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5.

On the correction of the Elements of Gerda.

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The planet Gerda was discovered by Dr. Peters at Clinton N. Y. on the 31st of July 1872. The observations of that year extended over a period of nearly two months, and furnished the materials for a pretty accurate determination of its orbit. Since then the writer has computed the perturbations of its motion produced by the attraction of Jupiter and Saturn, and has also prepared the various ephemerides of its motion. Since the year of its discovery we find observations as follows: In 1873 the observations extend from Sept. 27 to Nov. 12; in 1876, from March 31 to April 13; and in 1877 only two observations were made, and these were made by the discoverer of the planet,

on June 30, and July 2. The materials would thus seem to be available for a pretty accurate determination of the elements of the orbit by means of equations of condition. For this purpose normal places were deduced from the observations for the following dates: viz. 1872 Sept. 1, 1873 Nov. 8, 1876, April 10, and 1877 July 3. The normals were represented by the provisional elements with the following errors:

	$\Delta \alpha \cos \delta$	$\Delta \delta$
1872	— 1'36	0'00
1874	+ 400.9	+ 134.4
1876	— 2367.4	+ 945.4
1877	— 1598.1	+ 113.2

If we substitute these residuals in the equations of condition computed for the same dates they will become, in which $(\Delta \mu) = 100 \Delta \mu$;

$$\begin{aligned}
 (1) & + 1.3479 \Delta \pi - 0.01020 \Delta \Omega - 0.2261 \Delta i + 2.4390 \Delta \varphi + 1.3101 \Delta M_0 - 0.21931 (\Delta \mu) - 1'36 = 0 \\
 (2) & + 1.3239 - 0.01083 + 0.2712 - 0.2177 + 1.2318 + 5.1746 + 400.9 = 0 \\
 (3) & + 1.3195 - 0.01530 + 0.0543 - 1.2312 + 1.4103 + 18.3020 - 2367.4 = 0 \\
 (4) & + 1.4263 - 0.00074 + 0.1013 + 2.3508 + 1.4961 + 25.9870 - 1598.1 = 0 \\
 (5) & + 0.4113 + 0.03161 + 0.7424 + 0.7414 + 0.3995 - 0.05366 - 0.0 = 0 \\
 (6) & + 0.4558 + 0.02977 - 0.7860 - 0.0626 + 0.4240 + 1.7609 + 134.4 = 0 \\
 (7) & - 0.5276 - 0.03690 + 0.1355 + 0.3935 - 0.5644 - 7.5717 + 945.4 = 0 \\
 (8) & - 0.0984 - 0.00143 + 1.4359 - 0.1524 - 0.1039 - 1.7534 + 113.2 = 0
 \end{aligned}$$

If we reduce these equations to six by the method of least squares, they will become,

$$\begin{aligned}
 (9) & + 8.0100 \Delta \pi - 0.003149 \Delta \Omega + 0.00473 \Delta i + 4.8115 \Delta \varphi + 8.0577 \Delta M_0 + 72.718 (\Delta \mu) - 5323'' 9 = 0 \\
 (10) & - 0.003149 + 0.003705 - 0.008522 + 0.001848 - 0.003162 + 0.02042 + 2.03 = 0 \\
 (11) & + 0.00473 - 0.008522 + 3.3870 - 0.00510 + 0.00366 + 0.1118 + 3.60 = 0 \\
 (12) & + 4.8115 + 0.001848 - 0.00510 - 13.7697 + 4.7711 + 34.033 - 586.0 = 0 \\
 (13) & + 8.0577 - 0.003162 + 0.00366 + 4.7711 + 8.1296 + 75.958 - 5726.0 = 0 \\
 (14) & + 72.718 - 0.02042 + 0.1118 + 34.033 + 75.958 + 1100.93 - 89912.0 = 0
 \end{aligned}$$

Equations (10) and (11) are practically satisfied supposing each of the unknown quantities to be zero. It follows therefore that two of the unknown quantities in the above equations are indeterminate. Moreover the

coefficients of $\Delta \Omega$ and Δi in each of the other equations are very small, so that any finite value of these quantities would have an insensible effect on the resulting equations.

If we therefore suppose that $\Delta \Omega = 0$, and $\Delta i = 0$ and omit equations (10) and (11), the other four equations will give

$$\Delta \pi = + 49112'' = 13^\circ 38' 32'', \quad \Delta M_0 = - 51834'' = - 14^\circ 23' 54'', \quad \Delta \varphi = - 197'' 04, \quad \text{and } (\Delta \mu) = + 420'' 14.$$

These values are entirely inadmissible, and indicate that some incompatible conditions have been introduced into the equations. The discovery of these incompatible conditions has been the occasion of an unusual amount of trouble and annoyance; and will be the source of future mortification, should the explanation at which I have arrived ultimately prove to be erroneous.

$$\begin{array}{rclclclcl}
 (15) & 6.0495 & \Delta\pi & + & 5.1282 & \Delta\phi & + & 6.2336 & \Delta Mo & + & 65.064 & (\Delta\mu) & - & 5915''.0 & = & 0, \\
 (16) & 5.1282 & & + & 13.7184 & & + & 5.0658 & & + & 35.270 & & - & 490.3 & = & 0, \\
 (17) & 6.2336 & & + & 5.0658 & & + & 6.4325 & & + & 68.837 & & - & 6276.8 & = & 0, \\
 (18) & 56.064 & & + & 35.270 & & + & 68.837 & & + & 1071.05 & & - & 92223.0 & = & 0.
 \end{array}$$

The solution of these equations gives the following values

$$\Delta\pi = + 302''.52, \quad \Delta Mo = + 359''.58, \quad \Delta\phi = - 354''.888, \quad (\Delta\mu) = + 56''.304$$

If we apply these corrections to the provisional elements we shall obtain the following values of the

Corrected Elements of Gerda,
1872 August 24.0 M. T. Washington.

$$\begin{array}{lcl}
 L & 320^{\circ} 44' 24''.55 \\
 M & 112.27.14.27 \\
 \pi & 208.17.10.28 \\
 \Omega & 178.56.41.89 \\
 i & 1.36.16.85 \\
 \phi & 2. 5 23 74 \\
 \mu & 614.08484 \\
 \log a & 0.5078521.
 \end{array} \left. \begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \end{array} \right\} \text{Mean Equinox 1872.0}$$

These elements will almost perfectly represent the observations of the planet which were made in the years

$$\begin{array}{lcl}
 \text{Washington M. T.} & & \alpha \\
 1873 \text{ Sept. 27.0} & & 35^{\circ} 54' 18''.30 \\
 \text{Oct. 20.0} & & 32.15.11.55 \\
 \text{Nov. 12 0} & & 28. 7.12.00
 \end{array}$$

From these observations I have computed the following elements which represent the middle place within 1'' in longitude and perfectly in latitude; and I have placed

Elements of planet observed in 1873,
1873 Nov. 7.0 M. T. Washington.

$$\begin{array}{lcl}
 L & 35^{\circ} 4' 56''.6 \\
 M & 181.50.19.1 \\
 \pi & 213 14.37.5 \\
 \Omega & 178.53. 9.4 \\
 i & 1.36. 2.5 \\
 \phi & 1.58 39.8 \\
 \mu & 613''.9390 \\
 \log a & 0.5079209
 \end{array}$$

In order to obtain any admissible values of the unknown quantities from the above equations (1—8), I have been obliged to omit equations (2) and (6), which correspond to the normal place for 1873. If we omit these equations and also suppose that $\Delta\Omega = 0$, and $\Delta i = 0$, as in the former case, we shall have the following equations for the determination of the other unknown quantities.

1872, 76, 77; but for the observations made in 1873 the following large residuals remain.

C — O

$$\Delta\alpha = + 27'6'', \text{ and } \Delta\delta = + 9'8''$$

The question then arises: Was not the planet which was observed for Gerda in 1873, and very near its computed place in the heavens, really a different planet which chanced to be mistaken for Gerda, but had in fact never before been observed?

In order to decide this question I have endeavoured to determine the orbit of that planet from the observation made in that year independently of any other observations. For this purpose I have deduced the following normal places which are extremely well adapted for giving trustworthy results.

$$\begin{array}{lcl}
 \delta & & \\
 + 13^{\circ} 8' 30''.2 \\
 + 11.42.45.3 \\
 + 10. 7.36.5
 \end{array} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{Mean Equinox 1873.0.}$$

the revised elements of Gerda in juxtaposition after having brought them to the same epoch by applying the perturbations, for the sake of comparison.

Elements of Gerda.

$$\begin{array}{lcl}
 35^{\circ} 47' 13''.55 \\
 187.27.44.50 \\
 208.19.29.05 \\
 178.56.39.87 \\
 1.36.18.97 \\
 2. 0.51.09 \\
 614''.3842 \\
 0.5077209
 \end{array} \left. \begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \end{array} \right\} \text{Mean Equinox 1873.0}$$

If we compare these elements we shall at once perceive that four elements of the two orbits are almost identical, namely, the mean distance, the eccentricity, the inclination and the longitude of the node; while the transverse axes form an angle of about five degrees with each other, and the mean anomaly of Gerda is greater than that of the other planet by more than five and a-half degrees, and its mean longitude is greater by about seven tenths of a degree. Although the two bodies are apparently so close together we must remember that an arc of their orbits amounting to seven tenths of a degree implies a distance of about 3600000 miles. It is perhaps im-

possible to decide with certainty in regard to the identity of the planet observed in 1873 with that of Gerda; the close similarity of most of the elements would seem to indicate an identity, while the dissimilarity of the other elements and the impossibility of representing the observations indicate, perhaps with equal force, a dissimilarity. If there are really two planets moving in orbits so extremely near together it must happen in the course of time, unless their mean distances are exactly the same, that they will approach each other so closely that their mutual perturbations will cause them to unite and form a single planet.

Cleveland, Ohio, 1877 August 7.

Schreiben des Herrn Prof. R. Wolf an den Herausgeber.

Die so eben erschienene Nr. 44 meiner „Astronomischen Mittheilungen“ enthält zunächst ein einlässliches Referat über eine von mir in den Jahren 1874 bis 1877 am Kern'schen Meridiankreise der Züricher Sternwarte durchgeführte Polhöhenbestimmung. Ich benutzte für dieselbe 76 Sterne des Nautical-Almanac, von welchen 62 in weniger als 60° Zenithdistanz durch den Meridian gingen und direct zur Bestimmung der Polhöhe dienen sollten, während die 14 übrigen in sofort anzugebender Weise als Refractionssterne benutzt wurden. Dabei wurden 160 unabhängige, d. h. je mit eigener Ablesung des Nadirpunktes am Quecksilberhorizonte versehene Serien aufgenommen, — zur einen Hälfte bei normaler Zusammensetzung des Fernrohrs, zur andern Hälfte nach Umsetzung des Objectives und Oculars. Dieselben ergaben im Ganzen 1158 Polhöhenbestimmungen.

Zur annähernden Elimination der Biegung wurde wiederholt mit normalem und mit umgesetztem Fernrohr die Zenithdistanz einer Nachtmire gemessen, — daraus im Mittel die Biegungsconstante

$$b = 2'20 \pm 0'48$$

abgeleitet, — und nun jede gemessene Zenithdistanz z um b . Sin z vermindert oder vermehrt, je nachdem sie mit normalem oder umgesetztem Fernrohr erhalten worden war. — Zu den auf solche Weise für die Refractionssterne erhaltenen corrigirten Zenithdistanzen wurden die betreffenden Declinationen des Naut. Alm. zugefügt, — die Differenzen der erhaltenen Beträge von der laut früheren Bestimmungen provisorisch zu 47°22'40"00 angenommenen Züricher Polhöhe als wahre Refractionen betrachtet, — sodann aus diesen Bestimmungen durch Vergleichung derselben mit den der

Bessel'schen Tafel entnommenen mittleren Refractionen Werthe für die Verhältnisszahl α abgeleitet, mit welcher diese Letztere für den betreffenden Abend multiplicirt werden müssen um die wahren Refractionen zu ergeben, — und schliesslich die sämmtlichen im Verlaufe der ganzen Operation erhaltenen Werthe von α benutzt um für diese Grösse eine Formel

$$\alpha = 1 - 0.00230 \cdot \Delta b - 0.00406 \cdot \Delta t$$

aufzustellen, in welcher Δb die Anzahl Millimeter bezeichnet, um welche das Mittel der um 9^h und 11^h Abends abgelesenen und auf 0° reducirten Barometerstände unter den von Bessel für seine Refractionstafel angenommenen Normalstand 751,5^{mm} fiel, — dagegen Δt die Anzahl Centesimalgrade, um welche das Mittel aus den zu denselben Stunden abgelesenen Angaben eines im Meridiansaale und eines im Freien aufgehängten Thermometers über den Bessel'schen Normalwerth von 9°3 emporstieg. — Zur Bestimmung der definitiven Refractionen wurde alsdann in folgender Weise vorgegangen: Für Abende, an welchen zwei oder mehr directe Bestimmungen erhalten worden waren, wurde ohne weiteres das aus ihnen folgende mittlere α verwendet; war dagegen nur eine einzelne Bestimmung vorhanden, so kam das Mittel aus ihr und dem nach der Formel berechneten Werthe zur Verwendung; für einzelne Abende endlich, an welchen gar keine directe Bestimmung erhältlich gewesen war, hielt man sich natürlich ganz an die Formel.

Für die eigentliche Bestimmung der Polhöhe wurde zunächst an jeder, für die Biegung bereits vorläufig corrigirter Zenithdistanz in angegebener Weise die Refraction angebracht, sodann mit Hülfe der Declination