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NOVEMBER 5, 1877.

R. H. M. BOSANQUET, Esq., M.A., F.R.A.S., F.C.S.
IN THE CHAIR.

MUSIO A SCIENCE OF NUMBERS.

'Numeros memini si verba tenerem.'—*Virgil*.

By WILLIAM CHAPPELL, Esq., F.S.A., Vice-President.

THE subject which I submit for your consideration this afternoon is the influence of numbers in music, as in the various combinations of consonances and dissonances which we hear every day, and to show how these are explained by the fundamental laws of the science.

Although music has appeared to many persons a difficult subject, it is really one of the most easily intelligible and one of the most firmly grounded of sciences. It is purely a science of numbers.

The consonances which charm the ear, such as the octave, twelfth, fifth, fourth, and the major and minor thirds, have two concurrent sets of vibrations; the one set produced by the lower string or pipe, and the other by the upper. Although they vibrate at different rates, yet there are periodical coincidences of vibration between them, and these coincidences sound with much more power upon the ear than the vibrations which are non-coincident, or sound apart. It has been calculated that two hammers striking simultaneously upon an anvil have, through the greater displacement of air, fourfold loudness, instead of merely double. The same law applies to musical sounds. Coincidence of vibration is more briefly expressed by its synonyme, 'consonance'; and all non-coincident vibrations are included in 'dissonances,' meaning only that they sound apart. In a musical sense, dissonance is the medium between concord and discord, running from one into the other; for in the most pleasing intervals there are some non-coincident vibrations, and when these become very numerous, they overpower all concord. This will be shown in the sequel.

Suppose we take one long pianoforte-string or an organ-pipe. The lowest sound it can produce will be that of its whole length, and this may be made the foundation of an entire scale of conso-

nant notes, for every aliquot part of its length, or such as will measure without any remainder, will be also a multiple of the vibrations of No. 1. Thus No. 2, the octave, is half the length and vibrates twice as fast as the whole string. No. 3, the so-called twelfth, or octave and fifth, is a third of the length of No. 1, and it vibrates thrice as fast. Then if we sound No. 3 with No. 2 instead of No. 1, we throw off the lower octave and have the fifth only, or 3 to 2. It is essential for consonance that the intervals should be aliquot parts of No. 1, for if otherwise, we should only create discord. The musical law is expressed very simply, that the number of vibrations is in inverse ratio to the length of a string.

The scale of all consonances is called the harmonic scale, copies of which are before you. It is exemplified by string or pipe. Let us consider, first, the *Æolian* harp, on which the winds alone produce the consecutive sounds. The strings are tuned in unison, except the two outmost, one on each side, and those are covered with wire, and tuned an octave lower. When the wind blows quickly enough to sound the bass strings, which we will suppose to have been tuned to C in the bass clef, with 128 vibrations in a second of time, it is the whole string which sounds first, and the rapidity of the wind must be doubled before the harp will sound any change of note, and that note will be the octave above the first. It has already been said that the octave is produced by half the length of a string, and that it vibrates twice as fast as the whole—but mark the coincidence between music and consecutive numbers! As 1 and 2 have no intermediate figure, so they have no note between them, although the sound jumps from the whole length of the string to that of the half! When the bass strings sound the half length they have divided themselves into equal halves by a node, and that node creates tension in opposite directions; one ventral segment pulling as it were against the other. These self-forming nodes may be easily seen by daylight, and at night by throwing a light upon the string. They were shown at our first *conversazione* in these rooms by Mr. Spiller, and at the Edinburgh meeting of the British Association by Mr. Ladd. The gust of wind which sounds the octave, or half length of the bass strings of the *Æolian* harp, sounds at the same time the whole length of the gut strings because they are tuned to that pitch. Then, as the wind rises, subdivision goes on in both, with every multiple of 128 vibrations for the bass, and of 256 vibrations for the other strings.

The reason for tuning the *Æolian* harp to a low pitch is, that the strings may be more easily acted upon by the wind. We read, poetically, of hanging one in a tree, but it requires a much stronger draught than it will get there, except during a hurricane, when no one will care to go to listen. Our late lamented Vice-President, Sir Charles Wheatstone, F.R.S., fixed a single violin string under a very draughty door, as an *Æolian* harp, and he calculated the increase of draught caused by lighting a fire in the

room and by the opening of an outer door, by the rising pitch of the note. The varieties produced by this string have been described as 'simultaneous sounds,' but they were purely consecutive. Any one may satisfy himself that it could only be so, by repeating the experiment with a good violin string. The change of note is simultaneous with the change of nodes in the string, Longitudinal undulations, or irregularities of vibration, will not change the note, but injure the quality of the tone. All the curves that a string may describe in vibration have been calculated by mathematicians, but only when nodes are formed do they effect a change of note in music.

Often have I experimented upon harmonics, or natural sounds, in former years, and have watched the changes of node, and have heard the simultaneous change of note. Such experiments may be tried by any one who has access to a harpsichord, or a very old grand pianoforte. The tension is too great in modern instruments to allow free play to the string. Raise the damper and strike one of the longest uncovered strings with a hard pianoforte hammer near the bridge. The changes follow in numerical order, 1, 2, 3, 4, 5, as in the scale before you, and the sounds ascend by octava, fifth, fourth, major and minor third, harmonic seventh, to the third octave, and then to the major and minor tones. It is difficult to attain the highest of these numbers, but the harmonic seventh, No. 7, is readily distinguished by its unusual sound.

In the *Æolian* harp the rising pitch of the sounds is caused by the increasing rapidity of the wind: but it is not so on a pianoforte. It is there due to gradual contractions of the string till it ceases to vibrate, and sinks to rest. The vibrations of a long string are widely discursive, but they become gradually more and more contracted as the nodes of the strings diminish in length. The point to be remarked is that the sounds jump over intermediate discords—all are consonances—all aliquot parts: all the sounds are multiples of No. 1. It matters not whether it be wind, string or pipe; in each of them Nature teaches us the scale which is to resolve all musical doubts, all disputed chords. She indicates all the bases for musical intervals, the high numbers adapted only for melody, and the lowest for the most consonant harmony.

To prove the case further, we may take an illustration from a pipe. It must not be from those which have lateral openings, or keys, because they shorten the column of air artificially, but from such instruments as the coach horn, or hunting horn, the so-called French horn (which is older than either the French or English nations), or the trumpet without valves.

The fundamental tone, No. 1, or lowest sound it can produce, is derived from the whole column of air within the tube. To produce No. 2 the rapidity of the breathing must be doubled, and then the column of air within the horn divides itself into two equal halves, and the sound is an octave above; so that, if the first note be tenor C with 256 vibrations in a second of time, this treble C requires to be blown at the rate of 512 vibrations to pro-

duce it. Here again we arrive at the identification of sounds with numbers; for just as there is no intermediate number between 1 and 2, so is there no intermediate sound between 1 and 2, its double in vibrations, produced by half its length, upon the horn. The numbers run both ways. The fractions as to length of tube are the corresponding multiples as to vibrations. Again, just as there is an intermediate number between 2 and 4 (the second octave), so is there one intermediate sound, and one only; it is No. 3, which is produced by a third of the length of the tube, and is the fifth above No. 2. The fifth and fourth divide the vibrations of the octave equally between them, so that the fifth is three times No. 1, and the fourth immediately above it is four times. This, notwithstanding the diminution of the musical interval. The names which we have adopted for musical intervals are usually calculated from the key-note, as from C to E a third, from C to F a fourth, and from C to G a fifth, but these names are not real quantities and are rather confusing than an assistance. The octave is not an eighth but half, and the double octave is not a fifteenth, but a quarter of the length of No. 1, and vibrates four times as fast. Octaves are powers of 2; thus 2, 4, 8, 16, and 32 are successive octaves. But the octave 4 to 8 has only four sounds, and these are our major and minor third, and two others, divided by the harmonic seventh, which we do not use. From 8 to 16 are eight sounds, of which we use five, viz. C, D, E, G, and B, with the so-called diatonic semitone, from B to C. It is really the smallest of the eight tones, and not a semitone. The next octave is from 16 to 32, and that is all of semitones, while 32 to 64 is all of quarter-tones. After that, the octave is divided into eighths, sixteenths, and thirty-second parts of tones, among which it is only useful to note (and that only among musicians and mathematicians), that the so-called 'comma,' having the ratio of 80 to 81, is the eighth of a tone above the third of any key—as it is above E in the key of C. We have lately had mathematicians among us unskilled in musical acoustics, who have proposed to divide an octave into 'twelve equal semitones.' This is pure geometry and not music. In music there cannot be, even two, equal semitones within an octave. If our friends will only change their theme from twelve equal semitones into twelve *equally tempered* semitones, and give us their experience of the proposed sounds when heard *with the bass* (which seems not to have yet been taken into account), we shall gladly avail ourselves of their research, on the grounds of modern expediency. In the meantime we must be content to leave the tempering of a scale in the hands of experienced practical men, who, judging only by their ears, as they always will, have hitherto satisfied our immediate requirements. Tempering may be allowed within certain limits without changing the consonance of the note. That is proved by Wheatstone's Resonating Tube, the piston of which may be moved until it arrives at another consonant interval. It will not respond to any discord. Greater variation may be made in the bass than in the

treble, where the intervals run closely together. Thus, an uncertain 32-foot C may be kept in tune by sounding the 16-foot pipe with it.

The interval of a fifth is 2 to 3 in ascending and 3 to 2 in descending; but as the figures are usually placed over the upper note in scales, the 3 is written above the 2, as in the scale in your hands, where it appears over G, referring to C as 2.

And now for the practical use of these figures; for although the harmonic scale can be referred to when necessary, they are very easily remembered. All young pupils are taught the difference between an octave, a fifth, a fourth, and a third upon the pianoforte, and it is only to associate the numbers with those intervals to find out the best bass, and *every admissible bass*.

All octaves are in the ratio of 2 to 1, whether it be 4 to 2, 8 to 4, or 16 to 8 in a second. All fifths are in the ratio of 3 to 2, all fourths in that of 4 to 3, all major thirds 5 to 4, and minor thirds 6 to 5.

For instance, in the key of C; from C to the F above it is a fourth, and F is No. 4: therefore the F, two octaves below, is the consonant bass; whereas, if we strike G with the C above, C becomes No. 4, and is the natural bass to that interval. The most consonant basses are always found in the lowest numbers, because the proportion of consonant vibrations is there greatest. Thus, from D to G is also a fourth, in the scale of C, but its numbers are 9 to 12, having a remote bass in C, and there will be 12 vibrations of the one and 9 of the other, of which only two will coincide in every cycle—1 of the 12 with 1 of the 9. Then, the proportion of non-coincidence will be so great as to make the bass displeasing to the ear. But as 9 to 12 is in the ratio of 3 to 4, we have the best bass in these lowest numbers, and take G. By such various basses to intervals we modulate into other keys without the intervention of discord.

At the International Exhibition held at South Kensington in 1862, Mr. Saxe, the eminent inventor of Saxe Horns, exhibited an immense horn with an exceedingly long coil of tube, and perhaps standing six feet in height. When asked by the jury the object of this excessive size and length, he answered '*C'est pour jouer dans le cinquième étage*'—'It is for playing in the fifth octave,' and he produced with facility any of the sixteen tones and semitones of that octave from it. Half the length of any open conical tube is expended upon its second note, the octave. No human power could have blown the low notes of that horn. Supposing it to have been tuned to the lowest C upon the pianoforte, with thirty-three vibrations in a second, as then the usual French pitch, it would have had 66, 132, 264, and 528 for its first, second, third, and fourth octaves. Thus the fifth octave would commence on treble C, with 528, and extend to C above the lines with 1056 vibrations in a second of time. It would then be within the power of the lungs. He utilised only from the 16th to the 32nd part of his enormous tube, but it gave him the command of the semitones.

This great encumbrance of length is not necessary in a cylindrical *stopped* tube. It will take up its own octave according to the ratio of its length to its diameter. We have an example of this in a resonating tube invented by Sir Charles Wheatstone just fifty years ago. The lecture for which he invented it was afterwards reported in the twenty-fifth volume of the *Quarterly Journal of Science, Literature, and Art*, January to June 1828. Both he and I knew Eulenstein, an accomplished musician, whose admirable skill in playing upon the Jew's harp was the inducing cause of that particular lecture. Eulenstein had a peculiar facility for contracting and expanding the cavity of his mouth, through the pliability of his very thin cheeks and by the management of his tongue, so that he could fit them for any harmonic note within a certain compass. Wheatstone then gave the law, that a perfect harmonic scale might be drawn from a single tuning-fork, or from the vibrating tongue of the Jew's harp, by resonators adapted, or adapting themselves, to *multiples* of the original number of vibrations. 'I took,' said Sir Charles, 'a tube, closed at one end by a movable piston, and placed before its end the branch [or prong] of a vibrating tuning-fork of the ordinary pitch—C. The length of the column of air [within the tube] was six inches. On diminishing the length of the column of air to three inches [by moving up the piston], the sound of the tuning fork was no longer reciprocated [in unison], but its octave was produced.' 'It is therefore evident from experiments,' says he, 'that a column of air may vibrate by reciprocation, not only with another body whose vibrations are isochronous [or in unison] with its own, but also when the number of its own vibrations is any multiple of the sounding body.' Again he says: 'No other sounds can be produced by reciprocation from a column of air but those which are perfectly *identical with the multiplications* of the original vibrations of the tuning-fork or the tongue of the Jew's harp.' I produced his original tube in this room about two years ago; to check a recent theory—that resonators strengthened the ear, and answered only in unison, and Sir Charles ordered this to be made for me, by Mr. Groves, under his own superintendence. The improvement in this is, that the piston now works in a groove and is not liable to stick. Four octaves are produced from the tongue of one Jew's harp as rapidly as the piston can be moved up and down. There is no slurring between one sound and another, but clear jumps from one multiple to another, and every one of them may be arrested and heard by itself by checking the piston. But, although I am glad to produce this tube before those who were not present on the last occasion, and to do honour to the memory of our eminent Vice-President, who declined to refer in any way to himself, I have another motive also. This is a principle which has never been utilised. We have had pipes stopped at the top, like the usual pitch-pipe, but they have been found too slow in action to be suitable for any other purpose. This is rapidity itself, and

might surely be utilised for some such purpose as pedal pipes for an organ. The piston can be balanced outside to the greatest nicety, and one such pipe will take the scale of C, and another that of F. The F pipe is required because F and A (as we tune A for the sake of making it a third to F, and thereby cause it to be a false fifth to the D below) are not aliquot parts of C. All then required is to blow across the edge at the top, in the manner of the Pandean pipes, or perhaps, better still, to set free a fan or cogged wheel at the mouth tuned to each of the two fundamental notes. The wheel might be set free by the action of the foot upon the pedal as the piston reaches its place. It is now well known that the length of a 32 or a 16-foot pipe may be greatly reduced by breadth of scale. We Europeans have made little, if any, use of resonators, and yet they have been long in use in Java. The drawing on the wall is of an instrument brought from Java by Sir Stamford Raffles more than half a century ago. There is one of the same kind in the British Museum. But this is perhaps of greater interest, as it may have suggested to Wheatstone the *principle* of the resonating tube. The natives of Java cast metal plates, which they suspend in a row upon strings, and strike them with drum-sticks, which are fitted into circular heads. As all cast metal is more or less false in tone, owing to inequalities and lack of homogeneity, they place some of the largest bamboos, cut to short lengths and placed upright under the metal, to make the true sounds of these resonators to overcome the false harmonics of the metal plates.

Resonators were used in the theatres of ancient Greece—we here find them used in Java; but these powerful auxiliaries to tone still await their development in modern Europe.

And now, in conclusion, permit me draw your attention to a harmonium with two keyboards, the upper one having four octaves of our scale tuned without tempering, and the lower with the five octaves of the harmonic scale, and the sixteen notes in the fifth octave. Much has been said of the harmonic scale, and this is perhaps the only instrument on which the harmonics can be fully heard and sustained for experimental use.

[The following are the harmonic scales of C and F which were handed round to the members who were present.]

HARMONIC SCALE, OR SCALE OF CONSONANCES, OF C,

From the lowest C upon the Pianoforte, having 33 Vibrations in a second of time, according to the Stuttgart and Society of Arts Pitch.

The notes which do not agree with our scale are bracketed thus [].

FIRST OCTAVE.

Ratios.		Vibrations
1.	C ₁ . Fundamental note; whole length of string, the 16-foot C pipe of an Organ	
2.	C. Octave above No. 1 (half the length and twice as fast)	

SECOND OCTAVE.				Vibrations.
Ratios.				
2.	C.	Octave above No. 1		8 feet pipe 66
3.	G.	Fifth above 2 ($\frac{3}{2}$), and Twelfth above 1 ($\frac{3}{2}$)		99
4.	c	in the bass staff, Double Octave to 1		132

THIRD OCTAVE.				
4.	c	in the bass staff	4 feet pipe	132
5.	e.	Major Third to c ($\frac{4}{3}$), and Major Sixth to g ($\frac{3}{2}$)		165
6.	g.	Minor Third to e ($\frac{6}{5}$), and $\frac{2}{3} = \frac{4}{6}$ of c		198
7. [bb].		The true Seventh [flatter than our bb by $\frac{23}{32}$]		231
8.	c'	in the tenor clef, Minor Sixth to 5 ($\frac{8}{5}$)		264

FOURTH OCTAVE. (The eight Tones.)				
8.	c'	Tenor c	2 feet pipe	264
9.	d'	Major tone to c' ($\frac{9}{8}$)		297
10.	e'	Minor tone to d' ($\frac{10}{9}$)		330
11.	[f'].	Harmonic Fourth to 8 [sharper than our f' by $\frac{23}{32}$]		363
12.	g'	Fifth above c' ($\frac{3}{2} = \frac{6}{4}$)		396
13. [a' b].		Harmonic Sixth to 8 [sharper than our a' b by $\frac{19}{32}$]		429
14. [b' b].		Harmonic Seventh to 8		462
15.	b'	Major Seventh to 8 ($\frac{7}{4}$), Fifth to 10 ($\frac{5}{2}$), and Major Third to 12 ($\frac{3}{2}$)		495
16.	c''	in the treble clef		528

FIFTH OCTAVE. (The sixteen Semitones.)				
16.	c''.	Treble c	1 foot pipe	528
17.		Semitone above 16		561
18.	d''.	Octave to 9		594
19.		Semitone above 18 [only less than a Minor Third to c'' by $\frac{25}{32}$]		627
20.	e''.	Octave to 10		660
21. [f'' b*].		Semitone to 20, Harmonic Seventh to 12, $\frac{7}{4}$ to 14 [only flatter than our f'' by $\frac{23}{32}$]		693
22.	[f''].	Octave to 11		726
23. [f''].		Semitone above Harmonic Fourth		759
24.	g''.	Octave to 12		792
25. [g''].		Semitone to 24, $\frac{5}{4}$ to 20		825
26.		Octave to Harmonic Sixth, 13		858
27. [a''].		Semitone to 26, major tone to 24, and $\frac{3}{2}$ to 18 [our a is a minor tone to 24, a difference of $\frac{23}{32}$]		891
28. [b''].		Octave to Harmonic Seventh		924
29.		Semitone above 28 [too sharp for our b'' b]		957
30.	b''.	Octave to 15, $\frac{3}{2}$ to 20		990
31.		Semitone above b''		1023
32.	c'''.	Octave above 16	6 inch pipe	1056

* This may be named e sharp or f flat. In science there is no difference between the sharp of a note and the flat of the one immediately above it.

HARMONIC SCALE OF F,

With 22 Vibrations. To agree with that of C, of 33 Vibrations.

*N.B.—All marked * agree with the scale of C.*

FIRST OCTAVE.				Vibrations.
Ratios.				
1.	F ₁ .			22
2.	F ₁ .	Octave		44

Ratios.	SECOND OCTAVE.										Vibrations.
2.	F.	Octave	.	.	.	.	.	.	.	.	44
3.	C.	Fifth	.	.	.	.	.	.	.	.	66
4.	F.	Double Octave	.	.	.	.	.	.	.	.	88

THIRD OCTAVE.

4.	F.	Double Octave	.	.	.	.	.	.	.	.	88
5.	A.	Major Third, $\frac{4}{3}$ to F, also $\frac{3}{2}$ to C.	.	.	.	.	.	.	.	.	110
6.	c.	Minor Third, $\frac{3}{4}$ to A	.	.	.	.	.	.	.	.	*132
7.	[eb].	Harmonic Seventh	.	.	.	.	.	.	.	.	154
8.	f.	Third Octave	.	.	.	.	.	.	.	.	176

FOURTH OCTAVE.

8.	f.	Third Octave [our f]	.	.	.	.	.	.	.	.	176
9.	g.	Major tone ($\frac{9}{8}$)	.	.	.	.	.	.	.	.	*198
10.	a.	Minor tone ($\frac{16}{9}$) [our a]	.	.	.	.	.	.	.	.	220
11.	[b].	Harmonic b	.	.	.	.	.	.	.	.	242
12.	c ¹ .	Fifth	.	.	.	.	.	.	.	.	*264
13.		Harmonic Sixth to f	.	.	.	.	.	.	.	.	286
14.	[e ¹ b].	Harmonic Seventh to f	.	.	.	.	.	.	.	.	308
15.	e ¹ .	Major Seventh	.	.	.	.	.	.	.	.	*330
16.	f ¹ .	Fourth Octave	.	.	.	.	.	.	.	.	352

FIFTH OCTAVE.

16.	f ¹ .	Fourth Octave [our f ¹]	.	.	.	.	.	.	.	.	352
17.		The true Semitone above f ¹ [not our Semitone]	.	.	.	.	.	.	.	.	374
18.	g ¹ .	Octave to g.	.	.	.	.	.	.	.	.	*396
19.		Semitone above g ¹ [not ours]	.	.	.	.	.	.	.	.	418
20.	a ¹ .	Octave to No. 10 [our a ¹]	.	.	.	.	.	.	.	.	440
21.		Semitone above a ¹ , Harmonic Seventh to c ¹	.	.	.	.	.	.	.	.	*462
22.	[b ¹].	Harmonic Fourth to f ¹ [flatter than b ¹ ♯]	.	.	.	.	.	.	.	.	484
23.		Semitone above Harmonic Fourth	.	.	.	.	.	.	.	.	506
24.	c ¹¹	in the treble staff	.	.	.	.	.	.	.	.	*528
25.		Semitone above c ¹¹	.	.	.	.	.	.	.	.	550
26.	[d ¹¹].	Harmonic Sixth to f ¹ [wide from our d ¹¹]	.	.	.	.	.	.	.	.	572
27.		Semitone above Harmonic Sixth [this is d ¹¹ , No. 18 in the scale of C]	.	.	.	.	.	.	.	.	*594
28.	[e ¹¹ b].	Harmonic Seventh to f ¹	.	.	.	.	.	.	.	.	616
29.		Semitone above Harmonic Seventh	.	.	.	.	.	.	.	.	638
30.	e ¹¹ .	Octave to 15	.	.	.	.	.	.	.	.	*660
31.		Semitone above e ¹¹ [wide from the other e ¹¹ ♯]	.	.	.	.	.	.	.	.	682
32.	f ¹¹ .	Fifth Octave	.	.	.	.	.	.	.	.	704

Our A, B, C, D, E, F, G, A Scale, which we borrowed entire, and without any change of ratio, from the Greeks. (Claudius Ptolemy.)

	$\frac{9}{8}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$
A,	B,	C,	D,	E,	F,	G,	A.

The same intervals changed into our Major Scale, by beginning on C instead of on A.

$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$
C,	D,	E,	F,	G,	A,	B,	C.

Our Major Scale of C, with the ratios of each interval to the lower C.

1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	$\frac{2}{1}$
C,	D,	E,	F,	G,	A,	B,	C.

DISCUSSION.

Dr. BRIDGE said he did not quite understand about the horn mentioned by Mr. Chappell as made by Mr. Saxe, whether it gave all the intervals between the 16th and 32nd harmonic.

Mr. CHAPPELL said that was so. It would sound lower notes, because any horn of that length would have five octaves upon it. He was not present himself when its capacities were heard, but he saw at once that the extreme length would produce the semitones, and that the lower notes would be too low for the human breath.

Mr. C. E. STEPHENS remarked that in the trombone the lower tones, which were called the pedals of the trombone, were very seldom obtained.

The CHAIRMAN said he had often met with the term the pedals of the trombone, but did not quite understand its meaning.

Mr. STEPHENS said, properly speaking, there were seven positions of the trombone, which gave seven semitones, on each of which a scale could be played; but the fundamental notes were very seldom played, and, in fact, could not be obtained except under peculiar circumstances. It struck him that there was a slip in one of the figures mentioned in the paper, where it stated that 33 vibrations gave the lowest C of a grand pianoforte. He should say 66.

Mr. CHAPPELL said he was speaking of English vibrations. The French would call it 66 vibrations.

The CHAIRMAN said the French counted each half of the swing of a pendulum as one vibration, whereas in England, in America, and he believed in Germany, the double swing, or as Professor de Morgan called it, the 'swing-swang' formed the vibration. The Association probably knew by this time that there were slight differences of opinion between Mr. Chappell and himself, and those who thought with him, on the point mentioned in the paper. He did not propose to go into details, but many of them knew what was the nature of those differences. As to the general question of Wheatstone's views on the subject of what he called reciprocation, he considered those views of great interest, and had devoted a good deal of attention to them, especially since the interesting paper by Professor Adams, and had read a paper before the British Association at Glasgow last year, containing a discussion of the question from a theoretical point of view. He also showed there an instrument which demonstrated in a very simple way the nature of the conditions under which it happened that the fundamental vibration of a tuning-fork, or any similar body, did get transformed into the vibration of its harmonics. It did not always happen that this transformation took place, but required peculiar conditions. His

apparatus was made of small pendulums of different lengths, the longest swinging at such a rate as to represent the fundamental note, whilst there were little ones on either side that would swing in multiples of the former. Thus, if the first swung in one second, the next would swing in half a second, and the one beyond that in one-third, and so on, eight such little pendulums being provided. Knowing what the conditions should be under which, according to the laws of mechanics, impulses upon the large pendulum should set any of the smaller ones in motion, he arranged various modes of connection by elastic webs, so as sometimes to give a connection of one type and sometimes of another type. Under these circumstances it was perfectly easy to set the large pendulum swinging while there were tight connections existing between it and the smaller ones, and yet that they would not be set swinging at all. Whilst, on the other hand, directly the connection was slackened in another way so as to bring in a different set of conditions, the whole impulse not being transmitted but only certain portions of it, then all the small pendulums were set in vibration directly. Now, under ordinary circumstances, there were always irregularities in the transmission of impulses, by which much the same effect as those of the irregularities in the elastic connections between the pendulums, caused the very faint resonance of the harmonics perceived in the instrument of Wheatstone—a most remarkable instrument, and one very well worthy a good deal of attention. He only mentioned this, not to attempt to explain his own views, but to point out that he did not concur in the statement of the general principle of reciprocation, and that he thought that there was a great deal to be said from the point of view of the more advanced theory, which they now knew a great deal about, but of course they did not profess to fully understand it. He could not, however, accept, either as an experimental or theoretical result, the idea that any vibration whatever could, if its impulses were transmitted without transformation, support vibrations of its harmonics. That was not true generally. It was true that some small amount of transmission went on, an amount generally very small, but there might be occasions in which it was very large, and therefore more noticeable. That was all he would say on that particular question. With regard to the instrument from Java, a sketch of which had been shown, it was very interesting. The metal plates being placed upon tubes of bamboo, so arranged as to resound to the fundamental note, the effect was that when the plate was struck a perfectly smooth and prolonged tone was produced. It was an instrument in some respects more perfect than any in use in civilised countries. He did not know how far the tuning was accurate, but any one at all interested in the music of uncivilised nations could have no better material for experimenting on than that instrument. He added that he had often had an idea of

making a small pianoforte with tuning-forks and small resonators at the bottom.

Mr. STEPHENS said he thought that would be an admirable way of increasing the sustaining power of the instrument at a small cost.

The CHAIRMAN said the tones would be smooth and not brilliant, but it would be very expressive.

Mr. HERMANN SMITH said an instrument such as that spoken of was made by Mustel some years ago; it was rather an expensive instrument, called the diapason pianoforte.

Mr. CHAPPELL said he was not aware that he had anything to add to what he had stated in the paper. [The main difference of opinion between the Chairman and himself was as to the universal application of Wheatstone's law. In a paper read before the British Association in 1876, Mr. Bosanquet excepted to it that 'the stopped pipes of an organ are incapable of supporting the vibrations of resonators tuned to their octaves and double octaves, while open pipes are capable of doing so.' To this objection Mr. Chappell demurred, as not being one which affected the law, since it was due only to mechanical interference. The cap upon the stopped organ-pipe forced the vibrations back upon the ascending column at the octave or half distance; thus the ascending and descending vibrations neutralised each other. This neutralisation did not occur in other instruments, where the channel was continuous, as, for example, in the bassoon. Again, Mr. Bosanquet consulted the late Sir Charles Wheatstone about certain new musical theories, and admits that Sir Charles told him they were 'all wrong.' Nevertheless he applied the term 'more advanced theory' to them this evening. At the same time he admits that he does not fully understand this 'advanced theory,' and the question will naturally arise, why then does he judge? Wheatstone's law is proved by Wheatstone's resonating tube—a stopped pipe in which vibrations are free from interference. It supports all its harmonics, octaves included, with equal force. The continuous vibrations of the one fundamental note produce four octaves of harmonics by the simple process of diminishing or enlarging the resonator by pushing the piston up or down. This cuts away the basis of Professor Helmholtz's deductions, and, with it, the 'advanced theories' must fall also.]

Mr. SALAMAN announced that at the next meeting a paper would be read by Mr. W. H. Cummings, on 'The Formation of a National Musical Library.'