SCHOOL SCIENCE AND MATHEMATICS

VOL. XIV, No. 2

FEBRUARY, 1914

WHOLE No. 112

SCIENCE AND MATHEMATICS IN VOCATIONAL SCHOOLS: A RETROSPECT.

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To be invited to address the Central Association is a great honor; for me to speak before the Association in this city, on any phase of present day American education is a precarious undertaking, for many of you are national leaders, and Des Moines is the seat of five universities and colleges. I consider it, therefore, the better part of valor to take refuge into the past and translate myself to foreign lands.

Vocational schools are not of recent origin. The oldest vocational institution in America was Harvard College, founded in 1636 as a place for the training of young men for the ministry. Schools preparing for the British Navy existed at the time of Sir Isaac Newton, whose advice was sought as to the best courses to be offered.² Vocational training goes back as far as Greek antiquity.³ To write the history of vocational institutions would be to fill ponderous volumes. In recent years a new type of vocational school has sprung into existence, as the result of the rapid progress of science, the new industrial, social and economic conditions. The disappearance of the old system of apprenticeship, the introduction of the factory, the intense struggle for industrial supremacy, the growth of the cities, the greater laxity of home discipline, have thrust new responsibilities and burdens upon the state and the school for the training of youths during the early adolescent period. It is of these recent movements, as originated in Europe, that I desire more particularly to speak on this occasion.

In America a boy has been permitted by the state, and very often by his parents, to leave school at the age of 14, and to map out his own course of life, in spite of his ignorance of life and his

¹ Read before the Central Association of Science and Mathematics Teachers at Des Moines, Iowa, Friday, November 28, 1913.

² J. Edleston, *Correspondence of Sir Isaac Newton and Professor Cotes*. London, 1850, pp. 279-287.

³ L. F. Anderson, *School Review*, Vol. 20, 1912, pp. 191-201.

inefficiency as a workman. Such a boy is in danger of becoming a recruit in the army of the unemployed, and to become a menace to society. Have the parents and the state done their duty? No. Such a boy is like a horse harnessed to a wagon and left unhitched in the street when the railroad train rushes by. The horse does not want to run away, yet cannot help it. If capable of utterance it would say, "Somebody please stop me, please hold me." He feels the need of superior guidance. He must feel the bit now and then. And so the state should step in and say, "Boy of fourteen, you must either go to a high school, or if you go to work you must attend a continuation school to learn your trade and to prepare for good citizenship."

The first effective solution of the problem of guiding and restricting young boys and girls was given in Germany. The Germans have fully recognized the importance of employing the period of adolescence for consolidating and fixing permanently the knowledge acquired during childhood, and of training efficient and intelligent workmen. The continuation school does this for some boys and girls. For others who wish to continue in a regular full-time school and who are not attracted by the old form of academic or classic secondary curriculum, but desire courses that are more immediately practical, there was formed the intermediate technical school, or technical high school. Munich has been in the lead in this great movement.

On January 1, 1876, obligatory continuation schools were opened in Munich for boys. At first boys between 13 and 16, who were working, were required to attend a continuation school from two to five P. M. on Wednesdays and for five hours on Sundays, the course of instruction being reading, writing, arithmetic and drawing. As the work of these schools was far from satisfactory, Schulrath Kerschensteiner matured plans of reorganization. In 1900 the schools were remodeled in accordance with the ideas set forth in Kerschensteiner's famous Prize Essay. The course of instruction was modified so as to fit the occupations of the pupils, who were classified according to their trades. In this reorganization, the vocational feature was introduced, though even then there was little attempt at practical work. This new scheme was only moderately successful. A few years later Kerschensteiner introduced further changes. By laying much greater emphasis upon practical work, he aimed to secure to a higher degree the interest and good-will of pupils and, in the end, to accomplish really more along the line of the theoretical subjects of

drawing, arithmetic and the elementary sciences. Kerschens-
steiner secured also the active sympathy of employers and trade
associations. The employers were encouraged to visit the schools,
to give advice regarding the schemes of instruction, the choice of
teachers, the time of day when the pupils could best be spared
by the employers. Compulsory attendance on Sundays and even-
ings was done away with. The last touches in the scheme of re-
organization were effected in 1910. As now constituted, the
courses of study, though severely practical, pay attention to the
training of citizenship. *Lebens-und Buergerkunde* are taught in
every continuation and trade school in Munich.

These new schools have led to new problems. Attended by
pupils of less maturity than those of the higher technical schools,
new courses of study and modified modes of teaching were called
for. The psychologic side demanded careful consideration.
Eager to get at what is immediately practical, and fretting under
the task of learning that which seems to them of no value, they
must be led to see the motive of every step in their training.
Much experimentation has been and is still in process as to the
best mode of conducting such schools. What subjects should be
taught, and how? What branches of science and mathematics
should be taken up, and in what way can the instruction be made
most effective?

The psychological necessity of keeping all work in science and
mathematics in constant touch with practical problems, the neces-
sity of avoiding, in fact, all theoretical instruction except when
forced upon the pupil and teacher by the demands of a practical
problem, makes it often impossible to develop a topic in a sys-
tematic manner. The clue to the instruction must be sought in
the boy's interest and aptitude, rather than in any logical ar-
rangement of the subject matter. Moreover, students in electrical
work may encounter wholly different problems from students
in plumbing. These considerations, combined with difficulties
arising from the lack of uniformity in preparation, and the im-
maturity of the pupil, give point to a remark made in an Austrian
report⁴ to the effect that the "instruction in trade schools makes
indeed the greatest demands upon the individuality of the teach-
er." The problem of securing properly prepared teachers has
been found to be a difficult one. In reply to the question—"Do
you prefer to convert a tradesman into a teacher or a teacher into
a tradesman?"—Dr. Kerschens-
steiner declared unhesitatingly in

⁴ *Bericht über den Mathematischen Unterricht in Oesterreich.* Heft 4, Wien, 1910, p. 40.

favor of the former course.⁵ However, such instructors have usually attended courses in the art of teaching. As a rule compelled to proceed without the use of a text-book, the teacher's power of adaptability to various exigencies that may arise is taxed to a degree as nowhere else. Nowhere is the demand so imperative for truly great teachers, as there is in the teaching of mathematics and science in vocational schools.

Here is the general plan as given by an English teacher of practical mathematics, Mr. Sanderson of the Oundle School⁶: "An attempt is made to look upon the class as a staff of workmen, actually engaged in some 'live' work. . . . Mathematics come in incidentally and are learnt as need arises. The various mathematical principles and methods are thus acquired indirectly by continually applying them, and the boy learns his elements of mathematics in much the same way as he learns to walk. At a later stage he may put his knowledge into a logical frame-work."

Here are the words of a teacher in a building trade school in Leipzig⁷: "In our opinion, the mathematical instruction in technical schools, in order to lead to satisfactory results, must grow out of the practice and then grow into the practice."

What are some of the fundamental principles which are essential according to European experience?

First: A boy will grasp a mathematical process or a scientific law with much greater ease if it is introduced to him in connection with a practical problem.

Second: The average boy can much more readily be brought to *undertake independent thinking* when he is placed in contact with a practical problem; the average boy usually abhors abstractions.

Third: The elementary trade school should aim to develop power of technical thinking, and also to impart technical knowledge and skill. The mind of the boy is to be exercised and trained by direct contact with the concrete and practical rather than by the more formal study of mathematics and scientific law, as given in the non-trade high school.

Fourth: The contents of the instruction in a continuation or trade school must be determined by the practical need of the community and the special tastes and aptitudes of the pupil.

Proceeding to the consideration of details, we find some things

⁵ *Leith School Board. Report on Visit to Continuation and Trade Schools in Germany, Paris and London, 1912, p. 9.*

⁶ *The Teaching of Mathematics in the United Kingdom. Report No. 33, London, 1912, p. 5.*

⁷ G. Ehrig, *Mathematischer Unterricht an Baugewerkenschulen*, Leipzig, 1904, p. 16.

in the old country on which we ought to be able to improve, and other things which we can well afford to adopt. During the past decade a tremendous impetus has come from Germany, under the leadership of Felix Klein, of Göttingen, toward the establishment of a more perfect union between different branches of elementary mathematics—arithmetic with algebra, algebra with geometry and trigonometry—and toward more graphic work and greater stress upon functional relations. It is somewhat surprising that these radical reforms in mathematical teaching have as yet hardly reached down to the mass of elementary, German technical schools which we are considering. To be sure, geometry and drawing are quite extensively correlated, so are arithmetic and bookkeeping, particularly in Munich. But the examination of German courses of study reveals the fact that arithmetic and algebra are taken up quite independently; so are algebra and geometry. The idea of unification has not yet been practically tested in the German schools now under consideration. That a certain amount of unification is desirable, few of us would deny. That any mode of unifying mathematics is an improvement on the old method, is far from self-evident. Over-sanguine reformers may easily commit serious blunders. Fusion may mean confusion. There is room here for careful and thoughtful experimentation, to ascertain to what extent unification may be beneficially adopted. There is probably greater need of unification in elementary technical schools for the reason that in these schools mathematics are not taken up except when absolutely needed, and arithmetic, algebra, geometry and drawing are most likely needed at one and the same time in the study of some one practical trade problem. The trade school invites not only for the observance of the organic interrelation between different branches of elementary mathematics, but also for the intimate correlation of mathematics with the work of the shop. It makes a wide appeal for the exercise of self-activity on the part of the pupil. In Munich, "nothing is drawn that is not made in the workshop, and nothing is made that is not drawn." Nowhere is the appeal for the teaching of mathematics as one subject made more strongly than in England. Says one of the reports of work in evening technical schools⁸:

"The main principle is that mathematics is taught as one subject and not as many. . . . Results are only recognized when reduced to a numerical form. Every aid to obtain numerical re-

⁸ *The Teaching of Mathematics in the United Kingdom*. Report No. 24, London, 1912, p. 5.

sults, or to express them clearly, is made use of. Thus the use of logarithms, the use of the slide rule, the graphical representation of results on squared paper, are all made prominent in the work, the range of which covers the elements of algebra, trigonometry, dynamics, and statics."

Germany can give us lessons in the teaching of arithmetic. In 1872 Germany adopted the metric system. The bitterness engendered by the Franco-Prussian war did not prevent the conquering Germans from adopting an improvement introduced by the French. The adoption of the metric system resulted in greater emphasis being placed on decimal fractions and less on common fractions.⁹ The study of the intensely practical subject of denominate numbers is simplified enormously; the metric system is one of the greatest inventions ever made along the line of greater efficiency and mental economy. It is humiliating that England and the United States have not followed the lead of other civilized nations in relieving children of the great burden of mastering antiquated and discredited systems of measures. No one other reform can aid a boy of 14 as effectively in preparing himself for industrial life as the introduction of the metric system. Of course, there are inconveniences and expenses in making the change. So there is in moving from an old house into a new house. But where is the country which, having once adopted the metric system, has seriously considered a change back to the old one. In our judgment, the failure of the United States to introduce the metric system is due to the apathy of its scientific men and its teachers. Have we as teachers carried on an effective propaganda? No. The American Association for the Advancement of Science, the National Educational Association, the American Mathematical Society, and other scientific bodies should exert their influence by yearly petitions, repeated petitions, until the object is attained. We need the persistency of the Roman censor who, in season and out of season, came forth with the declaration: "*Delenda est Carthago*—Carthage must be destroyed."

The subject of proportion is still awaiting to be recast in German trade schools. The antiquated terms of arithmetical, geometrical and harmonic proportion have not wholly disappeared from books.¹⁰ The treatment of proportion in Germany, as well as in the United States, is still encumbered by a clumsy notation; the quantities involved are looked upon as rigid. Proportion should

⁹ W. Lietzmann, *Stoff u. Methode des Rechenunterrichts in Deutschland*, Leipzig u. Berlin, 1912, p. 88.

¹⁰ H. Grünbaum, *Der mathematische Unterricht an den d. mittleren Fachschulen der Maschinenindustrie*, Leipzig u. Berlin, 1910, p. 71; G. Ehrig, *op. cit.* P. 22.

be brought into closer union with the simple equation. The advance guard of German teachers is emphasizing the concept of variation, of functional relation, with its graphic representation, by the use of squared paper. Can the graphic representation of *empirical* data not be taught earlier than customary? The experiment is worthy of careful trial. In a few schools it has been tried. Here is the testimony from the head of the manual training school of Christ's Hospital¹¹: "Compare for example the mental attitude towards graphs of a boy whose first experience of the linear graph is derived from the set obtained in an experiment performed by himself with that of a boy whose first knowledge of a linear graph is obtained from a text-book of algebra. . . . The author who, in the desire to be 'practical,' introduces in his book graphs of results obtained in experiments which boys have *not* themselves performed or seen, runs the risk of being charged with attempting to force a false and artificial correlation between mathematics and another science."

The lack of inner connection between topics in German commercial arithmetics is deplored in a report prepared by Penndorf.¹² A unifying element for the various parts of arithmetic is the equation. The introduction of it in a simple, careful way, at first without the use of negative numbers, has been found not only to impart a coherence between different parts of arithmetic, but also to supply an easy symbolism, $y=ax$, for the expression of the functional idea. This mode of exposition is just beginning to find wider consideration in elementary German texts.¹³ Other countries appear to have been in the lead in this particular respect. An Austrian report speaks of the emphasis early placed upon the solving of easy literal equations with respect to each of the letters contained therein.¹⁴ An English teacher¹⁵ suggests that algebra "should have its starting point at problems leading to simple equations, or at mensuration formulae such as: area of a triangle= $\frac{1}{2}bh$."

Of importance is the emphasis placed upon mental arithmetic, especially for commercial demands. In this respect foreign countries have been in the lead. The money-earning significance of mental arithmetic, even for girls, has come to the notice of American teachers through the criticisms passed by heads of our department stores and similar establishments. Many a high-mind-

¹¹ *The Teaching of Mathematics in the United Kingdom*. Report No. 12, London, 1912, p. 10.

¹² B. Penndorf, *Rechnen u. Mathematik im Unterricht der Kaufmännischen Lehranstalten*, Leipzig u. Berlin, 1912, p. 95.

¹³ H. Grünbaum, *op. cit.*, p. 72; G. Ehrig, *op. cit.*, p. 44.

¹⁴ *Bericht über den math. Unterricht in Oesterreich*, Heft 4, Wien, 1910, p. 43.

¹⁵ *The Teaching of Mathematics in the United Kingdom*. Report No. 3, London, 1911, p. 9.

ed girl of attractive appearance has been found wanting because of her inability to do the arithmetical computations connected with ordinary sales and the making of change. The same problem arose in Berlin and was attacked in this wise. A committee composed of representative business men mapped out a course of study to be pursued for three years in a compulsory continuation school for sales girls. The course was officially approved by the continuation school authorities. It went into effect in April of the present year. The course includes arithmetic and bookkeeping for three years, stress being placed upon rapid calculation. It includes also German and correspondence, the science of living, labor conditions, household arts, etc.¹⁶

A closer union of plane and solid geometry has been maintained in the old country than in the United States.¹⁷ In elementary technical schools the limited time available for instruction, and especially for the study of problems growing out of the work-shop practice, renders such a union especially desirable.

The use of logarithms and the slide-rule can be introduced much earlier than is done at present. Simple considerations like $10^2 \times 10^3 = 10^5$, and $10^5 \div 10^2 = 10^3$ are sufficient to convey an idea of the theory. Then abundant drill should be given in the actual use of logarithmic tables and the slide-rule. The most difficult part in the use of the tables is interpolation. From Sweden comes the recommendation that four place tables be so constructed that interpolation is done away with.¹⁸ Experiments carried on in Sweden with parallel classes showed greater speed and accuracy resulting from the abandonment of interpolation.

The use of concrete multipliers has found little favor in elementary schools in Europe,¹⁹ though the subject has been agitated a few times. In my judgment there is an opportunity for American trade schools to take the lead in this matter. There seems to be no greater objection to the introduction of expressions like 7 cm. $\times$ 8 cm. = 56 □ cm., or 7 kg. $\times$ 5 m. = 35 kgm. than there is to the introduction of negative numbers into algebra. We have, in fact, been in the habit of using concrete multipliers, without perhaps being aware of it, in saying that 1-half times 3-eighths are

¹⁶ Report of the Commissioner of Education for 1912, Washington, 1913, Vol. I, p. 548.

¹⁷ Bericht über math. Unterricht in Oesterreich. Heft 1, Wien, 1910, p. 37; Schotten's Zeitschrift, Vol. 42, 1911, p. 570.

¹⁸ Der math. Unterricht in Schweden, Stockholm, 1911 (Carl Heüman, p. 21).

¹⁹ W. Lietzmann, *op. cit.*, p. 67; Hoffmann's Zeitschrift, Bd. 15, 16; O. Lodge, *Easy Mathematics*, London, 1906, Chap. V, XXVI, Appendix II; A. Lodge in *Association for the Impr. of Geom. Teaching*, 14th Report, Bedford, 1888, pp. 47-70, also 17th Report, 1897, pp. 10-22.

3-sixteenths. Practical convenience seems to demand this extension.

Of some interest, possibly, may be the attitude toward examinations maintained in elementary trade schools across the Atlantic. In a general way it may be said that the system of examinations is much more thoroughly entrenched in schools abroad than here, though in recent years old country rigidity has been lessened somewhat.²⁰ The experience in teaching of practical mathematics in England offers interesting information on this point. It is found that practical work cannot be readily tested by the ordinary type of examinations. "Results as tested by examinations may be disappointing," writes Mr. Bell.²¹ "It is hard to devise an examination which is a fair test, but this does not condemn the work. The boy understands what he is doing, as shown by the fact that he can make use of previous experiments, and make correct deductions from them; but it needs a much higher standard of training before he can show an examiner that he understands." We read further, "If possible, we must suit our methods of examination to the methods of teaching, though it is admitted that there may be serious difficulties in doing this. The difficulties may be surmountable; but even if they are not, it would seem to be undesirable to alter sound methods of teaching to suit examinations. And meanwhile we must be prepared for anomalies wherever examinations come into contact with new methods." Whatever the actual *practice* may be, few teachers champion the *theory* that methods of teaching be altered to suit the examinations. Their sane judgment is that we should modify the examination so that it becomes a real test of the kind of efficiency aimed at. If the aim of the instruction is not primarily to develop power of expression, but to develop a certain manual dexterity, or to make a certain type of drawing, or to carry out some observation or experiment, or to perform computations of costs, then the examination should follow strictly the same lines. We do not test the quality of foot-ball coaching by requiring the team to take a written examination. In Germany this feature appears to be recognized. The examinations are conducted by practical men. "An apprentice does not become a journeyman . . . merely by serving a specified number of years. For apprentices who have served their time, examinations are held in the continuation schools by the guilds or by the Chamber of Trades. Those who have a bad record at the continuation schools, or who fail in

²⁰ Bericht über den math. Unterricht in Oesterreich. Heft 1, Wien, 1910, pp. 49-60; *Nature*, 1912, pp. 587-589.

²¹ *The Teaching of Mathematics in the United Kingdom*, Report No. 33, London, 1912, p. 8, 9.

these examinations, may be kept an extra year at the continuation schools. Those who show sufficient knowledge and skill receive a journeyman's certificate."²²

In German continuation schools, the technical instruction generally includes the elements of geometry, drawing, the handling of materials, tools, machines and working models. In some few large industrial continuation schools the rudiments of algebra, physics, chemistry, natural science and mechanics are taught. The problem how to make science teaching in industrial high schools contribute toward a greater immediate efficiency gives rise to interesting questions. In the teaching of chemistry in secondary schools England has emphasized the pedagogical side and the training in scientific method. In America the aim has been to offer a scientific treatment of the elements of the science.²³ For the purposes of those continuation schools and industrial high schools which teach chemistry or physics, neither type of instructional method is satisfactory. The latter schools demand differentiation and specialization. The selection of such topics as are of immediate value to the pupil or the trade, is the primary consideration. The keynote to the instruction in the Leather-sellers College at Bermondsey in London has been the chemistry of tanning, in the L. C. C. School of Photo-engraving it has been the "three-color" process.²⁴ The German attitude is indicated by the outline of the elementary course in a Munich trade school for tanners. Pupils conduct experiments touching the production of white leather, the glacé- and chamois-dressed leather; they study the chemistry of hard and soft water, of the detection of injurious material in water, and of other technical matters of importance in tanning.²⁵

In the selection of topics and of their sequence, the question of logical order and of mental gymnastics is receiving less and less attention. Instead of beginning with definitions and general principles, the tendency now is to begin with such problems and apparatus as the pupil meets in daily life, as, for instance, a water motor, a hoisting frame or a gas stove. The formulation of principles involved comes last. This modern tendency may result in much good and also in great harm. There is danger of going too far. There seems no good reason why one should abandon the old practice of leading up gradually by a series of well-ordered preliminary exercises to the study of the more difficult topics.

²² *Leith School Board, op. cit.*, p. 4.

²³ "Industrial Education" in Monroe's *Cyclopedia of Education*, 1912.

²⁴ *Nature*, Vol. 91, 1913, p. 281.

²⁵ *Organization und Lehrpläne der obligatorischen Fachu. Fortbildungsschulen für Knaben in München. Mit einer Einleitung von Schulrath Georg Kerschensteiner.* München, 1910, p. 132.

The sequence of topics is often a vital question. For example, should the study of electricity be begun with static electricity or current electricity? Static electricity still maintains its place of priority in some old country technikums of good standing. Mr. C. E. Ashford, formerly at Harrow, now of the Royal Naval College at Dartmouth,²⁶ makes a convincing plea in favor of current electricity because of the practical applications which appeal to the ordinary boy, and the mastery of which adds to the boy's immediate efficiency.

It might be argued that not only should we begin, as suggested, with current electricity, but we should start the course in electricity with a study of the dynamo. Thereby the young boy is confronted at once with the subtleties of this most highly developed practical electric machine. Such a procedure raises the question of driving young boys too rapidly, of over-estimating their capacity. Here, as elsewhere, it is highly necessary to observe moderation.

The charge has been made that American eagerness to reach immediate and startling reforms has led to great mental epidemics, which have periodically swept the American people. Great excesses are committed, followed by reactions to opposite extremes. I greatly fear that American education has not always been free of this charge. But it is now generally recognized that we cannot afford to expose ourselves to such reckless procedure. The suggestions from abroad are for good or for evil, according to the degree of mental equipoise with which we receive them. The departure from logical order of development, to secure the immediate consideration of practical topics, calls for the exercise of no little restraint. The stress laid upon the applications of science may degenerate into the study of rules, devoid of attention to underlying principles. The avoidance of abstractions may lead to the avoidance of all thought. The unification of mathematics is a bane or a blessing, according to our degree of precipitous haste, or of progressive conservatism.

In this brief address I have endeavored to present to you some of the thoughts and experiences of educators in the old world. I have looked back rather than forward. I have somewhat illuminated the track behind. You, as sober, practical men, who are advancing the great chariot of educational progress, will be able in your wisdom to reflect this light and use it as a searchlight for the exploration of the way ahead.

²⁶ *Nature*, Vol. 88, 1912, p. 393.