DERIVATION OF EQUATION OF DECAYING SOUND IN
A ROOM AND DEFINITION OF OPEN WINDOW
EQUIVALENT OF ABSORBING POWER.

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IT is required to find the number of units of sound energy which pass out of a room per second through a window of area a . Let V be the volume of the room and i the sound energy per unit volume at time t . It is assumed that the sound energy is the same at each point of the room and that the energy is traveling equally in all directions on the average.

The case under consideration is covered by the flow of energy in one direction through a hole in a wall built across the room so as to divide the room into halves. Describe a sphere of radius r with its center at the hole. The area of this sphere is $4\pi r^2$. We may imagine the sound energy per unit volume, i , to be divided into $4\pi r^2$ equal parts, each part traveling at the normal velocity of sound, 340 meters per second at ordinary room temperature, towards a separate unit of area of the sphere. Consider a diameter D of the sphere drawn through the hole perpendicularly to the wall. Consider the zone of the spherical surface which lies between θ and $\theta + d\theta$, where θ is an angle measured from the diameter D . The area of this zone is $2\pi r^2 \sin \theta \cdot d\theta$. Therefore the fractional part $\frac{2\pi r^2 \sin \theta \cdot d\theta}{4\pi r^2}$ of the total sound energy per unit volume is streaming towards the various elements of this zone at a velocity of $340 \cos \theta$ normal to the wall and $\frac{2\pi r^2 \sin \theta \cdot d\theta}{4\pi r^2} \times i \times 340 \cos \theta \times a$ is the rate at which sound energy is streaming through the hole towards the various elements of the zone. So that the total rate at which sound energy is streaming through the hole from right to left is

$$E = 170ai \int_0^{\pi/2} \sin \theta \cdot \cos \theta \cdot d\theta \quad (a)$$

or

$$E = 85ai. \quad (b)$$

This rate of flow of sound energy out of a room through a window of area a is equal to the rate, $\frac{d(Vi)}{dt}$, at which the total sound energy in the room is decreasing, the walls and objects in the room being assumed for the purposes of the immediate discussion to absorb no sound energy, therefore :

$$\frac{d(Vi)}{dt} = -85ai \quad (c)$$

whence

$$i = Ie^{-\frac{85a}{V} \cdot t} \quad (d)$$

in which e is the naperian base, and I is the value of i when t equals zero.

In an actual room the loss of sound energy due to absorption by the walls and objects in the room is sensibly proportional at each instant to the intensity i of the sound at that instant. That is equation (c) expresses the rate of loss of total sound energy at each instant in case of an actual room, a being the open window area which is equivalent to the walls and objects in the room, and equation (d) expresses the value of the decaying sound intensity i at time t .

Sabine has adopted as the measure of the degree of reverberation in a room the time t_1 that is required for a sound to decrease from a given initial intensity I to an intensity one millionth as great. Therefore, substituting $10^{-6} I$ for i and t_1 for t in equation (d) we have :

$$t_1 = 0.1625 \frac{V}{a}. \quad (e)$$

This result is interesting inasmuch as Sabine finds experimentally the value 0.164 for the factor in this formula.

It seems not to be correct to assume that the absorption of sound in a room is due to incomplete successive reflections between which the sound travels over a certain mean path. Such an assumption

does not take account of dissipation of sound energy due to the friction which occurs when the air particles near a wall move to and fro parallel to the wall. In general the dissipation of energy is due to this action, to friction which occurs when the air particles move to and fro perpendicularly to a porous wall, and to imperfect elasticity of walls and objects as they partake more or less of the motion of the contiguous air. All these losses would be sensibly proportional to the sound intensity i for given frequency.