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XXII. *On the Curvature of the Spectrum Lines in a Concave Grating.* By ALBERT EAGLE, B.Sc., A.R.C.S., Imperial College of Science and Technology.

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It is well known that in the case of spectra obtained with prisms or plane gratings the lines are curved and formulæ for the curvature have been given in both cases. It is obvious that since a plane grating is a particular case of a concave grating the spectrum lines must be curved in the latter case also. The astigmatism of a concave grating, combined with any curvature of the spectrum lines, will, it is clear, readily spoil the definition, and it becomes of importance to inquire whether any appreciable resolving power is lost under the conditions of usual practice, and also whether such loss is more detrimental in a new form of mounting described by the author * than in Rowland's mounting.

Lord Rayleigh has recently called attention to the fact that the curvature of the spectrum lines in a concave grating does not appear to have been determined.† I have, therefore, thought it of sufficient interest to publish the following investigation.

Let the rulings of the grating be defined by the intersection of the series of parallel planes $y = ne$, where n is any positive or negative integer, with the sphere

$$x^2 + y^2 + z^2 = 2Rx, \quad \dots \dots \dots (1)$$

so that the plane $x=0$ is a tangent at the centre of the grating and the rulings are parallel to Oz . Let $P=(a, b, c)$ be a point in the source and $P'=(a', b', c')$ in the image. Let $A=(x, y, z)$ be a point on the surface of the grating. Let P and P' lie on the cylinder

$$x^2 + y^2 = Rx, \quad \dots \dots \dots (2)$$

described on the radius of curvature of the grating as diameter, and having its axis parallel to the rulings. Let y, z, c, c' be small quantities of the first order compared with R ; x will then be a small quantity of the second order.

It is well known that with such gratings as are employed in practice the effect of the fourth order terms is negligible. We will hence omit terms higher than the third order throughout.

* A. Eagle, "On a New Mounting for a Concave Grating," "Astrophysical Journal," XXXI., p. 120, 1910.

† "Encyclopædia Britannica" (eleventh edition), Vol. VIII., p. 249.

We have

$$AP^2 = a^2 + b^2 - 2ax - 2by - 2cz + y^2 + z^2 + c^2.$$

Putting $a^2 + b^2 = \rho^2$, and substituting the value of x from (1) we get

$$AP^2 = \rho^2 - 2by - 2cz + \frac{b^2}{\rho^2}(y^2 + z^2) + c^2,$$

since $1 - a/R = b^2/\rho^2$ by (2).

This may be written

$$AP^2 = \left(\rho - \frac{by + cz}{\rho} \right)^2 + \frac{b^2 z^2 - 2bcyz + c^2 \rho^2}{\rho^2}.$$

Hence

$$\begin{aligned} AP &= \rho - \frac{by + cz}{\rho} + \frac{b^2 z^2 - 2bcyz + c^2 \rho^2}{2\rho(\rho^2 - by - cz)} \\ &= \rho - \frac{by + cz}{\rho} + \frac{c^2}{2\rho} + \frac{b^2 z^2}{2\rho^3} + \frac{bc^2 y}{2\rho^3} - \frac{2bcyz}{2\rho^3} + \frac{b^3 y z^2}{2\rho^5}, \end{aligned}$$

correct to terms of the third order. A similar equation holds for AP' . The optical path $PA + AP'$ must be stationary along the same ruling, hence to find the point A we differentiate with respect to z and equate to zero, which gives

$$(b^2/\rho^3 + b'^2/\rho'^3)z = c/\rho + c'/\rho' \dots \dots \dots (3)$$

Substituting this value of z , the third order terms become terms in y , which have to be added to the main term, and, since the total coefficient of y must be equal to $n\lambda/e$ for a spectrum line, we see that b'/ρ' , the sine of the angle of diffraction, must vary for different points in the spectrum line, it being assumed that b/ρ is constant for all points in the source.

Two cases only are of sufficient interest to consider. The first is when the grating is mounted in Rowland's manner, and the light is diffracted along the normal. The second is when the diffracted light returns along the line of incidence as in a mounting recently described.*

In the first case we may put $\rho = R \cos i$, $b = \rho \sin i$, $\rho' = R$, and b' a small quantity of the second order. Equation (3) then becomes

$$b^2 z / \rho^3 = c / \rho + c' / R. \dots \dots \dots (4)$$

Substituting this value of z in the expression for $PA + AP'$ we get, since the coefficient of y is constant,

$$b'/R = \rho c'^2 / 2bR^2 - (\rho^2 - b^2)c^2 / 2b\rho^3 = (c'^2 - c^2) / 2R^2 \sin i. \dots (5)$$

This equation represents a series of parallel arcs, the arcs arising from $+c$ and $-c$ overlapping if c be not too great. It must

* A. Eagle, *loc. cit.*

FIG. 1. Diagram showing structure of spectrum lines in a concave grating. The continuous black lines represent light coming from five points at $\frac{1}{2}$ in. intervals on the slit. The dotted line is the line joining the centres of the others and to which the spectrum line reduces in a plane grating. The red end of the spectrum is to the right in all cases. The horizontal scale is 50 times the vertical scale.

FIG. 2. Slit, showing the points to which the different arcs correspond.

FIG. 3. Structure with Rowland's mounting $i = 30$ deg.

FIG. 4. " " $i = 60$ " "

FIG. 5. " " new " $i = 15$ " "

FIG. 6. " " " $i = 30$ " "

FIG. 7. " " " $i = 45$ " "

The short black lines represent the diffraction width, taken as equal to the distance from the central maxima to the first minima.

be borne in mind that the value of z cannot exceed $l/2$, where l is the length of the rulings. Hence, putting $c=0$, we see from (4) that a point at the slit is drawn out into a line of length $b^2R/\rho^3=l \tan i \sin i$. For a constant value of c , (5) gives an arc of a circle whose radius of curvature $c'^2/2b'=R \sin i$. The middle point of each arc corresponds to $z=0$, for which $c=\rho c'/R$ by (4); substituting this value of c in (5) we see that the centres of all the arcs lie on the curve $b'=c'^2 \sin i/2R$, and is thus an arc of a circle whose radius of curvature is $R/\sin i$. From the sign of b' it can easily be seen that the *concavity* of the lines as well as of the arc on which their centres lie is towards the red.

If this curvature is not seriously to affect the definition, the path error between rays falling on the centre of the grating and one edge must not exceed $\lambda/4$. Hence the displacement b' must not exceed $\lambda R/4s$, where $2s$ is the width of the ruled space. Putting $c'=0$ we see from (5) that the displacement at the centre of the line is $-c^2/2R \sin i$. Hence we must have $c^2 \leq R^2 \lambda \sin i/2s = R^2 e \sin^2 i/2ns$, where n is the order of the spectrum. Taking the formula for the astigmatism we see that the centre of the spectrum line will not be illuminated if $2c > l \tan i \sin i$. Applying these conditions to a 6 in., 21 ft. grating, with rulings 3 in. long and 15,000 lines to the inch, we find that the length of the slit illuminated need not be considered in the first order. In the second and third orders the length of slit used should not exceed $0.8 \sin i$ inches ($20 \sin i$ mm.) and $0.7 \sin i$ inches ($17 \sin i$ mm.) respectively. Figs. 2 and 3 show how the spectrum line is built up for angles of incidence of 30 deg. and 60 deg. in the grating considered.

Consider next the case in which the light is diffracted along the angle of incidence. Let $b'/\rho' = b/\rho + \beta/R$, where β is a small quantity of the second order. β is then the lateral displacement of any element of the spectrum line. Equation (3) gives us

$$2b^2z/\rho^2 = c + c'. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Substituting this value of z in the third order terms for the expression of the total path length and equating the total coefficient of y to a constant, we get

$$\beta = -\frac{(c^2 + c'^2) \cos 2i + 2cc'}{2R \sin 2i \cos i}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

For a constant value of c this gives an arc of a circle whose radius of curvature is $c'^2/2\beta = R \tan 2i \cos i$, with its convex

side towards the red if the angle of incidence be less than 45 deg. The length of each of these arcs is obtained by putting $z = \pm l/2$ in (6), which gives $2l \sin^2 i$. To find the locus of the centres of the different arcs we put $z=0$ and, therefore, $c = -c'$ in (7), which then becomes $\beta = c^2 \tan i \sec i/R$, and is thus an arc whose radius of curvature is $R \cos^2 i/2 \sin i$, with its concavity towards the red.

The system of arcs given by (7) for the angles of incidence 15 deg., 30 deg. and 45 deg. is shown in Figs. 4, 5 and 6, which are drawn for the same grating as Figs. 2 and 3.

The displacement at the centre of the spectrum line, *i.e.*, at $c'=0$ is $\beta = -c^2 \cot 2i \sec i/2R$, but it must be borne in mind that light from c will not illuminate the point $c'=0$ if $c > l \sin^2 i$. Considering the same grating as before it is easily found that the definition will never be affected at the centre of the spectrum line in the first order spectrum, while in the second, third and fourth orders the length of slit illuminated should not exceed 1.2, 0.95 and 0.8 in. each multiplied by $\sin 2i \sec^{\frac{1}{2}} 2i$, respectively. These restrictions, it can be seen, are not so severe as in Rowland's mounting.

If the slit of a concave grating be rotated in its own plane, the arcs into which different points of the slit are drawn out into in the spectrum will obviously remain parallel to their original direction, while the arc joining their mid points will be rotated through an angle of the same order of magnitude as the slit, hence a very slight rotation will merely shift the point where this latter arc is tangential to the first set (*i.e.*, the point of best definition) up or down the spectrum line. From this we can see that if the slit were entirely below the plane $z=0$ and the spectrum line entirely above it, the structure of the line could be made exactly the same, by turning the slit a little out of the vertical, as if both slit and spectrum line were symmetrical about the plane $z=0$. Thus no resolution is lost by having the slit entirely below the plane $z=0$, which is necessary in the mounting now considered, but the best definition is secured when the spectrum lines are not quite at right angles to the run of the spectrum. This angle for the grating whose mounting I have described is about $0.6 \tan i \sec i$ degrees. Angles of incidence greater than 45 deg. are not used in this mounting, since an angle of incidence of 30 deg. will give the same position in the spectrum as an angle of 90 deg. in Rowland's mounting. Hence this inclination never exceeds 1 degree.