

Interference Alignment Based Resource Management in MIMO Cognitive Radio Systems

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Abstract—In this paper, interference alignment (IA) is utilized to obtain an efficient and fair resource allocation algorithm in MIMO cognitive radio (CR) systems. In the proposed algorithm, IA enables all secondary users to share the available spectrum without affecting the quality-of-service of the primary system. The considered methodology increases the total degrees-of-freedom of the CR systems and achieves fairness among CR users. An optimal power allocation based on IA is formulated in order to maximize the total sum-rate while keeping the interference introduced to the primary system lower than the prescribed interference threshold. Furthermore, a sub-optimal power allocation scheme is proposed to overcome the high computational complexity of the optimal scheme. Simulations reveal that IA technique achieves significant sum-rate increase of CR systems compared to frequency division multiple access (FDMA) CR systems. Moreover, the sub-optimal algorithm approaches the optimal sum-rate performance.

Keywords—Cognitive Radio, Interference Alignment, Multicarrier, MIMO, Resource Allocation.

I. INTRODUCTION

The tremendously increasing demand of wireless data traffic requires excessive spectrum usage. Therefore, the scarcity of the spectrum has become a serious concern. Recently, cognitive radio (CR) has been proposed to better utilize the available spectrum, for instance, by opening the possibility of accessing under-utilized spectral resources. Basically, CR allows unlicensed (secondary) users to reuse frequency bands that are originally allocated to licensed (primary) users without affecting the quality of service (QoS) of the licensed networks [1]. Multiple-input and multiple-output (MIMO) technology and multicarrier communications have been recognized as desirable candidates for CR systems, where MIMO significantly improves the spectral efficiency by sending independent data streams simultaneously over multiple antennas, while multicarrier (MC) communications offer an important flexibility in allocating different resources between users and subcarriers [2], [3].

In CR networks, maximizing the throughput of the secondary users (SUs) while ensuring QoS of the primary user (PUs) is considered as a fundamental challenge. In order to handle this issue, many resource management approaches have been proposed in different scenarios. The problem of resource allocation for CR networks with single antenna employed at both PUs and SUs has been widely considered (e.g. [4], [5] and references therein). Furthermore, there have been many studies

on CR networks with multiple antennas at the SUs in order to achieve CR regulations by cognitive beamforming (e.g. [2], [6]). Recently, a cooperative paradigm for CR networks has been proposed for CR network with multiple antennas at the PUs and SUs in order to utilize the MIMO advantage to cooperatively relay the traffic for the PUs (e.g. [7]–[9]).

In this context, resource management on the base of interference alignment (IA) has been recently considered in order to improve the spectral efficiency of CR networks. IA is a proficient cooperative transmission approach that aims ideally to provide each user with half degrees-of-freedom of the system in order to achieve an optimal sum-rate for K -user interference channels [10]. The basic idea of IA in MIMO interference channels is to use a combination of linear precoder at the transmitter side and interference suppression decoder at the receiver side in order to minimize the dimensionality of the interference subspace [11], [12]. Meanwhile, the existing works of IA in CR have considered MIMO employment on both of PUs and SUs in order to allow SUs to utilize the free and non-free eigenmodes of PUs. In this scenario, MIMO employment at PUs provides extra degrees of freedom to null the interference at the PU receivers and to exclude interference constraints from the optimization problem (e.g. [13]–[16]). However, to the best of our knowledge, single antenna employment at PUs in IA based CR systems has not been considered.

In this work, we use IA technique to improve the spectral efficiency of CR systems, where each SU node with MIMO employment is co-located with PUs each with a single antenna. In this scenario, PUs doesn't provide MIMO's spatial degrees of freedom. This limitation increases the optimization problem complexity as several interference constraints should be added to the optimization problem. Therefore, a spectrally efficient resource management scheme based on IA is proposed for this scenario aiming at achieving the optimal throughput of the CR system. The fairness between SUs is guaranteed as the SUs are allowed to share the available spectrum resources. Moreover, power allocation is managed to maximize the sum-rate of the CR system under the total power budget and interference constraints. A sub-optimal resource management algorithm is proposed to reduce the computational complexity. Performance evaluation of the proposed algorithm is presented in multicarrier based CR systems.

In the following section, the system is modeled and the CR optimization problem with IA is reformulated. Section III shows the optimal and the proposed low-complexity power

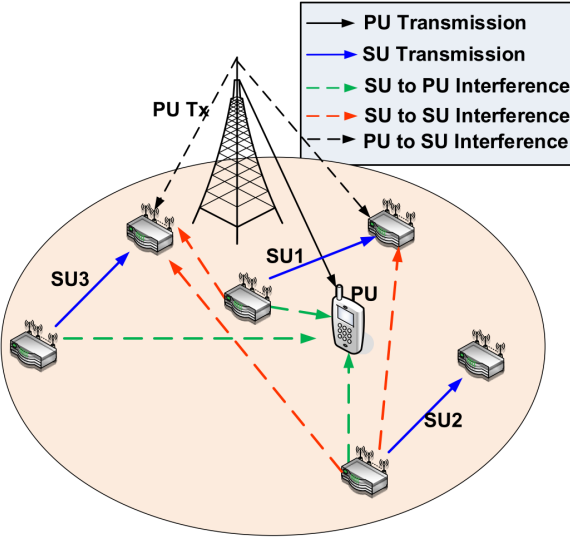


Fig. 1: Cognitive Radio Network.

allocation algorithms. Simulation results are illustrated and discussed for multicarrier based CR systems in section IV. Finally, Section V concludes the paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this work, we consider K pairs of secondary transmitters and receivers that coexist with several PUs in the same geographical area as seen in Fig.1. SUs and PUs use the side-by-side frequency bands, where the L active primary bands (W_1, W_2, \dots, W_L) have been occupied by the PUs. The available spectrum is divided into N subcarriers with a Δf separation as seen in Fig. 2. It is assumed that the CR network can utilize the non-active and active PU bands subject to the constraint that total generated interference to the l^{th} PU active band does not exceed I_{th}^l .

It is assumed that all PU nodes are equipped with a single antenna while each secondary pair has M_T transmit antennas and M_R receive antennas. Spectrum sharing among different SUs is allowed on a given subcarrier instead of restricting the transmission to one user at a given time. Hence, MIMO IA technique is used in order to manage the interference between SUs pairs via precoding matrices. We can apply IA independently on each subcarrier, thanks to the frequency orthogonality introduced by the multicarrier techniques. Therefore, for the n^{th} subcarrier, the d symbol data streams $\mathbf{x}_k^n \in \mathbb{C}^{d \times 1}$ are pre-multiplied with a precoder matrix $\mathbf{V}_k^n \in \mathbb{C}^{M_T \times d}$ before being transmitted from the k^{th} transmitter in order to align the desired data at its own receiver in interference-free subspaces while the interference signals from other SU transmitters are aligned at interference subspace [10], [12]. Assuming the channel state information CSI is perfectly known at each node, the discrete-time complex received signal at the k^{th} receiver over the n^{th} subcarrier is represented as

$$\mathbf{y}_k^n = \mathbf{U}_k^{nH} \mathbf{H}_k^n \mathbf{V}_k^n \mathbf{x}_k^n + \sum_{j=1, j \neq k}^K \mathbf{U}_k^{nH} \mathbf{H}_{kj}^n \mathbf{V}_j^n \mathbf{x}_j^n + \mathbf{U}_k^{nH} \mathbf{z}_k^n \quad (1)$$

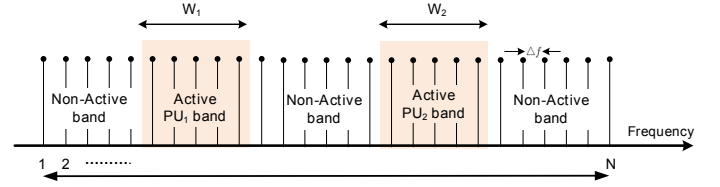


Fig. 2: Active licensed bands and cognitive bands.

where $\mathbf{U}_k^n \in \mathbb{C}^{M_R \times d}$ is an orthonormal linear interference suppression matrix applied at the k^{th} SU receiver, $\mathbf{H}_{kj}^n \in \mathbb{C}^{M_R \times M_T}$ denotes the channel frequency response between j^{th} SU transmitter and k^{th} SU receiver, and $\mathbf{z}_k^n \in \mathbb{C}^{M_R \times 1}$ is the zero mean unit variance circularly symmetric additive white Gaussian noise (AWGN) vector at SU receiver k .

At the k^{th} SU receiver, only the interference from undesired SU transmitters will be aligned and cancelled if perfect IA is achieved. That is [10]

$$\text{rank}(\mathbf{U}_k^{nH} \mathbf{H}_{kk}^n \mathbf{V}_k^n) = d \quad \forall k \quad \text{and} \quad \forall n \quad (2)$$

and

$$\mathbf{U}_k^{nH} \mathbf{H}_{kj}^n \mathbf{V}_j^n = 0 \quad \forall j \neq k \quad \text{and} \quad \forall n \quad (3)$$

The received signal in (1) becomes after perfect IA

$$\mathbf{y}_k^n = \mathbf{U}_k^{nH} \mathbf{H}_k^n \mathbf{V}_k^n \mathbf{x}_k^n + \mathbf{U}_k^{nH} \mathbf{z}_k^n \quad (4)$$

Referring to MIMO IA feasibility, perfect interference alignment would be proper with the condition $M_T + M_R - d(K + 1) \geq 0$ [17], [18]. Therefore, if the number of SUs exceeds the feasibility condition, SUs are clustered into disjoint groups where IA is performed to users within each cluster [19], [20]. Accordingly, without loss of generality, a proper CR cluster is assumed throughout this work, where $K = 3$ with $M_T = M_R = 2$ and $d = 1$.

According to the proposed cluster scenario, IA solution can be obtained with a closed-form solution, where the interference is completely eliminated at each SU receiver. Therefore, the sum-rate of the SUs over the n^{th} subcarrier is [21]

$$R^n = \sum_{k=1}^K \log \left| \mathbf{I}_d + \frac{1}{\sigma_{\text{AWGN}}^2 + J_{l,k}^n} \mathbf{U}_k^{nH} \mathbf{H}_{kk}^n \mathbf{V}_k^n \mathbf{S}_k^n \mathbf{V}_k^{nH} \mathbf{H}_{kk}^n \mathbf{U}_k^n \right| \quad (5)$$

where $\mathbf{S}_k^n \in \mathbb{R}^{d \times d}$ is the input covariance matrix for the k^{th} SU user at the n^{th} subcarrier, which can be described as

$$\mathbf{S}_k^n = \mathbb{E} [\mathbf{x}_k^n \mathbf{x}_k^{nH}]. \quad (6)$$

Therefore, the transmitted power by the k^{th} SU user over the n^{th} subcarrier is $P_k^n = \text{Tr}(\mathbf{S}_k^n)$. Moreover, J_l^n is the total interference introduced by the l^{th} PU transmitter in the n^{th} subcarrier to the k^{th} CR user, and can be expressed as [22]

$$J_{l,k}^n(Dn) = \sum_{m=1}^{M_R} \left(\int_{Dn-\Delta f/2}^{Dn+\Delta f/2} |y_{l,k}^{n,m}|^2 \psi_l(e^{j\omega}) d\omega \right) \quad (7)$$

where Dn represents the spectral distance between the n^{th} CR subcarrier and l^{th} PU band. $\psi_l(e^{j\omega})$ is the power spectral density (PSD) of the l^{th} PU signal and $y_{l,k}^{n,m}$ is the channel

gain between the m^{th} SU antenna at the k^{th} SU receiver and l^{th} PU signal over the n^{th} subcarrier. Moreover, the interference introduced by the k^{th} SU transmitter over the n^{th} CR subcarrier transmission to the l^{th} PU receiver can be expressed as [22]

$$I_{l,k}^n(Dn, P_k^n) = \sum_{m=1}^{M_T} \left(\int_{Dn-W_l/2}^{Dn+W_l/2} |g_{l,k}^{n,m}|^2 P_{m,k}^n \Phi^n(f) df \right) \quad (8)$$

where $g_{m,k}^n$ denotes the channel gain between the m^{th} antenna of the k^{th} SU transmitter and the l^{th} PU over the n^{th} subcarrier, $P_{m,k}^n$ denotes the power transmitted from the m^{th} transmit antenna of the k^{th} SU over subcarrier n and Φ^n is the PSD of the n^{th} subcarrier. Eq. (8) can be reformulated into

$$I_{l,k}^n(Dn, P_k^n) = \text{Tr} \left(\Omega_l^n \mathbf{G}_{k,l}^n \mathbf{V}_k^n \mathbf{S}_k^n \mathbf{V}_k^{nH} \mathbf{G}_{k,l}^{nH} \right) \quad (9)$$

where $\mathbf{G}_{k,l}^n \in \mathbb{C}^{1 \times M_T}$ denotes the channel gain between the k^{th} SU transmitter and the l^{th} PU over the n^{th} subcarrier and Ω_l^n is the interference factor of the l^{th} subcarrier to the l^{th} PU, which is represented as

$$\Omega_l^n = \int_{Dn-W_l/2}^{Dn+W_l/2} \Phi^n(f) df. \quad (10)$$

Our objective is to maximize the total throughput of the CR system subject to the interference introduced to the PUs and total transmit power budget constraints. Since each PU has only single antenna, the interference from SUs to PUs should be considered. Moreover, all SUs share the spectrum resources simultaneously using IA technique, which means that a heuristic algorithm to allocate the subcarriers to the different users, as in [23] and references therein, is no further required. Therefore, the problem can be formulated as

$$P1 : \quad \max_{\mathbf{S}_k^n} \sum_{n=1}^N R^n \quad (11a)$$

$$\text{s.t.} : \quad \sum_{n=1}^N \text{Tr}(\mathbf{S}_k^n) \leq P_k \quad \forall k \quad (11b)$$

$$\mathbf{S}_k^n \succeq 0, \quad \forall n \text{ and } \forall k \quad (11c)$$

$$\sum_{n=1}^N \sum_{k=1}^K \Omega_l^n \text{Tr} \left(\mathbf{G}_{k,l}^n \mathbf{V}_k^n \mathbf{S}_k^n \mathbf{V}_k^{nH} \mathbf{G}_{k,l}^{nH} \right) \leq I_{th}^l, \quad \forall l \quad (11d)$$

where (11b) represents the k^{th} SU total power constraint (P_k), while a positive transmission power at each antenna is guaranteed using the constraint in (11c). The constraint in (11d) ensures that the total interference induced by SUs to the l^{th} PU is below the prescribed interference threshold (I_{th}^l).

III. OPTIMAL AND SUP-OPTIMAL POWER ALLOCATION ALGORITHMS

According to our CR system assumptions in the last section, the sum-rate in (5) can be simplified into

$$R^n = \sum_{k=1}^K \log \left(1 + \frac{1}{\sigma_k^{n2}} P_k^n x_k^n \right) \quad (12)$$

where

$$x_k^n \triangleq \mathbf{U}_k^{nH} \mathbf{H}_{kk}^n \mathbf{V}_k^n \mathbf{V}_k^{nH} \mathbf{H}_{kk}^{nH} \mathbf{U}_k^n$$

and $\sum_{l=1}^L J_n^l$ can be modeled as AWGN. This is a general assumption in this research area (e.g. [4], [24]). This assumption is justified using central limit theorem. Therefore, we can write

$$\sigma_k^{n2} = \sigma_{AWGN}^2 + \sum_{l=1}^L J_n^l \quad (13)$$

Accordingly, the optimization problem P1 can be reformulated as

$$P2 : \quad \max_{P_k^n} \sum_{n=1}^N \sum_{k=1}^K \log \left(1 + \frac{1}{\sigma_k^{n2}} P_k^n x_k^n \right) \quad (14a)$$

$$\text{s.t.} : \quad \sum_{n=1}^N P_k^n \leq P_k \quad \forall k \quad (14b)$$

$$P_k^n \geq 0, \quad \forall n \text{ and } \forall k \quad (14c)$$

$$\sum_{n=1}^N \sum_{k=1}^K \Omega_l^n P_k^n \text{Tr} \left(\mathbf{G}_{k,l}^n \mathbf{V}_k^n \mathbf{V}_k^{nH} \mathbf{G}_{k,l}^{nH} \right) \leq I_{th}^l, \quad \forall l \quad (14d)$$

The problem P2 is a convex optimization problem. The Lagrangian can be written as

$$\begin{aligned} G = & - \sum_{n=1}^N \sum_{k=1}^K \log \left(1 + \frac{1}{\sigma_k^{n2}} P_k^n x_k^n \right) + \sum_{k=1}^K \beta_k \left(\sum_{n=1}^N P_k^n - P_k \right) \\ & + \sum_{l=1}^L \alpha^l \left(\sum_{n=1}^N \sum_{k=1}^K \Omega_l^n P_k^n \text{Tr} \left(\mathbf{G}_{k,l}^n \mathbf{V}_k^n \mathbf{V}_k^{nH} \mathbf{G}_{k,l}^{nH} \right) - I_{th}^l \right) \\ & - \sum_{n=1}^N \sum_{k=1}^K P_k^n \mu_k^n \end{aligned} \quad (15)$$

where β_k , α^l and μ_k^n are the non-negative Lagrange multipliers. (15) can be solved as

$$P_k^n = \left[\frac{1}{\sum_{l=1}^L \alpha^l \Omega_l^n \text{Tr} \left(\mathbf{G}_{k,l}^n \mathbf{V}_k^n \mathbf{V}_k^{nH} \mathbf{G}_{k,l}^{nH} \right) + \sum_{k=1}^K \beta_k} - \frac{\sigma_k^{n2}}{x_k^n} \right]^+ \quad (16)$$

where $[y]^+ = \max(0, y)$. The optimal solution of problem P1 requires high computational complexity and its complexity grows exponentially with the number of subcarriers, which is unacceptable in practical application. This motivates to propose a less complexity algorithm to approach the optimal solution.

By ignoring the per-SU power constraints and considering only the l^{th} PU interference constraint, the problem is reduced to

$$P3 : \quad \max_{\hat{P}_k^n} \sum_{n=1}^N \sum_{k=1}^K \log \left(1 + \frac{1}{\sigma_k^{n2}} \hat{P}_k^n x_k^n \right) \quad (17a)$$

$$\text{s.t.} : \quad \sum_{n=1}^N \sum_{k=1}^K \Omega_l^n \hat{P}_k^n \text{Tr} \left(\mathbf{G}_{k,l}^n \mathbf{V}_k^n \mathbf{V}_k^{nH} \mathbf{G}_{k,l}^{nH} \right) \leq I_{th}^l, \quad (17b)$$

$$\hat{P}_k^n \geq 0, \quad \forall n \text{ and } \forall k \quad (17c)$$

where $(\hat{\cdot})$ represents the variables that are optimized under the interference constraint only. By solving P3, we obtain

$$\hat{P}_k^n(l) = \left[\frac{1}{\hat{\alpha}^l \Omega_l^n \text{Tr}(\mathbf{G}_{k,l}^n \mathbf{V}_k^n \mathbf{V}_k^{nH} \mathbf{G}_{k,l}^{nH})} - \frac{\sigma_k^{n2}}{x_k^n} \right]^+ \quad (18)$$

where the Lagrange multiplier $\hat{\alpha}^l$ is evaluated using (18) and (17b) as

$$\alpha^l = \frac{|NK|}{I_{th}^l + \sum_{n=1}^N \sum_{k=1}^K \frac{\Omega_l^n \sigma_k^{n2} \text{Tr}(\mathbf{G}_{k,l}^n \mathbf{V}_k^n \mathbf{V}_k^{nH} \mathbf{G}_{k,l}^{nH})}{x_k^n}} \quad (19)$$

If (18) and (19) satisfy the per-SU power constraint in (14b), then the optimal solution for P2 problem is found using optimization problem of P3 considering only interference constraints. Otherwise, P2 can be solved by modifying the mechanism that were proposed in [23] to fit with our IA scenario. We assume that the maximum power $P_k^{n\max}$ that can be allocated for the k^{th} user over the n^{th} subcarrier is decided by considering only the interference constraint as follows

$$P_k^{n\max} = \min \left\{ \hat{P}_k^n(l) \right\}_{l=1}^L \quad (20)$$

Then, if the relation $\sum_{n=1}^N P_k^{n\max} \leq P_k$ for all SUs is satisfied, then the solution for P2 is $P_k^n = P_k^{n\max}$. If not, the power budget for each SU P_k is distributed among all the subcarriers subject to be lower or equal to the power upper-bound of each user at each subcarrier $P_k^{n\max}$. The problem can be reformulated to get a cap-limited waterfilling [25] as

$$P4: \quad \max_{\tilde{P}_k^n} \sum_{n=1}^N \sum_{k=1}^K \log \left(1 + \frac{1}{\sigma_k^{n2}} \tilde{P}_k^n x_k^n \right) \quad (21a)$$

$$\text{s.t. :} \quad \sum_{n=1}^N \tilde{P}_k^n \leq P_k \quad (21b)$$

$$0 \leq \tilde{P}_k^n \leq P_k^{n\max} \quad (21c)$$

where \tilde{P}_k^n is the allocated power by the problem P4. This problem is proceeded using the conventional waterfilling concept. As a starting point, we find the waterfilling solution as [26]

$$P_{k,WF}^n = \left[\lambda - \frac{\sigma_k^{n2}}{x_k^n} \right]^+ \quad (22)$$

where $P_{k,WF}^n$ is the allocated power by waterfilling solution for the k^{th} user at the n^{th} subcarrier and λ is the waterfilling level. Thereafter, if the power allocated by waterfilling solution $P_{k,WF}^n$ is greater than $P_k^{n\max}$, the power is readjusted to $P_k^{n\max}$ and the total power budget is reduced by the difference between them. Then, successive waterfilling is proceeded over the users and subcarriers that did not exceed the maximum power $P_k^{n\max}$ in the last step until the allocated power \tilde{P}_k^n doesn't exceed $P_k^{n\max}$ in any user at any subcarrier in the new iteration. Since we constraint the solution \tilde{P}_k^n of P4 problem to be less than or equal $P_k^{n\max}$, some of the allocated power \tilde{P}_k^n doesn't not reach the maximum allowed power. This leads the system to lose some of the allowed degrees of freedom and, hence, decrease the capacity of CR system. Therefore, we utilize this available power gap by allocating

Algorithm 1 Sup-Optimal Power Allocation Algorithm

- 1: $\forall l \in \{1, \dots, L\}$, Find $\hat{P}_k^n(l)$ using (18) and (19).
- 2: $\forall n$ and $\forall k$, Evaluate $P_k^{n\max} = \min \left\{ \hat{P}_k^n(l) \right\}_{l=1}^L$.
- 3: **if** $\sum_{n=1}^N P_k^{n\max} \leq P_k; \forall k$ **then**
- 4: Let $\tilde{P}_k^n = P_k^{n\max}$ and stop the algorithm.
- 5: **end if**
- 6: $\forall n$ and $\forall k$, Execute the cap-limited waterfilling under the per-user constraint P_k and the maximum power that can be allocated to each subcarrier $P_k^{n\max}$ and find the set B_l where $\tilde{P}_k^n = P_k^{n\max}$.
- 7: Evaluate the residual interference I_R^l using (23) and the updated interference constraints I_{th}^l using (24).
- 8: Perform Steps (1-2) to update $P_k^{n\max}$.
- 9: $\forall n$ and $\forall k$, Execute the cap-limited waterfilling under the per-user constraint P_k and the updated maximum power that can be allocated to each subcarrier $P_k^{n\max}$ and set $\tilde{P}_k^n = \tilde{P}_k^n$.

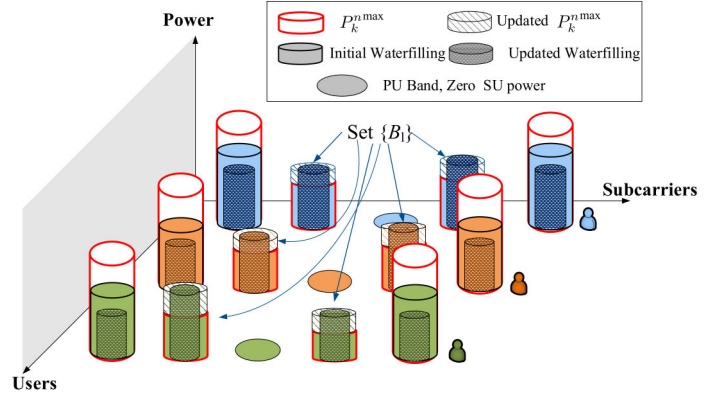


Fig. 3: Graphical representation of the proposed power allocation algorithm.

some power from one subcarrier to another one in order to enhance system throughput. This can be achieved by updating the maximum power that can be allocated to each subcarrier $P_k^{n\max}$ depending on the residual interference, which can be calculated as follows

$$I_R^l = I_{th}^l - \sum_{n=1}^N \sum_{k=1}^K \tilde{P}_k^n \Omega_l^n \text{Tr}(\mathbf{G}_{k,l}^n \mathbf{V}_k^n \mathbf{S}_k^n \mathbf{V}_k^{nH} \mathbf{G}_{k,l}^{nH}) \quad (23)$$

Assuming that B_l is the set of users that reach the maximum allowed power at some subcarriers, i.e. $\tilde{P}_k^n = P_k^{n\max}, \forall k$ and $\forall n \in B_l$, then, $P_k^{n\max}, \forall k$ and $\forall n \in B_l$ can be updated by applying the equations (18)-(20) on the users in the set B_l with the updated interference constraints, which is

$$I_{th}^l = I_R^l + \sum_{n \in B_l} \sum_{k \in B_l} \tilde{P}_k^n \Omega_l^n \text{Tr}(\mathbf{G}_{k,l}^n \mathbf{V}_k^n \mathbf{S}_k^n \mathbf{V}_k^{nH} \mathbf{G}_{k,l}^{nH}) \quad (24)$$

Finally, the procedures of the cap-limited waterfilling that were used to solve problem P4 is re-performed to find the final solution $\tilde{P}_k^n = \tilde{P}_k^n$. At this point, the solution \tilde{P}_k^n is approaching the optimal solution that achieving interference constraints with equality as well as guaranteeing that the total power budget constraints are satisfied. This approach is described in Algorithm 1 and Fig. 3.

IV. SIMULATION SETUP AND RESULTS

In our simulation, we evaluate the performance of MIMO orthogonal frequency division multiplexing (MIMO-OFDM) CR network using IA algorithm (CR-IA) compared to MIMO-OFDM that uses frequency division multiple access (FDMA) technique (CR-FDMA) as an orthogonal transmission scheme under the scenario given in Fig. 1. A ($K = 3$) SUs CR system with $N = 64$ subcarriers is considered. The PSD of the n^{th} subcarrier of the OFDM system can be written as [3]

$$\Phi_n(f) = T_s \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 \quad (25)$$

where P_n is the total transmit power emitted by the n^{th} subcarrier and T_s is the symbol duration which equals 4μ seconds in this simulation. The values of Δf and σ_k^{n2} are assumed to be 0.3125 MHz and 10^{-6} respectively. $M_T = M_R = 2$ antennas at each SU node and single antenna at each PU node are assumed. A 10 MHz active PU band is assumed. All the results have been averaged over 1000 iterations, where channel realizations have been drawn from independent and identically distributed Gaussian distribution with zero mean and unit variance. In our simulations, we consider the sum-rate performance of the optimal resource allocation of CR-IA by simulating problem P1 compared to the optimal resource allocation of CR-FDMA that is simulated using the work in [2]. CVX toolbox is used in our simulation to obtain the optimal solutions [27]. Moreover, the performance of the proposed sub-optimal approach of Algorithm 1 is evaluated.

In terms of complexity, the optimal CR-IA enumeration scheme has the complexity of $\mathcal{O}(N^3)$, which is practically complex to be implemented. For this reason, the sub-optimal approach is proposed. Step 1 in Algorithm 1 has a waterfilling like computational complexity of $\mathcal{O}(N \log N)$. Step 1 should be performed for L primary bands, hence the complexity of step 1 is $\mathcal{O}(LN \log N) \leq \mathcal{O}(KLN \log N)$. Steps 6 and 9 in the algorithm execute the cap-limited waterfilling for all SUs with a complexity $\mathcal{O}(N \log N)$. Accordingly, the complexity of steps 6 and 9 is $\mathcal{O}(KN \log N) \leq \mathcal{O}(KLN \log N)$. Step 8 has a complexity of $\mathcal{O}(|B_l| \log |B_l|) \leq \mathcal{O}(KLN \log N)$ considering all SUs. Hence, The complexity of the proposed sub-optimal algorithm is lower than $\mathcal{O}(KLN \log N)$.

Fig. 4 presents the average sum-rate against the interference thresholds when the per-SU power budget is set to be $P_k = 50$ mWatt. In general, for all scenarios, the average sum rate increases as the interference threshold levels increase since each SU has more flexibility to allocate more power on its subcarriers. It can be observed that the optimal CR-IA algorithm achieves higher sum-rate gain compared to optimal CR-FDMA algorithm. It is further shown that the sub-optimal algorithm presents close sum-rate performance to the optimal CR-IA, and better sum-rate performance than the optimal CR-FDMA algorithm.

Fig. 5 plots the instantaneous data rate for a given user over time for the optimal and the sub-optimal CR-IA compared to CR-FDMA when $I_{th} = -30$ dBm. Assuming that our rate target per SU is $R_{min} = 160$ bits per OFDM symbol, It is noted that the proposed algorithms keep the instantaneous rate mostly above our target, while CR-FDMA rate changes dramatically and causes deep rate degradation at some time samples.

Fig. 6 presents the outage sum-rate probability of the different algorithms when $I_{th} = -30$ dBm, where the minimum

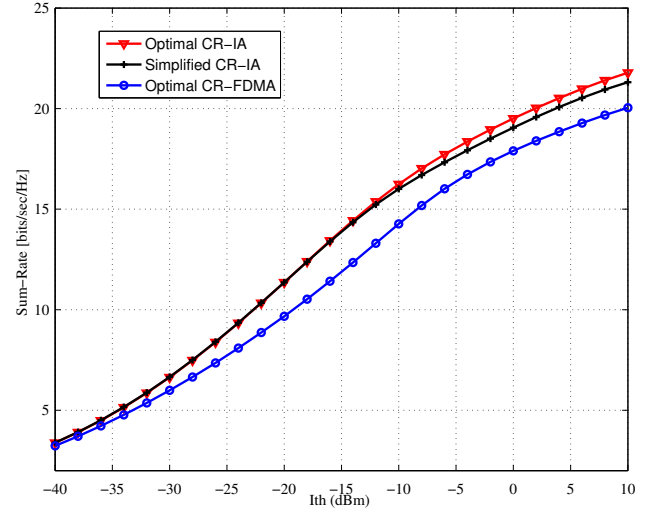


Fig. 4: Achieved sum-rate vs allowed interference threshold.

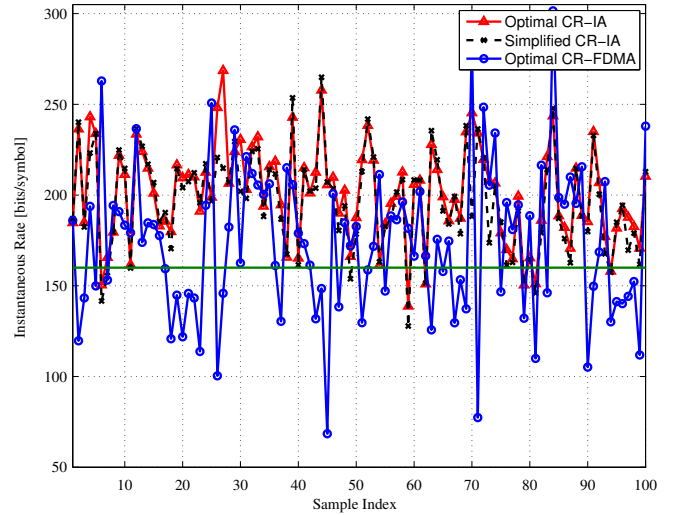


Fig. 5: Achieved Instantaneous Rate.

rate for each SU is set to be 40 Mbits/sec, i.e. $R_{min} = 160$ bits per OFDM symbol. Generally, outage probability decreases as interference constraint increases since the ability of the algorithms to give the minimum instantaneous rate for the different users increases. Furthermore, the outage probability of the sub-optimal CR-IA is very close to optimal CR-IA, and both are much lower than that of the CR-FDMA. It is clearly observed from Fig. 5 and Fig. 6 that CR-IA algorithm is able to achieve a high-level of fairness among the different users.

V. CONCLUSION

In this paper, an efficient resource management algorithm based on IA in MIMO CR systems is presented. In the proposed algorithm, all SUs share the available spectrum using IA technique without affecting the QoS of the primary system. An optimal power allocation among subcarriers on the base of IA is formulated so that the total data rate is maximized while the interference introduced to the primary system remains under the prescribed interference limit.

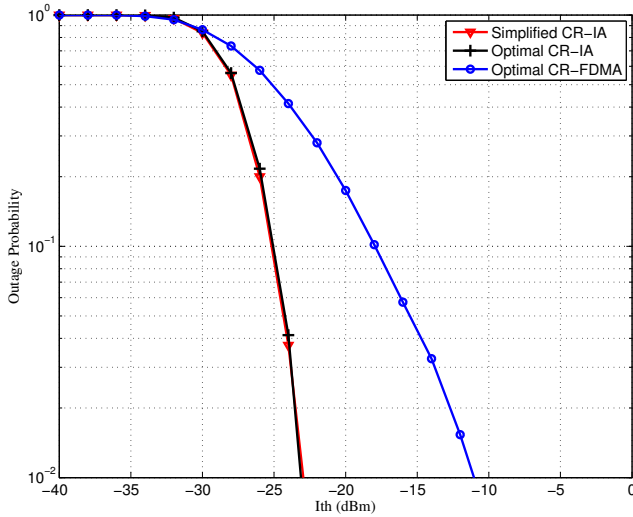


Fig. 6: Outage probability vs interference thresholds, $R_{min} = 40\text{Mbits/sec}$.

Furthermore, a sub-optimal power allocation algorithm is proposed with lower complexity. Simulations show that IA technique achieves significant sum-rate increase of CR systems compared to traditional CR systems (FDMA CR). Moreover, CR-IA algorithm can attain the fairness among the users compared to CR-FDMA. The sub-optimal method approaches the optimal sum-rate performance with less computational complexity.

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