THE CONDITION OF SECONDARY MATHEMATICAL
INSTRUCTION WITH SOME HINTS AS
TO REMEDIES.

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It is a truism that in all education the attitude of the learner should as nearly as possible parallel that of the investigator. He should not so much be getting told what others have done as himself finding out what to do by doing, making use of what knowledge he has or thinks he has to lead to further ideas, further knowledge, and this both in the way of amplification and correction.

Yet what do we find? Geometry, for example, is presented as a completed and perfected whole substantially as it came from Euclid. If problems are given, these are based not upon the student's ideas and conceptions, but upon the text and with the object mainly of getting the student to learn the text. To be sure, there are manuals written upon what may be called the leading-question method, but the questions always have in mind the old stereotyped form and could hardly be imagined to lead a student to build up a geometry of his own, selecting his own axioms, arranging his own sequence of propositions, supplying his own analysis and criticisms, in brief, asking his own questions, and himself supplying the answers. Nor is instruction in algebra one whit better than in geometry. To the majority of students it must seem the art of juggling with symbols that have nothing but the vaguest of meanings. Almost on a par is it with the old method of learning Latin by memorizing the grammar. I can remember as a boy using an algebra by an author different from that used in the class and yet finding in it the same topics in the same order, and even the same numbers of examples to a topic!

No wonder the physicists have found fault with our teaching. With them the laboratory has been revolutionizing instruction. The investigating spirit has even crept into the high school. But when a next step would be taken, when the stu-

dent should apply the arithmetic, geometry, and algebra that he has been worrying over for years, he has no grasp and fails ignominiously. "We could do better with him by teaching him ourselves in our own laboratories," say the physicists. "Moreover, we could teach him physics at the same time."

Surely it behooves us mathematicians to be up and doing. Either we must turn to and help the physicists or they will supplant us. We must realize the connection that our science has with the other sciences and with life. We must keep ourselves and our science in touch with what is going on around us. Isolation means pedantry, and pedantry means death.

A step forward is it to correlate the arithmetic, algebra, and geometry, to introduce inventional geometry in the grades, to weed out useless technicalities, to simplify demonstrations. All this is good, much of this sort has been done. But it is only a beginning. Inventional geometry should be continued into and through the high school and we should have inventional algebra as well. Observing facts before reasoning upon them, guessing at other facts but always testing our guessing. Nor should this testing cease when the student reasons out his conclusions; for reasoning is but an outgrowth of guessing and, like it, is liable to error. Most useful is that test of conclusions, derived either by computation or by argument, that is furnished by various students working independently, using each his own method. We have in this, what the scientist values above all else, the concurrence of independent investigation, something that the routine following of a text totally fails to give.

Some may object that a text is necessary to give unity and system to the work. In such a way it can be made useful; but the guiding mind of the teacher, by the aid of conferences and the tabulation of results that the members of a class have themselves worked out and which they themselves help to tabulate, can secure a better system, because it is one growing out of the daily work. Moreover, by correlation with other subjects a system can be developed to embrace these also, the student thus learning not merely the detached unity of some fragment of mathematics, but getting a glimpse at least of the unity of all mathematics and all science.

But mathematics has its correlation with art as well. What would music be without rhythm; what would architecture be without symmetry and proportion? Many patterns and designs are plainly geometrical and others only less obviously so. The science of perspective, so useful to the artist, is but a simple development of geometry. Taught in connection with space geometry and accompanied by actual drawings from observation it would halve the difficulty and double the interest and value of the latter subject.

Then there are the numerous practical applications, such as that of plane geometry to surveying, to graphics, to the study of machines.¹ Such tables as the student needs he could mostly construct himself; his working formulas his own algebra would lead him to; and his own common sense should teach him, with perhaps some slight suggestion, that computation carried beyond the limit of accuracy which the data warrant is not only worthless but false and misleading. He will thus be prepared to appreciate justly the help afforded in computation by the slide rule, by graphical construction, or by logarithm tables of a moderate number of places.²

Perhaps the most important correlation of all is that of mathematics with clear English. It is on clear statements and their just and complete apprehension that the student's power of deduction depends. The mere careful writing out of a difficulty will half the time cause the difficulty to vanish. The student should be permitted to use symbols and technical words only so far as it is certain that he exactly grasps their meaning. To this end there must be constant exercise in translation into ordinary language from the technical and back, while even the everyday talk must be freed and purified from all that is ambiguous, meaningless, or misleading. The essence of mathematics is exact truthfulness.

¹Castle, *Workshop Mathematics*. MacMillan.
Harrison and Boxandall, *Practical Plane and Solid Geometry*.

²Whapson and Gee, *Mathematical and Physical Tables*. Macmillan & Co.
Macfarlane, *Elementary Mathematical Tables*. Ginn & Co.
Von Zelzer & Shutts, *Place Tables*. Tracy, Gibbs & Co., Madison, Wis.
E. M. Longley, *Computation*. Longmans, Green & Co.

Besides the work and along with the work there may be mathematical play as well.³ The old "puzzles" and "catch problems" had their place. No better way to teach the meaning of words is there than a problem such as to hopelessly trip the student who has failed to catch that meaning. Very instructive are formal demonstrations proving absurdities as that $3 = 6$ or that all triangles are isosceles. Difficult problems or "riders" that none have to work but which any one can get extra credit by working serve to create and maintain enthusiasm. Occasionally a mathematical tournament might be arranged, the winner being he who can devise the greatest number of problems that he can himself solve but which no other can.

All this will require thought and trouble on the part of the teacher, but the results will amply repay it. Nor is it at all certain that more time will be required for such complete teaching than for the half or quarter teaching now so prevalent. On the contrary, the experiment in the correlation of mathematics and physics under Miss Long in the Lincoln High School,⁴ indicates a considerable saving of time. Nor need we be surprised at this; for a knowledge of sciences, as of things, is largely a knowledge of their relations to each other, their differences and their resemblances, their bearings upon each other. To teach them in segregation is to shut from the student a good part of this knowledge.

If these proposals be called revolutionary, we reply that even though there be crying need of revolution yet we advocate "revolution only by evolution." Out of the present must grow the better, and out of that, in succession, the better still and best. A good beginning can be made by the introduction of tests and quizzes which shall serve the double purpose of bringing to the student's consciousness his deficiencies and of setting his inventive and critical faculties to work. A recent series of observations on the teaching of arithmetic⁵ in the grades indicates the value of this method. I have elsewhere⁶ given some specimens of the

³Ball. *Mathematical Recreations*. MacMillan & Co.

Lucas. *Arithmétique Amusante*. Gauthier-Villars, Paris.

⁴*Educational Review*, December, 1902.

⁵*Forum*, Jan., Feb., 1903.

⁶*Bulletin New York Math. Soc.*, Oct., 1893.

sort of questions in geometry that could come into such a test. The object is not to find out which students may go on (of course those as good as the average are to go on), nor to in some more or less mechanical way determine a grade, though it will well serve these purposes; but to discover the habits of thought of all the students. Thus is one prepared to lead them to better habits. There should be tests in which the student answers on the spur of the moment, and tests in which he is given all the time he wants, a year, if need be. There should be problems suited to all capacities and every encouragement should be offered to all to work at them. Not infrequently a dull student will succeed when all the rest fail, showing, perhaps, that both his more usual failures and his exceptional success are due to his having simply a different mode of thought from the majority. it may be a better habit.

I said this would do for a beginning. It will be a break with formalism. The bond once broken, few, I imagine, will desire to return to the service of Dame Formality.

THE OUTLOOK FOR ARITHMETIC.

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The movement begun nearly twenty years ago in this country, looking to the elimination of obsolete matter from arithmetic, has been more or less successful. Like all such movements, reformatory in character, it has also been destructive. Destruction of a decayed structure not only requires less exertion than the construction of a solid edifice, but if one is to build on the same ground, it is almost a prerequisite. For some time, however, we have simply been contemplating the dust and rubbish pile, glad that the old building is gone, but uncertain as to what should go in its place.

There have not been wanting suggestions as to the new edifice. It has always been true, and it is true still, that the first thought at such a time is to move some house from up the street to the vacant spot. In years past they moved a so-called