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The Earth's Albedo. By Frank W. Very.

In the following research the albedo of the earth has been determined from visual photometric measurements of the earth-shine on the moon, in comparison with light from similar sun-illuminated areas on the moon's surface. The photometer, shown in plan in Fig. 1a and in front elevation in 1b, consists of a rectangular wooden case, having a

projecting shelf at *S* on which a lantern is placed, whose flame (furnishing the standard light) is viewed through an iris-diaphragm (*I*) by means of totally reflecting prisms ($P_1 P_2$) and a positive eye-piece (*E*). At *L* is an achromatic lens of 34.9 cm focal length and 1.94 cm effective aperture, which, being directed towards the moon, gives an image of that object, or of the neighboring sky at *M* (after reflexion by the prisms $P_3 P_4$). At the common focal point of the objective and the positive eye-piece, are twin circular apertures in black card, each subtending $7'$ of arc on the celestial sphere, one above, and the other below the horizontal line of junction of the last pair of reflecting prisms. On removing the black card, the upper half of the field is seen occupied by the moon and surrounding sky, while the lower half of the field is uniformly illuminated by light from the flame, variously

modified. It is seldom necessary to point the instrument at other than moderate elevations, since the earth-shine is best observed not far from new moon and therefore the moon's altitude is low. The instrument, as constructed, is not suitable for observations at large altitudes, but could easily be adapted to such work by the addition of a diagonal mirror in front of the iris, placing the lantern at one side on a rocking frame.

The ideal illumination on the right hand (as seen from behind) would have been that of a uniformly illuminated ground-glass disk whose luminosity could have been modified by the iris-diaphragm in exact proportion to the area of its aperture; but the exigencies of the work demanded a wider range of light. In some of the measures it was necessary to dispense with any ground-glass screen and to receive the light from the unobstructed flame through various colored glasses. Since the distribution of light in the flame is not uniform and the diffusion of its light not the same as that from the screen, the brightness of the lower and flame-illuminated half-field is then no longer proportional to the area of the aperture, and it becomes necessary to calibrate the instrument and to prepare curves of illumination corresponding to the opening of the iris by which the readings must be evaluated — curves which differ essentially from the typical iris-curve. A further need of several such curves of calibration will be seen when we consider the problems connected with the measurement of very faint light which also possesses distinct color.

The plan finally adopted was to place one or more modifying glasses at *G*, and if necessary a plate of parallel and plane-surfaced, neutral-tinted glass could be inserted in the path of the rays from the lens at *G'*, while in some cases the moonlight was cut down by diaphragming the objective. The final adjustment to equality in the two small illuminated circles (viewed in the center of the field as if they constituted the components of a double star) was made by opening or closing the iris-diaphragm which was graduated into fifteen divisions and read by estimation to tenths of a division.

The lunar measures were made easily with this instrument. Whatever discrepancies appeared were usually readily explicable, and resulted mainly from the inevitable variations of atmospheric transparency, rather than from any flaw in the apparatus, or in the method. Some difficulties were encountered in obtaining consistent and reliable measurements of the transmission-coefficients for the differently colored glasses, but these were finally overcome. After trying various arrangements, including a flicker-photometer, the most satisfactory method for measuring the very wide range of intensities included in the several transmitting pieces was found in a modification of the wedge-photometer, due to Dr. Charles H. Williams, of Boston, in which a long strip

of photographic plate receives an impression of progressively increasing density through a graduated exposure to light. This instrument, called a »simplex« photometer by its originator, is graduated in stellar magnitudes. The scale-reading in magnitudes is multiplied by 0.4, and the product subtracted from ten gives the tabular logarithm of the brightness.

An example will show the method used. The objective was directed to a lamp flame of whiter quality than the lantern flame, and in order to make the half fields of equal brightness and color, when the aperture of the lens was equal to the full aperture (I_{15}) of the iris, the lamp wick was repeatedly trimmed and the height of the flame altered as well as the position of the flame in respect to the lens aperture, until a slightly smoky flame was obtained which matched the normal lantern flame. This condition was maintained throughout these measures of the transmission by the colored glasses for light of lantern-flame quality, unless otherwise specified.

	$b_2 \times I_{15}$	$b_2 \times I_{12}$	$b_2 \times I_{10}$	$b_2 \times I_8$	$b_2 \times I_6$	$b_2 \times I_4$	$b_2 \times I_2$	$b_2 \times I_1$
Simplex (1) Illumination	$S_{3.5}$ 0.00505	$S_{4.2}$ 0.00265	$S_{5.0}$ 0.00127	$S_{5.7}$ 0.000667	$S_{6.0}$ 0.000505	$S_{6.5}$ 0.000319	$S_{7.0}$ 0.000202	$S_{7.5}$ 0.000127
Simplex (2) Illumination	$S_{3.2}$ 0.00667	$S_{4.0}$ 0.00319	$S_{4.5}$ 0.00202	$S_{5.0}$ 0.00127	$S_{5.5}$ 0.000801	$S_{6.2}$ 0.000420	$S_{7.0}$ 0.000202	$S_{7.9}$ 0.000088
Simplex (3) Illumination	$S_{3.8}$ 0.00608	$S_{3.9}$ 0.00349	$S_{4.4}$ 0.00221	$S_{4.8}$ 0.00152	$S_{5.3}$ 0.000963	$S_{6.0}$ 0.000505	$S_{6.8}$ 0.000243	$S_{7.5}$ 0.000127
Mean Illumination	0.00593	0.00311	0.00183	0.00115	0.000756	0.000415	0.000216	0.000114

Iris-factors for b_2 (Flame values).

	I_{15}	I_{12}	I_{10}	I_8	I_6	I_4	I_2	I_1
Series 1	1	0.525	0.251	0.132	0.100	0.063	0.040	0.025
» 2	1	0.478	0.303	0.190	0.120	0.063	0.030	0.013
» 3	1	0.574	0.363	0.250	0.158	0.083	0.040	0.021
Mean	1	0.526	0.306	0.191	0.126	0.070	0.037	0.020

If we include all of the twenty-four observations of b_2 in the three series, reducing each measure to full aperture by the mean iris-factors, we have

$$b_2 = 0.00592 \pm 0.00017.$$

The probable error of this result is small, but the observations with small opening of the iris and faint light are uncertain, and it is better to use none but comparisons made with full apertures. These gave

$$b_2 = 0.00561 \pm 0.00051,$$

which includes results obtained by several methods.

The transmission by this piece of blue glass is of importance in these lunar measures, and I have therefore given much time to its determination. The iris-factors drawn from the transmission-curve for light from the lantern flame through blue glass (b_2 Fig. 2) deviate from the smooth curve for a dark neutral-tinted glass (n Fig. 2), both being associated with the iris-diaphragm. The smaller apertures which yield relatively large values of the transmission are presumably more efficacious for blue light than the area of the aperture would indicate, on account of the great sensitiveness of the eye for faint light of this color, which gives the faint light

The simplex-photometer was now placed in front of the objective, reducing its aperture to 6.9 mm, or the aperture-factor $= (6.9/19.4)^2 = 0.127$. For simplex = 1 magnitude (S_1), the transmitted light is $1/2.5 = 0.4$, which, multiplied by the aperture-factor, gives for the brightness of the upper half field $0.127 \times 0.4 = 0.0508$. The corresponding iris-reading, giving equal illumination to the lower half of the field, was $I_{3.8}$, which by the flame-curve of calibration is equal to an illumination of 0.0550. This agreement was good enough for practical purposes, as the measures were to be comparative.

A piece of dark blue cobalt glass (b_2) was now put in the position marked G in the figure, and the following measures were taken [the brightness corresponding to the simplex-reading is in each case multiplied by the aperture-factor (0.127) to give the illuminations, and these, divided by the value for the full aperture of the iris, give the iris-factor].

an advantage that it would not possess if of the original yellow color. A similar deviation from the normal iris (direct flame) calibration-curve, obtained with neutral-tinted glass, is also found for cobalt glass of a fainter blue (b_1 Fig. 2). Consequently, there can be no doubt that these calibration-curves for the blue glasses include not only irregularities coming from the unequal distribution of light in the flame (the iris flame factor per se), but also physiological phenomena connected with peculiarities of vision for faint light. Each point of these curves has resulted from a comparison with the neutral-tinted simplex-photometer which is the ultimate standard to which all other means of measurement have been referred, although it is not itself convenient for use in the lunar work.

The following are the symbols designating the various transmitting glasses:

n , a faintly neutral-tinted glass, almost entirely colorless, and such as is used sometimes for spectacles. It has not been used directly in the lunar measures, but has been found convenient as an intermediary in testing the other glasses.

n is a darker, so-called »neutral« glass, but with a decided tinge of yellowish green.

b_1 is a light blue cobalt glass.

b_2 is a dark blue cobalt glass.

g_2 is a transparent glass which has been finely and evenly ground on both faces. It is not appreciably colored when viewed by direct transmission, but shows a blue green tint when held edgewise to the light.

These glasses have been used in various combinations. While the transmission by successive strictly neutral-tinted

media may be taken equal to the product of their individual transmissions, it will not do to assume this in the case of colored glasses, for there are bands of transmission in one glass for certain wave-lengths which may be nearly, or entirely absorbed by glass of another color, thus completely changing the quality of the sifted light presented for transmission by the next member of the series. Such combinations must therefore be tested directly in each case.

The ground glass differs from all of the other media

in being not only an absorbent, but a secondary source of illumination, diffusing a light whose intensity varies with the position of the glass in the path of the rays. For the normal position at G (Fig. 1), three series of measures gave for the effective transmission

(g_2) 0.00452, 0.00455, 0.00488; Mean = 0.00465.

I give next the mean results for n_s , n , b_1 and two of their combinations in conjunction with the iris. The curves of the iris-factors for b_1 and n are drawn in Fig. 2.

Transmission by colored glass with iris.

Iris =	I_{15}	I_{12}	I_{10}	I_8	I_6	I_4	I_2	I_1
n_s	0.187	0.108	0.067	0.040	0.021	0.009	0.003	0.001
n	0.131	0.078	0.051	0.032	0.019	0.008	0.003	0.001
b_1	0.098	0.057	0.036	0.024	0.014	0.007	0.003	0.001
$n \times n_s$	0.0245	0.0148	0.0097	0.0061	0.0035	0.0015	0.00056	0.00024
$b_1 \times n_s$	0.0183	0.0105	0.0066	0.0044	0.0026	0.0013	0.00058	0.00024

The following are the mean iris-factors for use whenever g_2 is a member of the transmissive series. The use of this piece substitutes a new secondary source of light in place of the lantern flame, and introduces a new set of conditions requiring the use of these special iris-factors from which the curve (g_2 of Fig. 2) is drawn.

Iris-factors.

Iris	I_{15}	I_{12}	I_{10}	I_9	I_8	I_6	I_5	I_4	I_3	I_2	I_1
g_2	1	0.674	0.468	0.434	0.361	0.225	0.155	0.119	0.076	0.050	0.033

Comparisons between the lantern flame, circumscribed by the iris and viewed without any intermediary, and the light diffused from the ground glass when illuminated through the iris, gave the following ratios:

Iris-factors (g_2) divided by iris-factors (flame).

Iris	I_9	I_6	I_5	I_4	I_3	I_2	I_1
$g_2/\text{Fl.}$	2.62	3.75	3.76	3.53	3.16	2.79	2.60

It may possibly be anticipated from an imperfect examination of this case that, although the illuminations

are different, their ratios should be equal; but the following considerations show that this is not so; because the illumination through the iris varies with the aperture and the distribution of brightness in the flame in either case; but with a very small aperture, each element of surface of g_2 receives light from a portion of flame only a very little larger than the aperture, because the illuminating cone of rays is almost a cylinder; while, as the aperture enlarges, the illuminating cone to g_2 acquires a wider angle, consequently a proportionately larger area of flame is visible from g_2 and the ratio

increases. Finally, the aperture approaches the dimensions of the flame and at last the cone of illumination on g_2 embraces more than the flame and the ratio diminishes.

I will now give two entirely independent measures of the faint light (originally of lantern-flame quality and intensity) transmitted by the combination $b_2 g_2$. The first is by comparison with the whiter and brighter flame of the lamp, placed at a distance of 64.5 cm and reduced to equal brightness, but not to identical color, by wire screens which transmit the fraction 0.477 as computed from sizes of mesh and wire. After reducing the lights to equality in this way, the wire screens were removed, the simplex was placed over the lens, reducing its aperture to an area of 0.127, the glasses b_1 and n were inserted at G' on the left, and g_2 with b_2 at G on the right, and the area of the iris = 0.359. Equality was then restored with the simplex at 5.3 mag. which corresponds to a reduction to 0.007812. The measured transmissions of b_1 and n are 0.099 and 0.131 respectively. What has been done is this: Lights were equal with two wire screens on the left. Removal of screens on the left increased the light to $L \times 1000/447$, and

$$L \times (1000/447) \times (0.099 \times 0.131 \times 0.007812 \times 0.127) = \\ = L \times g_2 \times b_2 \times 0.359,$$

from which we obtain

$$b_2 g_2 = 0.0000752.$$

In the second determination on a different occasion, lamp and lantern were placed at equal distances from the photometer. Equality was obtained with $I_{8.6}$, when the lens was stopped down to 6.9 mm. The simplex, which is also of aperture 6.9 mm, was then inserted on the left and $b_2 g_2$ on the right. Equality was obtained with the simplex at

10.3 mag. which gives log of light

$$= 10 - (0.4 \times 10.3) = 5.88,$$

and transmission $b_2 g_2 = 0.0000759$.

The transmission by the deeply colored b_2 has been difficult to measure, but the adopted value, $b_2 = 0.00561$ can not be far from the truth. *Ives*, in his paper on the »Photometry of Lights of Different Color« (Philosophical Magazine, July 1912, p. 149, and September 1912, p. 352), finds that, through individual measures of colored lights by ordinary means vary widely, their average agrees with the value by a flicker-photometer. In the comparison of differently colored lights, it is usually possible to get a middle position which is fairly close to the true ratio of brightness, by setting the apparatus successively at positions which make one of the compared fields first »certainly too bright« and then »certainly too dark«; while a sufficient number of repetitions on the middle position fixes the measure with reasonable precision. The product of the separate determinations of b_2 and g_2 , namely,

$$b_2 \times g_2 = 0.00561 \times 0.00465 = 0.0000261,$$

is only about one third of the observed value for the combination; but the transmission of light by conjoined colored glasses (and the ground glass is slightly colored) does not, in general, equal the product of the transmissions of the individual components, exception being made of neutral glasses which do not alter the true color of the colored and associated medium.

The iris-factor for g_2 and $b_2 g_2$ is the same as that already given (g_2 Fig. 2); but b_2 alone, that is in the absence of g_2 , requires the flame-factor. We have therefore for these glasses in conjunction with the iris the following table:

Transmission.

Iris =	I_{15}	I_{12}	I_{10}	I_8	I_6	I_4	I_2	I_1
b_2	0.00561	0.00295	0.00172	0.00107	0.00071	0.00039	0.00021	0.00011
g_2	0.00465	0.00313	0.00218	0.00168	0.00105	0.00055	0.00023	0.00015
$b_2 g_2$	0.0000756	0.0000510	0.0000354	0.0000273	0.0000170	0.0000090	0.0000038	0.0000025

Other combinations which have been used and which have had their transmission measured by methods similar to the foregoing are

$$g_2 \times b_1 \times n = 0.0000689, \quad g_2 \times b_1 = 0.000632.$$

Measures were also made of the transmission of n , b_1 and b_2 for sunlight with which that for moonlight ought to agree. The general result is that the transmission by the blue media which have been used varies with the light source, being greater for a white light (or for one verging on blue) than for the light from a yellow flame; but the greenish »neutral« glass has a uniform transmission of $n = 0.131$, except when used in combination with the other glasses, for which some slight variation of n as a factor occurs.

Transmission.

Light-source =	Sky	Sun	Lamp	Lantern
Transmitting piece				
b_2	0.119		0.0127	0.0056
b_1	0.396	0.295	0.099	0.098
Cu SO_4 Solution	0.224		0.093	
n		0.133	0.132	0.131

There is considerable alteration in the transmission by b_2 when the quality of the light changes. Equally wide variations are found for a light of constant quality and intensity when b_2 is used with the simplex by the disappearance method. Here the following changes were noted in the apparent transmission of the light from a distant lamp as the general daylight faded.

Day	$b_2 = 0.0128$	Day	$b_2 = 0.0330$
»	0.0152	»	0.1000
»	0.0272	Night	» 0.1657.

This is simply an example of the development of extra sensitiveness of rod-vision for blue light as the general illumination diminishes. There can be no question here of any variation in the quality of the original light. The phenomenon is purely physiological, and the peculiarities of the iris-factor curves for blue glass, already noticed, have a similar origin; but the variation in the transmission of light from such different sources as the sky and a lamp, involves a real difference in the composition of the light.

The simplex-photometer on which these measures are based, agrees in principle with the wedge-photometer, and shares its advantages as well as its disadvantages. The present instrument embraces a range of fifteen magnitudes.

In a study of *Pritchard's* wedge-photometer as ordinarily used, that is, when used to get the sensible extinction of the light to be measured (*Memoirs American Acad.*, Vol. VIII, p. 301-324, 1886), *S. P. Langley*, *C. A. Young*, and *E. C. Pickering* have considered the reliability of the method as affected by the variation of transmission within the limits of the visible spectrum.

A possible variation of another sort is briefly noted by Professor *Young* who says: »For some not very obvious reason, the extinction mean of the three stars varied very much on the four nights. It was respectively (in wedge readings) 2.051, 2.113, 2.326 and 2.494 inches. It looks very much as if the eye, after practice, became able to follow the star farther before extinction« (*Op. cit.*, p. 304). It is undeniable that the eye of a novice can be, and must, ordinarily, be trained to recognize faint indications, visible (that is, capable of being seen) but not otherwise noticed than as the result of education. Professor *Young*, however, was no novice in delicate observation, and possibly the change to which he refers was not so much a training of the eye, as an education of the observer to a recognition of the necessity for waiting longer for the eye to acquire the requisite sensitiveness. I have passed through the same experience in the measurement of the transmission of radium rays by means of phosphorescent effects, and have learned that the chief obstacle to success in this case was the observer's impatience to begin.

The neutral-tinted wedge and simplex deplete the light through selective scattering by small particles, the shorter wave-lengths being the ones most obstructed; and therefore these media are not absolutely neutral, though coming under this title as an approximate distinction. There is no appreciable line absorption, at least not within the limits of the visible spectrum. This is known, because the Allegheny spectrobolometric measures of the transmission of selected wave-lengths by the *Pritchard* wedge (described in Director *Langley's* part of the above report) showed no particular variation of the coefficients of transmission after sifting by successive layers. But the bolometrically determined transmission of the total beam by successive equal layers varied notably, because the composition of the rays was changed through successive rejections of particular rays, that is, the depletion was selective. The mean of nine series with sunlight, conducted on two days by Mr. *James Page* and the writer, gave the following results:

	Galvanometer deflections (a)	Trans- mission (b)	Ratios of successive transmissions (c)
No wedge	714.0	$t_0 = 1.0000$	
Wedge at 0.3 inch	305.7	$t_1 = 0.4282$	
» » 1.8 »	142.6	$t_2 = 0.1997$	$t_2/t_1 = 0.467$
» » 3.3 »	81.4	$t_3 = 0.1140$	$t_3/t_2 = 0.570$
» » 4.8 »	54.3	$t_4 = 0.0761$	$t_4/t_3 = 0.668$
» » 6.3 »	39.7	$t_5 = 0.0556$	$t_5/t_4 = 0.731$

In commenting on these measures, Director *Langley* said: »As the result of these measures, we have then, we

repeat, both in the tables for each day and in the final values, numbers in *a* and *b* which should — if the wedge be of uniform substance and figure, and apart from the consideration of reflection, etc. — increase in geometrical progression for equal increments of thickness, if there is no selective absorption in the wedge. That this is not so, is shown in *c*, where we find on the contrary that there is a very strongly marked progressive increase in the quotients there given«. It should be noted, however, that the bolometric measurement includes the entire range of wave-length, and that the variation in question is smaller in the limited range of radiations which chiefly affect the eye. Professor *Pickering*, in summarizing the tests, says (*Op. cit.*, p. 324) that »valuable results may be obtained with the wedge photometer in skilful hands. But the experiments described above [in the observations by *Young*, the Allegheny observers, and *Pickering*] show sources of error which must be carefully studied before we can safely apply it to stars of different colors, or to detecting small systematic errors in star catalogues«.

Among these tests, the only one which touches the real question, which is a physiological one, relating to the slow development of rod-vision for radiation of short wave-length, is that noticed in the quotation from *Young*. If this effect is noticeable in measures with ordinary light and neutral-tinted glass, it will readily be recognized that much greater effects are to be looked for where strongly colored glasses are used, or with lights of very different quality, and it is to such that the figures on page 275 relate. The causes of discrepancy which have been described here at some length, but still all too briefly for the thorough development of the subject, demonstrate the inapplicability of the extinction method in such work as this, but no serious fault can be found with the use of the wedge, or its equivalent, the simplex, as a standard absorption-piece in the method of equalization of adjoined images.

Record of Observations.

In the following record the times immediately recorded are of the Eastern Standard (five hours slow of Greenwich). The moon's true altitude is computed to the nearest minute for two or three observation-times on each date, and is interpolated for the rest. The phase-angles have been computed to seconds of arc, but it will be sufficient to state them to the nearest tenth of a degree for approximate mean epochs.

In the first observation on Sept. 26, 1911, the moon passed behind trees on the SW horizon before the sky had become dark enough for this work. Sunset, 5^h 36^m; moonset 7^h 31^m. Sky clear.

- 1) At 6^h 45^m, dark part of moon = 0.000511
- 2) » 6 50, sky outside dark limb = 0.000288
- 3) » 6 55?, » » » = 0.000209.

All of these relate merely to fading twilight. The light in 1) was distinctly reddish, or of a quality not very different from that of the unobstructed flame. Nothing of the sort has ever been noticed in earth-shine observations made after the last vestiges of the reddish sunset tints reflected by the atmosphere have vanished, and it seems probable that the reddish color attributed to the earth-shine by some

observers relates to an occasion similar to the present one. Professor *Young* says (General Astronomy, Art. 254): »Near the time of new moon the whole disk of the satellite is easily visible, the portion on which sunlight does not fall being illuminated by a pale ruddy light. This light is earth-shine, the earth as seen from the moon being then nearly full . . . . The ruddy color is due to the fact that light sent to the moon from the earth has twice penetrated our atmosphere and so has acquired the sunset tinge«. But this is a mistake; for, although light which has passed through the deeper and moisture-laden layers of the atmosphere and so has acquired a reddish color, is sometimes refracted upon the eclipsed moon, giving it a ruddy appearance, only a very small fraction of the light reflected by the earth comes from the solid surface, but most of the reflected light comes from the atmosphere and its clouds, and indeed much of the light comes from the upper layers which are neither dense enough nor moist enough to give to the light turned back a reddish color. The source of this error will be recognized when we notice that Professor *Young* supposes that the earth's albedo is less than 50 per cent greater than the moon's. He says: »Probably, taking the clouds and snow into account, the earth's surface on the whole is rather more brilliant than the moon's, so that near new moon, the earth-shine, by which the dark side of the moon is then illuminated, is from fifteen to twenty times as strong as full moonlight«. Noting that this ratio would be 13.4 : 1, if the albedos were the same, this conjecture, as we shall see, enormously underrates the reflecting power of the earth's atmosphere, and assumes that the earth, with its partly snow-covered surface, reflects as if there were no atmosphere, whereas the opacity of the atmosphere is such that only a small fraction of the light which falls upon the center of the hemisphere of the earth, visible from the moon, and scarcely any of that at the earth's limb, can penetrate to the solid or liquid surface and be reflected back again.

If it be objected that the planet Jupiter shows bands of a reddish color and a spot which is often strongly tinged with red, the answer to this objection is that Jupiter has a very much denser atmosphere than the earth and one which is much richer than the earth's in aqueous vapor. The α group in the red, for instance, is very broad and dark in Jupiter's spectrum. Hence the sunset colors may in this case be observed in light reflected from the center of the disk; but only, be it noted, when the reflection comes from belts where the sky of Jupiter is relatively cloudless and where the sun's rays can penetrate deeply. From the clouded surface and at the limb where, in spite of the oblique path, the rays are reflected by upper clouds and do not penetrate the more highly moisture-laden and absorbent layers, the light from Jupiter is not red. *Young's* description evidently applies solely to a remnant of sunset color in sky light. The genuine earth-shine is usually somewhat smaller than the superposed sky light, even in the early crescent phase when the earth-shine is strongest; and I find that the combined effect of the two is matched visually (after the twilight has passed away) by a bluish white light which is such as might come from white sunlight reflected to the moon by floating particles at various elevations in the atmosphere,

mixed with a moderate amount of outwardly scattered blue light (not distinguishable from ordinary sky light), reinforced by the similar and more powerful diffuse blue light of the night sky coming from the neighboring moon; but there is no evidence that the earth-shine is red. It would take too long to examine the literature of the subject extensively and critically; but if we were to believe all of the conflicting statements, we should have to suppose that the color of the earth-shine is as variable as that of *Aesop's* chameleon.

In the present investigation, each measurement of the earth-shine (plus sky light) is compared with that of an equal area of sky, similarly situated with respect to the bright part of the moon, but outside the moon's limb, and a correction for the sky light is obtained in this way.

1911 September 28. Sunset, 5^h 33^m; moonset, 8^h 40^m. Observations began at 7^h 30^m and continued until 8^h, the altitude of the moon changing from 8° 26' to 4° 33'. Moon's age = 6^d 9^h 9^m at 7^h 30^m. The sky was clear at first, but rapidly deteriorated, and at 8^h the fading yellow moon was obscured by thin cirrus cloud. Observed values were — Moon = 0.00277; successive readings on sky, 0.0000333, 0.0000544; earth-shine, 0.0000055. Thus the earth-shine was but a small fraction of the light from a hazy and changing sky, and was so far overpowered by the latter that the measurement is unreliable, as is evident if we consider that successive readings on the shifting sky showed a change four times as great as the apparent earth-shine value. At the last reading, sky light was $\frac{1}{52}$ moonlight. Two nights later, with a good sky, the sky light was about $\frac{1}{3000}$ moonlight. The ratio of sky light to moonlight thus forms an efficient test of the purity of the sky and of the availability of the observations for the earth-shine measurement. Those on this occasion are not available, but are useful to indicate causes of failure and the precautions which are needed.

1911 September 30. Sunset, 5^h 29^m; moonset, 10^h 18^m. Observations began at 6^h 50^m (moon's altitude, 19° 10') and continued until 7^h 30^m (altitude, 17° 15'). Moon's age = 8^d 9^h 6^m at 7^h 2. Sky clear.

The following are samples of some of the observations, and refer in each case to a circular area, 7' in diameter, circumscribing a portion of the moon's surface, or adjacent sky.

Illuminated surface; adjacent to terminator, $M = 0.0428$
 » » » » limb, $M = 0.1145$

Earth-shine (E) plus sky light (S) near East limb = 0.0000544
 Sky light outside East limb = 0.0000370
 $E = 0.0000174$

$E + S$ (near N.E. limb) = 0.0000605
 S outside N.E. limb = 0.0000482
 $E = 0.0000123$

These readings are uncorrected for atmospheric absorption
 Reduced to zenith, the earth-shine values are $\left\{ \begin{array}{l} 0.0000286 \\ 0.0000204 \end{array} \right.$

The moonlight through π was matched by flame through b_2 with adjustment by the iris.

In the earth-shine measures, the flame was modified by $b_2 + g_2$, adjusted by the iris.

1911 October 2. Sunset, $5^h 26^m$; moonset, $12^h 23^m$. Observations commenced at $7^h 10^m$ (moon's altitude, $23^\circ 38'$) with a fairly good sky, but clouds soon formed. Moon's age = $10^d 9^h 5'$ at $7^h 2'$.

1911 October 3. Sunset, $5^h 24^m$; moonset, $13^h 30^m$. Observations at 7^h (moon's altitude, $24^\circ 18'$). Phase-angle = $129^\circ 9'$. Sky clear until clouds stopped observations which were not for earth-shine, but were made on the bright parts of the moon for a phase-curve of intrinsic brightness.

1911 October 7. Sunset, $5^h 17^m$; moonrise, $4^h 58^m$; full moon, $11^h 11^m$. Observations for phase-curve at 9^h (moon's altitude, $38^\circ 4'$). Moon's age = $15^d 11^h 5'$. Phase-angle = $177^\circ 57'$. Sky clear.

1911 October 23. Sunset, $4^h 52^m$; moonset, $5^h 31^m$. Moon's age = $1^d 18^h 8'$ at 6^h . Successive observations of fading twilight sky adjacent to moon: 0.0000895, 0.0000573, 0.0000315, 0.0000139. The twilight at the last was still too strong for a measurement of the earth-shine. Red tints were not noted. Two measurements on the bright and yellow crescent were made with an altitude of $1^\circ 17'$. Light = 0.00396 (uncorrected for atmospheric absorption).

1911 October 26. Sunset, $4^h 47^m$; moonset, $7^h 17^m$. Observations from $5^h 43^m$ (altitude $11^\circ 1'$) to $6^h 30^m$ (altitude $5^\circ 27'$). Moon's age = $4^d 18^h 9'$ at $6^h 1'$. Sky clear. On this occasion, simply as an experiment, the earth-shine was measured tangent to the terminator where it was completely swamped in a diffused sky light of ten times its power, coming from the closely adjacent bright surface. At points of the dark surface more distant from the terminator and compared with the sky outside the dark limb, the combined earth-shine and sky light was about twice the sky light, which was itself $\frac{1}{3083}$ times the moon light; but the sky light adjacent to the bright limb was ten times as great as this.

1911 October 29. Sunset, $4^h 43^m$; moonset, $10^h 7^m$. Observations from $6^h 10^m$ (altitude, $22^\circ 27'$) to $6^h 48^m$ (altitude, $20^\circ 57'$). Moon's age = $7^d 19^h 3'$ at $6^h 5'$. Sky clear.

1911 November 5. Sunset, $4^h 34^m$; moonset, $16^h 58^m$; full moon, $22^h 48^m$. Observations at $6^h 25^m$ (altitude, $24^\circ 4'$) for phase-curve. Phase-angle = $170^\circ 53'$. Sky clear.

1911 November 15, astronomical reckoning (a.m. of the 16^{th} , civil). Sunrise, $18^h 35^m$; moonrise $14^h 3^m$. Observations from $15^h 23^m$ (altitude, $12^\circ 42'$) to $16^h 22^m$ (altitude, $23^\circ 6'$). Moon's age = $25^d 16^h 7'$ at $15^h 9'$. Sky clear.

1911 November 16 (a.m. of the 17^{th} , civil). Sunrise, $18^h 37^m$; moonrise, $15^h 10^m$. Observations from $15^h 38^m$ (altitude, $3^\circ 23'$) to $16^h 48^m$ (altitude, $15^\circ 42'$). Moon's age = $26^d 17^h 0'$ at $16^h 2'$. Exceptionally clear.

1911 November 27. Sunset, $4^h 15^m$; moonset, $10^h 4^m$. Observations $6^h 55^m$ to $7^h 7^m$; mean altitude at $7^h 2^m$ = $23^\circ 27'$. Moon's age = $7^d 3^h 2'$ at 7^h . Sky clear at first. The advance of a bank of haze from the SW was first noticed from a sudden increase in the sky light as measured, and before the haze was appreciable to eye observation.

1911 Dec. 13 (a.m. of 14^{th} , civil). Sunrise, $19^h 5^m$; moonrise, $13^h 3^m$. Observations $18^h 0^m$ to $18^h 6^m$ (mean altitude, $43^\circ 28'$). Moon's age = $24^d 2^h 3'$ at $18^h 1'$. Sky exceptionally clear. The preparations were made hurriedly, as the dawn was near. A smoked lantern chimney reduced the light of the flame and doubled all readings.

1912 Febr. 13 (a.m. of 14^{th} , civil). Sunrise, $18^h 44^m$; moonrise, $17^h 4^m$. There was just time for a single set of measures at $18^h 5^m$ (altitude, $4^\circ 31'$). Moon's age = $26^d 6^h 4'$ at $17^h 7'$. A clear cold morning. Temperature = $+2^\circ$ Fahr. Owing to the low altitude of the moon, the readings are all small and the result is perhaps less reliable than some others.

1912 February 20. Sunset, $5^h 22^m$; moonset, $7^h 57^m$. Observations from $6^h 49^m$ (altitude, $10^\circ 36'$) to $7^h 7^m$ (altitude, $7^\circ 21'$). Moon's age = $2^d 17^h 3'$ at 6^h . No clouds were seen at the start; but as the observations progressed, faint bars of cirrus were noticed, although none seemed to cross the moon.

As an example, the observations of November 16^{th} (morning of the 17^{th} , civil reckoning) are given in full.

1)	$3^h 38^m$ a.m.	Moon (obstructions away)	=	Flame	+ $I_{1.2}$
2)	3 43	" Earth-shine + sky (W limb)	=	" + $b_1 + g_2$	+ $I_{1.2}$
3)	3 48	" Sky (W limb)	=	" + $b_1 + g_2 + n$	+ $I_{0.9}$
4)	3 52	" $E + S$ (W limb)	=	" + $b_1 + g_2 + n$	+ $I_{3.8}$
5)	3 55	" $E + S$ (center)	=	" + $b_1 + g_2 + n$	+ $I_{4.2}$
6)	4 2	" S (near E limb)	=	" + $b_1 + g_2 + n$	+ $I_{2.2}$
7)	4 7	" $E + S$ (near terminator)	=	" + $b_1 + g_2 + n$	+ $I_{4.5}$
8)	4 10	" $E + S$ (NW limb)	=	" + $b_1 + g_2 + n$	+ $I_{3.2}$
9)	4 12	" S (NW limb)	=	" + $b_1 + g_2 + n$	+ $I_{1.1}$
10)	4 15	" Moon	=	" + b_1	+ $I_{5.6}$
11)	4 35	" Moon	=	" + b_1	+ $I_{8.3}$
12)	4 38	" $E + S$ (SW limb)	=	" + $b_1 + g_2 + n$	+ $I_{4.6}$
13)	4 40	" S (SW limb)	=	" + $b_1 + g_2 + n$	+ $I_{1.3}$
14)	4 44	" $E + S$ (center)	=	" + $b_1 + g_2 + n$	+ $I_{5.2}$
15)	4 48	" S (near bright limb)	=	" + $b_1 + g_2 + n$	+ $I_{2.4}$

No.	Moon's Altitude	Object	Photometric value	Reduced to zenith
1)	$3^\circ 23'$	Moon	$0.0120 \times 1 = 0.0120$	0.0623
10)	9 59	"	$0.118 \times 0.098 = 0.0116$	0.0322
11)	13 28	"	$0.257 \times 0.098 = 0.0252$	0.0516

(Moonlight) Mean = 0.0487

No.	Moon's Altitude	Object	Photometric value	E by difference	Reduced to zenith
2)	4° 15'	$E + S$	$0.027 \times 0.000632 = 0.0000171$	0.0000157	(0.0000730)
3)	5 10	S	$0.020 \times 0.0000689 = 0.0000014$		
4)	5 53	$E + S$	$0.110 \times \text{»} = 0.0000076$	0.0000062	0.0000243
5)	6 25	$E + S$	$0.130 \times \text{»} = 0.0000090$	0.0000053	0.0000297
6)	7 40	S	$0.054 \times \text{»} = 0.0000037$		
7)	8 33	$E + S$	$0.143 \times \text{»} = 0.0000099$	0.0000062	0.0000192
8)	9 6	$E + S$	$0.087 \times \text{»} = 0.0000060$	0.0000043	0.0000127
9)	9 27	S	$0.024 \times \text{»} = 0.0000017$		
12)	14 0	$E + S$	$0.147 \times \text{»} = 0.0000101$	0.0000080	0.0000158
13)	14 20	S	$0.030 \times \text{»} = 0.0000021$		
14)	15 0	$E + S$	$0.178 \times \text{»} = 0.0000123$	0.0000082	0.0000153
15)	15 42	S	$0.060 \times \text{»} = 0.0000041$		

Mean Earth-shine (omitting No. 2) = 0.0000195

Ratio Moonlight : earth-shine = 0.0487 : 0.0000195 = 2497 : 1.

The first earth-shine measurement is omitted, as the previously protected eye was unduly sensitive to faint blue light.

Comparing the original measures, the ratio of the sky light near the moon's bright limb to moon light is

$$0.0000041/0.0163 = 1/3976.$$

Taking the mean of 9) and 13) for the sky outside the moon's dark limb (NW and SW quadrants), the sky ratio is

$$0.0000019/0.0163 = 1/8597,$$

which is evidence of an uncommonly fine sky, and justifies the description »exceptionally clear« which was entered in the record.

I will not take space to give the other observations in full. Since the lantern flame, though constant enough for relative comparisons during the interval occupied by a single series of measures, could not be trusted to preserve identical luminosity on different dates, there would be little use in presenting the original readings. The final ratios only need be given.

In order to compare observations made with the moon at different altitudes, the variations of atmospheric transmission must be eliminated as far as possible, and to the extent that these depend upon the air masses penetrated by the rays, this has been done by reducing all observations to the zenith. The value 0.8 is adopted as an approximate coefficient of transmission of the most effective luminous rays by one atmosphere.

The photometric measurement relates to a limited area of the lunar disk, considered to be of average luminous quality, as nearly as may be estimated. Since the color and brightness of the lunar surface vary through a considerable range, this estimate at the best can be made with only a rough approximation. When to this we add the variation of the transmissive quality of the atmosphere, it is evident that we need not expect an exact agreement between the results of different dates. I have compared the adopted moon reading (M) of each date with *Zöllner's* phase-curve, and the differences, which include those from every source, are given in the last column of the next table. The photometric measurement gives the intrinsic brightness, and must be multiplied by the fractional part of the lunar disk occupied by the illuminated area, that is, by $0.5 - 0.5 \cos \varphi$, where φ is the phase-angle from new moon. This, being further

multiplied by the instrumental constant $2^{1/4}$ to get an equivalent of unity for full moon, gives

$$M' = 2.25 M \times (0.5 - 0.5 \cos \varphi),$$

which should be comparable with the total illumination. The original values of the moonlight measures, thus treated, are the ones which have been compared with *Zöllner's* curve. The distribution of the positive and negative residuals, and their amounts, are as satisfactory as we have any right to expect, since the lantern flame can not be taken as an absolute standard and must be added to the variables already mentioned.

Date	Phase-angle (φ)	Moon's intrinsic brightness (M)	(M')	<i>Zöllner's</i> phase- curve (Z)	Difference ($M' - Z$)
1911 Oct. 23	20° 39'6	0.0344	0.0025	0.0025	[0]
1912 Febr. 20	31 30.7	(0.0080)	Rejected	on account	of cloud
1911 Oct. 26	54 3.2	0.0156	0.007	0.018	-0.011
Sept. 28	74 33.6	(0.0080)	Rejected	on account	of cloud
Nov. 27	78 1.3	(0.0161)	"	"	" poor sky
Oct. 29	86 42.8	0.1233	0.131	0.050	+0.081
Sept. 30	95 49.4	0.1126	0.124	0.073	+0.051
Oct. 2	117 54.6	0.0689	0.114	0.204	-0.090
Oct. 3	129 8.7	0.3134	0.508	0.310	+0.198
Nov. 5	170 53.4	0.2299	0.514	0.870	-0.356
Oct. 7	177 57.4	0.4310	0.969	0.969	[0]
Dec. 14 a.m.	-69 16.3	0.1210	(To be multiplied by $1/2$ for smoked chimney)		
»		(0.0605)	0.044	0.026	+0.018
Nov. 16 a.m.	-51 53.3	0.0676	0.028	0.014	+0.014
1912 Febr. 14 a.m.	-42 5.6	0.0253	0.007	0.010	-0.003
1911 Nov. 17 a.m.	-39 58.8	0.0487	0.013	0.009	+0.004

The next table gives the intensity of the light of the sky at distances from the bright part of the moon, comparable to those at which the earth-shine is measured, and expressed as a fraction of moonlight, so that the smaller

the fraction, the purer is the sky. The third column contains the ratio of moonlight divided by earth-shine, being the average for the date.

1911-12	Sky light Moonlight	Moonlight Earth-shin.	Phase- angle	Remarks
Sept. 28	1/52	549	74.6	Haze
30	1/3095	4596	95.8	Clear
Oct. 2	1/1149	3313	117.9	Clear to cirro-stratus
26	1/3033	2889	54.1	Clear
29	1/3626	14540	86.7	"
Nov. 16 a.m.	1/1871	2290	-51.9	"
17 a.m.	1/8579	2735	-40.0	Exceptionally clear
27	1/1358	1932	78.0	Clear to hazy
Dec. 14 a.m.	1/9380	2224	-69.3	Exceptionally clear
Febr. 14 a.m.	1/10164	5769	-42.1	Clear to cirro-stratus ¹⁾
20	1/2476	698	31.5	Faint cirrus bars

The average ratio of earth-shine to moonlight (rejecting none of the observations) is 1:3776 for an average phase-angle of 67°.4. If we reject the three most discordant observations, the mean value becomes 1:3218 for an average phase-angle of 68°6.

The measures may be divided into two groups for phase-angles greater and less than 60°.

Earth-shine measures grouped by phase-angles.

Date	φ	Ratio M/E	Intrinsic brightness M	Residuals	
				$E(\text{ratio})$	M
1912 Febr. 20	31.5	698	0.0344	2178	0.0038
1911 Oct. 26	54.1	2889	0.0151	13	0.0231
Nov. 16 a.m.	-51.9	2290	0.0676	586	0.0294
1912 Febr. 14 a.m.	-42.1	5769	0.0253	2893	0.0129
1911 Nov. 17 a.m.	-40.0	2735	0.0487	141	0.0105
Mean.	43.9	2876	0.0382		
		± 491	± 0.0067		
1911 Sept. 28	74.6	549	0.0080	3977	0.0569
Nov. 27	78.0	1932	0.0161	2594	0.0488
Oct. 29	86.7	14540	0.1233	10014	0.0584
Sept. 30	95.8	4596	0.1126	70	0.0477
Oct. 2	117.9	3313	0.0689	1213	0.0040
Dec. 14 a.m.	-69.3	2224	0.0605	2302	0.0044
Mean	87.1	4526	0.0649		
		± 1271	± 0.0139		

The ratio of intrinsic brightness of the moon at $\varphi = 44^\circ$ and $\varphi = 87^\circ$ is

$$M_{44} : M_{87} = 0.0382 : 0.0649 = 1 : 1.699.$$

The variation is barely beyond the limits of possible error; and it will therefore be better to take this ratio from the smoothing curve (Fig. 3, p. 274) which is founded on a larger number of measures. This gives

$$M_{44} : M_{87} = 0.098 : 0.152 = 1 : 1.55.$$

The ratio of earth-shine (expressed as a fraction of moonlight) at these phases is

$$E_{44} : E_{87} = 1/2876 : 1/4526 = 0.0003477 : 0.0002210 = 1.573 : 1.$$

Reducing the earth-shine to what it would have been with constant moonlight for comparison, we get

$$E_{44} : E_{87} = (0.0003477 \times 1.55) : 0.0002210 = 0.0005389 : 0.0002210 = 2.438 : 1.$$

If we assume that the illumination cast upon the moon by the earth varies according to the same law that Müller found for the planet Venus, we find

$$E_{44} : E_{87} = 63 : 30 = 2.100 : 1 \text{ (computed).}$$

The albedo of the earth no doubt varies with the cloudiness of the hemisphere presented to the moon, and its phase law need not be identical with that of Venus.

I have been unable to consult a determination of the intensity of the earth-shine by *Dusejour*, and the only other visual photometric measurements which I have seen are those of *Arago* and *Laugier* (Compare Works of *Arago*, Vol. X, p. 294, 1858). I do not know how accurately these observations coincide with the epoch of quadrature (given without further specification at the reference cited); but assuming that the coincidence was meant to be exact, we should have from the mean of measures at first and last quarters, the following fractions representing earth-shine in terms of moonlight:

- 1) $\varphi = 44^\circ$, Earth-shine = $1/2876$ (F. W. V.)
- 2) $\varphi = 87^\circ$, " = $1/4526$ "
- 3) $\varphi = 90^\circ$, " = $1/5515$ (A and L.).

For $\varphi = 0^\circ$, $\frac{\text{Moonlight}}{\text{Earth-shine}} = 2876 \times 0.63 = 1812$ from 1)

$$\begin{aligned} & \text{"} = 4526 \times 0.30 = 1358 \quad \text{"} 2) \\ & \text{"} = 5515 \times 0.28 = 1544 \quad \text{"} 3) \end{aligned}$$

$$\text{Mean} = 1571$$

In round numbers we may say that the earth-shine at new moon has an intrinsic brightness about $1/1600$ of moonlight of average quality, such as we receive shortly before first quarter.

Discussion of the earth-shine observations.

The observations of the earth-shine by visual photometry are so delicate that the results must be examined with careful discrimination in order to reach a reliable conclusion. The color difference between the earth-shine and the comparison flame is very great, making the use of a blue glass over the flame necessary. The employment of such a medium, cobalt glass, for example, does not make the composition of light from the two sources identical in quality, and physiological effects still have some influence in producing unbalanced diversities which can only be entirely removed by spectro-photometric measures. These are not considered in the present comparison, but will be introduced later. It becomes necessary, therefore, to examine the remaining anomalies and uncertainties which are inherent where very faint light of a bluish color is to be measured photometrically. It is well known that the eye is especially sensitive to blue and violet light after prolonged rest in darkness, and as the faint

¹⁾ The moon reading is, if anything, too small; but the sky and earth-shine readings appear to have deviated even more on the side of deficiency. The appearance of the sky did not warrant so small an amount of sky light as that given by the fraction $1/10164$.

earth-shine was richer in these rays than even the modified lamp-light transmitted through a light blue glass, there was some reason to expect that variations attributable to this physiological cause might appear. The following anomalies are explicable in this way.

On several occasions, in the anxiety to get evening measures before the low-lying crescent moon had set, observations were commenced before the eye had acquired sufficient sensitiveness, and the first measurement of the earth-shine showed a very much smaller intensity than subsequent ones. On the other hand, when measures of the earth-shine were made in the morning, the eyes being exceptionally sensitive after the night's sleep, and the observing eye being kept shielded until observations began, the initial measurement gave to the earth-shine an intensity much above that of the subsequent measures. This is shown in the morning measures of Nov. 17, but not in that of Nov. 16, when the precaution of shielding the eye had not been followed. The initial observations of Sept. 30 and Nov. 17 have been rejected on this account. They would, in fact, be rejected by any rational criterion founded on probable errors.

Even after the correction for variation of atmospheric transmission on account of the very different altitudes of the moon in individual measures has been applied, there still remain diversities due to differences in the quality of the sky as to haziness, and the number of observations is perhaps rather small for the formation of an absolute phase-curve, but as my curve differs little from that of other observers, I shall retain it.

In order to be strictly comparable, all observations must be reduced to a common altitude. The air masses (ϵ), used in obtaining the transmission, have been taken proportional to the secant of the zenith distance for altitudes above 25° , but to the refraction in seconds of arc, multiplied by $0.0174/\cos \text{alt.}$ for altitudes below 25° . The observations are reduced to zenith on the assumption that the apparent transmission by one atmosphere for average moonlight is 0.8, and the transmission is taken equal to $(0.8)^\epsilon$ for altitudes above 10° . For altitudes below 10° , the increasingly yellow color of the moonlight corresponds to an average light of longer wave-length, requiring a larger coefficient of transmission than 0.8, and these values have been obtained by prolonging the transmission-curve between 10° and 20° altitude for the smaller altitudes. This is not intended as a definite mode of obtaining transmissions for the low altitudes. My object was to collect a consistent body of lunar measures freed from variations depending on change of altitude, and this process was found by trial to give the desired result.

From the nature of the research, the occasions on which it can be prosecuted are of limited duration, and are widely separated as to season. The accuracy is also especially liable to be infringed because of adverse weather conditions, since the measures frequently have to be made at a particular hour of the day. The comparisons of moonlight and earth-shine were not simultaneous, and therefore any observation on a given date which varies from the mean to an extent beyond what is to be attributed to error of observation is unsuitable for such a comparison, unless there is good reason to believe that the changes affect all of the measures equally.

In the main, it seems probable that this is the case in the present measures. There are obviously a few which could be improved, but it is always hazardous to make a selection among observations of this character, and I prefer to let them stand at their face value, rejecting none. An uneven sky can evidently give ratios which may be either too large or too small; but even under this unfavorable condition, such variations ought to cancel out in the average. A few words of comment may be desirable in regard to extreme cases.

With a single exception, the only earth-shine comparisons which differ widely from the mean of the date, or from that of the entire series, have been obtained on occasions which would be considered less auspicious on account of a large value for the light from the sky, or in some cases from the recognized presence of cirrus streaks. On two occasions (namely Febr. 14, 1912, and the first observation of Nov. 17, 1911), the readings were exceptionally small by reason of the very low altitude of the moon, and the earth-shine readings, being at the extreme range of the photometer, are less reliable than usual. When, on Sept. 28, 1911, the uniformly hazy sky was especially luminous (ratio of sky light to moonlight = 1 : 52), the ratio of earth-shine to moonlight had the large and impossible value of 1 : 549. This with the equally impossible earth-shine ratio, 1 : 698, on Febr. 20, 1912 (when there were visible cirrus streaks) would have to be rejected, if they did not serve to balance other results which err in the opposite direction, and of which an extreme instance is the measurement of October 29, 1911. Without attempting any further conjectural explanations of these anomalies, it seems probable to the writer that the averaging of the observations in only two groups will eliminate most of the fortuitous discrepancies.

The curve of average intrinsic brightness of the moon at different phases (lower curve of Fig. 3) is founded on a mean for selected areas, not discriminating first from last quarter, introducing the factor $2^{1/4}$ in order to reduce to unity for mean intrinsic brightness at full moon. The reduced values are these:

$\varphi = 20^\circ 7$	$54^\circ 1$	$78^\circ 0$	$86^\circ 7$	$95^\circ 8$
$2.25 M = 0.0774$	0.0340	0.0362	0.2774	0.1566
$\varphi = 117^\circ 9$	$129^\circ 1$	$170^\circ 9$	$178^\circ 0$	
$2.25 M = 0.1550$	0.7052	0.5173	0.9698	
$\varphi = -69^\circ 3$	$-51^\circ 9$	$-42^\circ 1$	$-40^\circ 0$	
$2.25 M = 0.1361$	0.1521	0.0569	0.1096	

Each of these photometric values in the bright part of the lunar image is the mean for several selected areas, so grouped as to give what the observer considered a fair value for a region of average quality. My observations, therefore are not immediately comparable with those of *Arago* and *Laugier*, who gave widely differing ratios of earth-shine to moonlight at first and last quarters, because they referred their values to the average for an entire hemisphere, whence their standard of reference for moonlight was low at the last quarter when the widely spreading dark areas of *Oceanus Procellarum* predominate, and high at first quarter when bright areas are in excess. My measures must be compared with the mean given by these observers, and I have drawn the upper

curve of Fig. 3 so that it shall pass between this mean at $\varphi = 90^\circ$ and my own observations at 87° .

The lower curve of intrinsic brightness (Fig. 3) gives 0.16 for $\varphi = 90^\circ$. One half of this, or 0.08, represents the fraction of the total light of the full moon received on the average at quadrature. Zöllner's curve for total moonlight, extended to quadrature, gives 0.06. An observation by Bond gives 0.106, and my own being 0.08, a mean of the three is 0.082. The low readings at 78° and 118° were caused by poor sky. Those at 42° and 54° were made at low altitudes and appear to have suffered extra absorption by the mists of the horizon, which has not been entirely compensated in the reduction to zenith. The high value at 129° is the result of a conventional selection in which it was necessary to harmonize values differing in the ratio of 1:30 between a position within, but adjacent to the terminator, in the dark Oceanus Procellarum, and the bright limb. This case illustrates the difficulty experienced in reaching consistent estimates of the moon's widely ranging intrinsic brightness. Similar difficulties of course attend the selection of areas for earth-shine observations, but by varying the selection, a fair average may be reached.

The wide departure of the law of reflection for moonlight with varying phase, when compared with that for the planet Venus, is due to the shadows cast by inequalities on the lunar surface and to their variation during the changing phases, and also to the variable presentation of specularly reflecting surfaces, which, however, occupy only a small portion of the moon's area. The earth-shine is not affected by either of these changes, since the rays from the earth return upon their own track. Consequently, no earth-shine shadows can be visible, and in this respect the rays reflected from the earth to the moon are constantly in that condition reached by the direct solar rays only at full moon. Hence, if for no other reason, the earth-shine must be compared with full moonlight in order to have both illuminations received under the same presentation. The fraction $1/5100$ expresses very well the relation of earth-shine to moonlight when both earth and moon are at quadrature. We wish to find their relation when each body is at the full, and thus at epochs separated by half a lunation. This will give us the data for the calculation of the earth's albedo, to which we now proceed.

The symbol E in what follows expresses the earth-shine on the moon in terms of the average intrinsic brightness of an equal area of the moon's surface, illuminated by sunlight when the moon is full. M is the intrinsic brightness of the bright part of the moon at the time of observation, and is the average for regions believed to be approximately comparable, as to reflecting power, with those from which the earth-shine comes. The value of M used in the present connection is the fraction for the lunar phase-angle φ , M for full moon being unity. Thus E and M are no longer given in arbitrary photometric units. Consequently, if we measure

M and the ratio M/E , which is the observed ratio of the moon's intrinsic brightness under solar illumination to the moon's intrinsic brightness under earthlight at lunar phase-angle φ , we can eliminate M and obtain the earth-shine at lunar phase-angle φ , or earth-phase $\varphi + 180^\circ$. This observed value of E must next be reduced to the full-earth value by dividing E by F , which is the fraction of earth-shine reflected at $\varphi + 180^\circ$ (reflection for full earth = 1).

The quantity E/F , or the ratio of full earth-shine to full moonlight, is now to be reduced to equal angular area for the two bodies, since we wish to compare the albedoes, which are their respective intrinsic brightnesses.

Let E_m = the value which the full earth-shine would have in terms of full moonlight if the earth's albedo were the same as the moon's (which of course it is not). A_e = earth's albedo. A_m = moon's albedo = 0.174, according to Zöllner. Then taking Zöllner's value of the ratio of sunlight to full moonlight, namely, 618000:1, since the earth has a section 13.4 times that of the moon,

$$E_m = 13.4/618000 = 1/46120,$$

and

$$A_e = A_m \times \frac{\left[\frac{M}{M/E} / F \right]}{E_m}$$

Having learned from a preliminary reduction that the earth's albedo is not very different from that of the planet Venus, I assume that the earth's reflection of light to the moon varies with the phase-angle of the earth according to the law determined for that planet by Dr. Gustav Müller (*»Helligkeitsbestimmungen von Planeten«*. Publ. d. Astrophys. Obs. Potsdam, Band 8, Seite 318, Tabelle VIII). This gives $F = 0.282$ at quadrature, from which, with my value, $M = 0.16$, and the earth-shine ratios: 1) $M/E = 7030:1$ at first quarter, obtained by Arago and Laugier, 2) $M/E = 4000:1$ at last quarter, according to the same observers, and 3) $M/E = 5100:1$ from the curve (Fig. 3) which assigns equal weight to the measures of these observers and my own, we get

$$\begin{array}{l} 1) A_e = 0.6477 \\ 2) A_e = 1.132 \\ 3) A_e = 0.8925 \end{array} \left. \vphantom{\begin{array}{l} 1) \\ 2) \\ 3) \end{array}} \right\} 0.8899$$

The first of these is a minimum value. The second is of course impossible; but as I have said, the two values given by Arago and Laugier should be combined into one mean, and as a final result we may adopt for the albedo of the earth

$$A_e = 0.89.$$

In an article in B. A. for January, 1913¹⁾, I intimated that the earth's albedo might be something like 70 per cent on the assumption that the solar constant is 3.0 cal./sq. cm min. in equivalent thermal units; and in two articles in the Ap. J. of the same date²⁾, I show reason to believe that the solar constant is as much as 3.6 cal./sq. cm min. The present determination of the earth's albedo favors the higher value.

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Westwood Astrophysical Observatory, 1913 June 28.

¹⁾ "Sur la détermination de la constante solaire récemment obtenue par les observateurs de l'Institution Smithsonianne". B. A. t. 30, p. 1.

²⁾ "A High-level Measurement of Solar Radiation", Ap. J. 37, p. 25-30; and "A Criterion of Accuracy in Measurements of Atmospheric Transmission of Solar Radiation", p. 31-47.