the principal ones. The distance between two pairs of these lines was very small."

The doublets (B C) and (E F) at times appeared as single lines. The single line was not the doublet badly defined; its width was not greater than that of one of the components of the doublet. Capacity and inductance in the primary and secondary had no effect on the satellites. They were usually seen best when the hammer of the induction-coil was going jerkily—when it was screwed up, so that its spring was short.

The bright yellow line of Hg 5790 was also investigated, and was found as in fig. 6, B being so close to C as sometimes not to be distinguished from it. Calculating the distance $d\lambda$ from an imaginary line between C and D from observations made at three different times, we get

A	-.203	-.133	-.180
B	-.072	-.081	
C	-.032	-.031	-.035
D	+.032	+.035	+.039
E	+.139	+.139	+.157

On one occasion while we measured the differences, the frequency of the hammer changed and the line A was seen to move further out. This was seen only once. The satellite systems are of course exactly the same when viewed across and along the direction of discharge ; and no component is ever plane-polarized.

§ 2. These two lines were also examined in the electric arc, the soft core of the lower carbon being bored out, a rubber tube being fastened to its lower end and connected to a mercury reservoir, by raising and lowering which the supply of Hg vapour in the arc could be altered, when some extraordinary cases of reversal were observed. These remained steady for long. Figs. 7 & 8 represent what the green line reversed into. Here the light maxima were rather bands than lines. $d\lambda$ is given as if the lines were satellites, for which they were at first taken.

Y_1	Y_2	A	C
-0.195	-0.153	-0.093	0.065
	X_1	X_2	
	0.140	0.167

The numbers represent distances from B. Similar measurements for fig. 8, taken from a point midway between L and M, are

G	H	L	M	N
-0.246	-0.130	-0.032	0.032	0.121

Finally, the satellite system was obtained as in the vacuum tube (fig. 5). The results were :—

A	B	C	E	F
-0.227	-0.106	-0.056	0.046	0.084

Fig. 9.

A striking reversal effect (fig. 9) was also obtained for the yellow line in the arc. E was very faint; E_1 is E in the next order. Measuring from a point midway between B and C we have

A	B	C	D	E
-0.14	-0.05	+0.05	+0.16	+0.29

The current varied from 5 to 7 amperes during the above measurements.

Readings were also made on the lines of zinc in the electric arc. The zinc was introduced into the arc by boring the lower carbon which was the anode, and packing strips of thin zinc sheeting into the hole. Again some very striking forms of reversal were obtained. They did not maintain themselves so long as in the case of Hg, and passed out of the one form into the other frequently when I was making measurements on them. It was easier to recognize the zinc lines as being reversed. Changes took place in the reversal system when the arc began to hiss. Also the distance between the different bands varied in the different parts of the arc, being greater and better defined near the anode A; being small but well-defined at the edges B, and being as a rule ill-defined towards C.

Each of the four lines, the red and the three blue (6361, 4812, 4721, and 4681) reversed roughly speaking in three types, figs. 10, 11, & 12. Let

Fig. 10.

Fig. 11.

Fig. 12.

us take the measurements for the case of 4812, the bright blue line. Measuring the wave-length differences in the three cases from B, E, and L, we obtained

Fig. 10. C... +0.05

Fig. 11. D... -0.06

F... +0.06

Fig. 12. G... -0.19

H... -0.08

M... +0.08

P... +0.16

P may, however, be G in the next order.

Using a different arc-lamp and a smaller quantity of Zn, I was able to get the lines without reversals. The red line ($\lambda=6361$) was single. The

three blue lines (4812, 4721, 4681) are all doublets, the components being of equal intensity. For 4812 $d\lambda$ is 0.09, for 4721 0.08, and for the third line something similar.

Examining Hg vapour under the same conditions, the green line appeared with two satellites, one corresponding to A (fig. 5) and the other to the doublet (B C), while the yellow line appeared as a double line with a weak component towards the red. For the green line $d\lambda = -0.24, -0.08$, and for the yellow $-0.06, +0.12$.

The nickel line (5476) and the Na D lines were single.

The red C line of hydrogen was double, $d\lambda$ being $=0.065$, or about one-half Michelson's value. This explains why the line does not appear double in the first-order spectrum of a Rowland's grating.

It seems on the whole tolerably certain that the satellites of the green line of Hg vary in number and position with the conditions of the source. What these determinative conditions are it is difficult to say.

We can look on the multiple reversals of the lines as being made up of simple reversals of the components of the complex spectral lines. This seems more probable than alternate layers of radiating vapours at different temperatures, as it is difficult to see how these alternate layers can arise. The above measurements were made with a micrometer eyepiece and not on a photographic plate. This explains probably why such reversal effects are not more frequently seen.

§ 3. The echelon used for the above observations was made and mounted by A. Hilger. The telescope and collimator were both fixed and mounted on the one stand. The echelon rested on a platform between them ; by moving this about a vertical axis, the different images could be brought into the field of view. The rays of light from the collimator do not fall normally on the echelon. In the discussion usually given it is assumed that they do so, that the collimator and echelon remain fixed and that the telescope moves. The formulæ are the same for both cases, though the discussion is slightly different. I have derived the formulæ for the case of the fixed telescope, using the graphical method given in Preston's 'Light,' and as the echelon spectroscope illustrates this method very neatly the discussion is given here.

It will be well to describe the appearances seen in the field. First of all, if the instrument is focussed on a well-defined line we see several images of it in the field. As the echelon is rotated these images cross the field, and as each crosses the centre of the field it gets very much brighter. In fact it is only the two or three in the centre of the

field that are usually noticed. This is the interference effect.

If a bright line upon a faint background is being examined, in addition to the above images we see a broad band in the centre of the field, and on each side of it a number of bands which are narrower and very much fainter. As many as eleven of these bands may be seen on both sides. This is the diffraction effect. The two effects are seen superimposed as

Fig. 13.

in fig. 13. If the echelon is now rotated the diffraction effect remains practically steady in the field, and the interference images move across the diffraction-bands, becoming very much brighter when they cross the centre of the chief diffraction-band. If we are examining a continuous spectrum we see only the diffraction effect. In the intensity-curve $\frac{bc}{ac}$ as $\frac{BC}{AC}$ &c.

Fig. 14.

Let the figure represent two adjacent steps of an echelon. Let f be the width of a step (E F) and e the length of a step (D E). Let there be n complete steps. Let the angle of incidence of the light-rays be i , and let us consider those rays that leave the echelon in the direction θ . Let us consider all the rays which cross the face C D. They will all suffer a change of phase relative to the extreme ray through C. The path through D is longer than the path through C by

$$f(\sin \theta - \mu \sin i).$$

If we denote the relative angular phase-difference by $2F$,

$$f(\sin \theta - \mu \sin i) = \lambda \frac{2F}{2\pi}.$$

Similarly, if we denote by $2E$ the angular phase-difference that the light which travels through E suffers relative to that

which travels through D

$$e(\mu \cos i - \cos \theta) = \lambda \frac{2E}{2\pi}.$$

Now all the rays for which θ is the same will terminate in one point in the focal plane of the telescope. The amplitude and phase in that point will be obtained by compounding the amplitudes and phases due to the different paths. Then by Preston's 'Light,' § 150, the resultant amplitude due to one aperture (i. e. e , the faces f are ground) is proportional to $\frac{f \sin F}{F}$. It will depend also on the angles i and θ , but we disregard that at present. By analogy with Preston's 'Light,' p. 266, I, the resultant intensity in the direction θ , is given by

$$I = \frac{f^2 \sin^2 F \sin^2 n(E + F)}{F^2 \sin^2(E + F)} \dots \dots (1)$$

The variation of the first factor with F is discussed in Preston, § 151. It gives the ordinary diffraction effect. If the intensity of the first maximum is 1, the intensities of the succeeding maxima are $\frac{1}{25}$, $\frac{1}{56}$, and $\frac{1}{116}$. The principal maximum corresponds to $F=0$, i. e.

$$\sin \theta - \mu \sin i = 0$$

This equation gives the "no deviation" position. For if we consider the ray represented in the diagram its deviation is $\theta - r$. But $\sin r = \mu \sin i$, hence its deviation is $\theta - \sin^{-1} \mu \sin i$, and this is zero when $\sin \theta - \mu \sin i = 0$. The principal diffraction maximum should always appear in the centre of the field, no matter what the angle r is, i. e. no matter how the echelon is rotated. I tested this by rotating the echelon, so that r increased from 0° to about 5° , which was all the range that the adjustment permitted. The diffraction maximum did not, however, remain always equally bright and equally broad exactly in the same place; the intensity diminished so that the visible breadth became less, and there was a slight motion of the maximum in the direction in which θ grows smaller. The width of the maximum decreased more on the one side. This apparent motion can be explained by adding an obliquity factor, depending on i and θ , to the expression for the intensity, to represent the effect that was disregarded above. For the positions of minimum

Fig. 15.

the angular distance of an image in the focal plane of the telescope from the no deviation position, the angle being subtended at the centre of the object-glass. Therefore

$$\begin{aligned} & \mu(e \cos i - f \sin i) - (e \cos \mu i - f \sin \mu i) \\ & + (e \sin \mu i + f \cos \mu i) \Delta \theta_1 = p \lambda. \quad (3) \end{aligned}$$

Suppose now we have also light of a wave-length slightly different from λ . Then let us suppose that the echelon remains in the same position. This new radiation will produce an image given by $\Delta \theta_2$. Thus we have

$$\begin{aligned} & (\mu + d\mu)(e \cos i - f \sin i) - (e \cos \mu i - f \sin \mu i) \\ & + (e \sin \mu i + f \cos \mu i) \Delta \theta_2 = p(\lambda + d\lambda). \quad (4) \end{aligned}$$

Subtracting (3) from (4),

$$d\mu(e \cos i - f \sin i) + (e \sin \mu i + f \cos \mu i) d\theta_1 = p d\lambda, \quad (5)$$

where $d\theta_1 = \Delta \theta_2 - \Delta \theta_1$, *i. e.* is the angular distance between the two images.

Since i is small this becomes

$$d\theta_1 = \frac{p d\lambda - e d\mu}{f}. \quad (6)$$

Let the next order image of the radiation λ be given by $\Delta \theta_3$. Then

$$\begin{aligned} & \mu(e \cos i - f \sin i) - (e \cos \mu i - f \sin \mu i) \\ & + (e \sin \mu i + f \cos \mu i) \Delta \theta_3 = (p+1)\lambda. \quad (7) \end{aligned}$$

Subtracting (3) from (7),

$$(e \sin \mu i + f \cos \mu i) d\theta_2 = \lambda, \quad (8)$$

where $d\theta_2 = \Delta \theta_3 - \Delta \theta_1$ = angular distance between two orders. If i be small we obtain

$$d\theta_2 = \frac{\lambda}{f}.$$

The value of p is approximately $\frac{(\mu-1)e}{\lambda}$. Therefore

$$\begin{aligned} \frac{d\theta_1}{d\theta_2} &= \frac{\frac{(\mu-1)e}{\lambda} d\lambda - e d\mu}{\lambda} \\ &= -e \frac{d\lambda}{\lambda^2} - e d\left(\frac{\mu}{\lambda}\right). \quad (9) \end{aligned}$$

This is the usual formula for evaluating an angular distance in Ångström units.

To find the resolving power: if we consider the interference factor $\frac{\sin^2 n(E+F)}{\sin^2(E+F)}$, it = 0 for $n(E+F) = m\pi$. If m be a multiple of n however, then $E+F$ is a multiple of π , and we obtain a chief maximum. The minimum adjacent to this maximum is then

$$E+F = \frac{m+1}{n}\pi,$$

$$\text{i. e. } e(\mu \cos i - \cos \theta) + f(\sin \theta - \mu \sin i) = \frac{m+1}{n}\lambda;$$

putting $\theta = \mu i + \Delta\theta_1$ for the maximum

$$\begin{aligned} \Delta\theta_1(e \sin \mu i + f \cos \mu i) &= \frac{m}{n}\lambda - \mu(e \cos i - f \sin i) \\ &+ (e \cos \mu i - f \sin \mu i), \quad (10) \end{aligned}$$

and for the adjacent minimum

$$\begin{aligned} \Delta\theta_2(e \sin \mu i + f \cos \mu i) &= \frac{m+1}{n}\lambda - \mu(e \cos i - f \sin i) \\ &+ (e \cos \mu i - f \sin \mu i). \quad (11) \end{aligned}$$

Hence $d\theta_3$, the angular separation of the maximum and minimum, is given by

$$d\theta_3(e \sin i + f \cos i) = \frac{\lambda}{n},$$

or, since i is small,

$$d\theta_3 = \frac{\lambda}{fn}. \quad (12)$$

By equation (6) $d\theta_1$, the angular distance corresponding to an increase of wave-length $d\lambda$, is given by

$$d\theta_1 = \frac{pd\lambda - ed\mu}{f}.$$

In order that two lines with a wave-length difference $d\lambda$ should just appear resolved, $d\theta_1 = d\theta_3$,

$$\text{i. e. } \frac{\lambda}{n} = pd\lambda - ed\mu.$$

$$\text{Now } p \doteq (\mu - 1) \frac{e}{\lambda},$$

$$\therefore \frac{\lambda}{n} = \frac{(\mu - 1)ed\lambda}{\lambda} - ed\mu. \quad (13)$$

The resolving power or $\frac{\lambda}{d\lambda}$ is thus given by

$$ne\left(\frac{\mu-1}{\lambda} - \frac{d\mu}{d\lambda}\right),$$

which is the value arrived at by the other method.

Let us now consider the secondary maxima more closely. Their directions are given by $n \tan (E+F) = \tan n(E+F)$, *cf.* Preston. As $E+F$ increases from 0 to π , we pass from the direction of one chief maximum to that of its neighbour. If we solve

$$y = n \tan (E+F)$$

$$y = \tan n(E+F),$$

$E+F$ varying from 0 to π in each case, the values of $E+F$ thus found give the directions of the secondary maxima within the range. The result for $n=26$ is approximately

$$E+F = \frac{3\pi}{52}, \frac{5\pi}{52}, \frac{7\pi}{52}, \dots, \frac{49\pi}{52}.$$

Applying the formula for the relative intensity, we find that if the intensity of the chief maximum be denoted by 1, that of the neighbouring maxima should be 0.05, 0.02, &c., but all greater than 0.001. There are twenty-four of these secondary maxima between two adjacent principal maxima.

Two of these secondary maxima are as a rule $\frac{\pi}{26}$ apart. Now

$$(E+F) \frac{\lambda}{\pi} = f(\sin \mu i - \mu \sin i) + e(\mu \cos i - \cos \mu i) \\ + \Delta\theta(f \cos \mu i + e \sin \mu i).$$

If the change in $(E+F)$ is approximately $\frac{\pi}{26}$, the corresponding change in $\Delta\theta$ is $\frac{\lambda}{26f}$, and this is exactly the theoretical limit of the resolving power. These secondary maxima, even if bright enough, should not be seen as lines but should form a continuous background. When using the electric arc on some occasions, however, the faint background between the orders appeared as if constructed of a great number of fine lines.

The ratio of the distance between the principal and adjacent secondary maxima to the distance between two principal maxima is $\frac{3}{26}$. The width of a principal maximum in a

typical case was found experimentally to be $\frac{46}{160}$ of the distance between two principal maxima; *i. e.* the adjacent secondary maxima are lost in the principal maxima.

The observations recorded in this paper were made in the Physical Laboratory of the University of Glasgow with Prof. A. Gray's echelon spectroscope, and I have to thank Prof. Gray both for encouragement and advice.

LVII. *Note on Mr. Jeans' Letter in Phil. Mag. for December.*

To the Editors of the Philosophical Magazine.

GENTLEMEN,—

MR. JEANS in his letter omits to take notice of the most important point which arises on his paper and my note. He claims, namely, to have proved Maxwell's law, which may be stated as follows. If $m_1 \dots m_n$ be the masses of n molecules, forming a group out of the much greater number N , $u_1 \ v_1 \ w_1$ &c. their velocities, $x_1 \ y_1 \ z_1$ &c. their space coordinates, then the chance that these velocities and coordinates respectively shall lie within assigned limits $u_1 \dots u_1 + du_1 \dots$ &c., $x_1 \dots x_1 + dx_1$ &c. is proportional to

$$e^{-h(\sum m(u^2 + v^2 + w^2) - 2\chi)} du_1 \dots dv_n dx_1 \dots dz_n,$$

where χ denotes the potential of external forces which in that configuration the n molecules have. Intermolecular forces are not considered.

But this expression may be put in the form

$$f(u_1)f(v_1)f(w_1) \dots f(w_n)f(2\chi);$$

in that form it asserts that "the chance of the velocities of any molecule, as m_1 , lying within assigned limits is independent of the positions and velocities of all the other molecules for the time being."

Maxwell's law cannot then be true unless at the same time that statement is true, and that statement I call assumption A. But Mr. Jeans has proved that this assumption is untrue in fact—namely, in paragraph 2 of his paper he points out that it is inconsistent with the continuity of the motion (and is therefore untrue because the motion is continuous), and in his letter he says "the laws of dynamics imply causation with no greater certainty than they imply the negation of assumption A." If, then, Maxwell's law cannot be true without A being true, and A is not true, it necessarily follows that Maxwell's law is not true.