Saturn's Rings.

By Percival Lowell.

In the calculation of the effects of perturbation the perturbative function is expressed by a series progressing according to sines and cosines of multiple arcs of the mean motions and to powers of the eccentricities. For the disturbance in the radius vector δr the typical form of one of these terms is:

$$e^x e'^x \frac{P}{(pn - qn')} \cos(pnt - qn't + Q)$$

where P is a function of a, a' , the radii vectores and Q is a function of the epoch and of the places of the perihelia.

(1). Since p and q must always be integers from the form of the series, if two integers can be found nearly expressing the ratio of the mean motions, the term depending on the argument $(pn - qn')$ will be very large since its coefficient is divided by $(pn - qn')$ which is very small.

(2). Secondly from the mode of formation of the argument it appears that the principal coefficient will contain powers of e and e' to $p - q$ dimensions. For these are the smallest powers multiplying n and n' by which the argument can be built up. Airy in his excellent treatise on the planetary theory here remarks: »Consequently p and q must both be small«. This does not necessarily follow. Thus if p were 360 and q 361 while n' and n were in the inverse ratios of these numbers the disturbance would be the same as if the ratios were expressible by small fractions $1/2$ or $2/3$; but the time taken to produce this disturbance would be correspondingly lengthened.

(3). Lastly the smaller both p and q are, the quicker will the perturbation be repeated and the greater be the effect in a given time.

It is to be noticed that for a commensurability of p and q there is a term in which the argument is raised to $(p-1)n - qn'$ and the sum of the eccentricities likewise to $(p-1) + q$. This amounts to going into one less power of the eccentricities though the numeral factor is less than in the term of the argument $pn - qn'$.

A body, then, will be greatly perturbed by another if (1) the periods of the two are commensurable, (2) in proportion as the numerator and denominator of the ratio differ by the least amount, and (3) if the proper fraction be the smallest possible.

The result of commensurability of period is well exem-

plified by the great inequality in the longitudes of Jupiter and Saturn. Owing to the periods being about as 5 to 2 Jupiter is perturbed in longitude by an amount resulting at its maximum to 49' of arc. For the same reasons an analogous perturbation takes place in the radius vector of the disturbed body and with this we have now to do.

One effect of it is the way in which the several planets have swept clear the space about them. In proportion as we approach a planet do the spaces in which a body having a period differing but one revolution from the disturber grow nearer together. Thus $1/2$ and $1/3$ represent orbits far apart, while $169/170$ and $170/171$ lie close and closer as the disturbing body is neared. But a difference of one between the numerator and the denominator of the ratio gives, if time be neglected, the greatest perturbative effect.

In this connection it is immaterial to the observed effect whether the perturbation be restitutive in the long run or not. For the effects will clearly be greatest during the epochs of commensurability and therefore the two bodies will be in a condition of incommensurability for a much longer time than when seemingly commensurable, and therefore bodies of commensurable periods will appear not to exist whether they really do in perpetuity or not.

A very interesting instance of the effects of this principle exists in the case of Saturn's satellites and rings. If we examine the ring-system in the light of commensurability of period with the orbits of Saturn's inner satellites a curious set of relations is exhibited from which it appears that the whole ring system owes its present formation to this secondary outcome of attraction. It is all the more remarkable that the result is not correspondent with the direct attraction of the several satellites. From estimates of the stellar magnitudes of Mimas, Enceladus and Tethys made at this observatory it appears that their diameters are about in the ratio of 1 to 2 to 4 and their masses probably, roughly speaking, in the cubes of these numbers. Thus allowing for distance Tethys should be the most powerful. Yet it is Mimas which has been the chief fashioner of the rings, aided it is true by Enceladus and still less by Tethys. In spite, therefore, of a less direct pull, the principle of commensurability of periods has been the determining factor in the evolution. To show this we will consider the ring-system and the satellites' orbits. The most modern measures give the dimensions of the ring-system as follows:

Radius from centre of Saturn	At distance 9.53887	In radii of Saturn actual reduced to Struve's major diameter		Authority
Outer edge ring A	19.68	2.253	2.253	W. C. Bond, H. Struve
» » » »	20.19	2.311	2.253	See, Barnard, Lewis and Dyson
Encke's division	18.89	2.16	2.11	See
» »			2.09	Lowell ¹⁾
Inner edge ring A	17.42	1.99	1.94	See, Barnard, Lewis and Dyson
Cassini's division				
Outer edge ring B	16.92	1.94	1.89	See, Barnard, Lewis and Dyson
Outer edge of shading in ring B			1.70	Lowell ²⁾
Inner edge ring B	12.86	1.47	1.44	See, Barnard, Lewis and Dyson
Inner edge ring C	10.40	1.19	1.16	See, Barnard, Lewis and Dyson
» » » »			1.23	Lowell ³⁾
Body of planet	8.735			H. Struve from observations of Titan and Rhea M. N. 54.456

It is proper to reduce the values to Struve's exterior diameter because the distances of the satellites are his got from measures of these satellites, the two, therefore, permitting of direct comparison.

Before going on to compare these positions with the structure of the ring-system, some further detections in the arrangement of that system recently made at this observatory must be mentioned⁴⁾. On Oct. 11, 1909 a division was suspected by me in ring B; it was on Nov. 18 evident enough to be drawn one-third way out in B from its inner to its outer edge. On Nov. 4, 5, 18, 19 dark stripes were seen by me in ring A, in the following positions from the inner to the outer edge of that ring:

- 0.40 Encke's division
- 0.67 new division A
- 0.86 new division B.

Now as the masses of the satellites and, still more, those of the particles of the rings are negligible with regard to Saturn, suitable substitution in the formula

$$\frac{a^3}{a'^3} = \frac{T^2}{T'^2}$$

will give us the radii of the orbital distances at which a particle with a given commensurability would circulate. The importance of any ratio depends upon the reasons we have stated; those where the difference between the numerator and denominator of the proper fraction expressing the ratio is least being the most potent. The following table exhibits these.

Ratio of periods	Distance from centre of Saturn in radii of Saturn	Importance graded from 1 down
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Mimas.

$a = 26^{\circ}8'14''$ at dist. 9.53887 = 3.07 radii of Saturn.

$\frac{5}{7}$	2.255	2
$\frac{3}{5}$	2.184	2
$\frac{4}{7}$	2.114	3
$\frac{1}{2}$	1.932	1
$\frac{3}{7}$	1.745	4
$\frac{2}{5}$	1.667	3
$\frac{1}{3}$	1.495	2
$\frac{2}{7}$	1.332	5
$\frac{1}{4}$	1.218	3

Enceladus.

$a = 34^{\circ}40'1''$ at dist. 9.53887 = 3.94 radii of Saturn.

$\frac{3}{7}$	2.239	4
$\frac{2}{5}$	2.138	3
$\frac{1}{3}$	1.893	2
$\frac{2}{7}$	1.708	5
$\frac{1}{4}$	1.563	3
$\frac{1}{5}$	1.347	4

Tethys.

$a = 42^{\circ}58'6''$ at dist. 9.53887 = 4.876 radii of Saturn.

$\frac{2}{7}$	2.115	5
$\frac{1}{4}$	1.935	3
$\frac{1}{5}$	1.667	4

Dione.

$a = 54^{\circ}54'3''$ at dist. 9.53887 = 6.244 radii of Saturn.

$\frac{1}{5}$	2.135	4
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¹⁾ Estimated 0.44 from inner to outer edge of A.

²⁾ Estimated $\frac{4}{7}$ from inner to outer edge of B.

³⁾ Estimated 0.55 from limb to edge C.

⁴⁾ Since the paper was written a good view on Dec. 8 has confirmed these observations and added another:

- 0.14 new division
- 0.41 Encke's division
- 0.65 new division
- 0.83 new division

giving for the new divisions A and B

A = 2.15 radii of Saturn
B = 2.20 radii of Saturn.

If now we compare these places where a particle would be greatly perturbed we find:

Importance

Mimas.

1. The most potent of Mimas commensurabilities falls in Cassini's division, near its outer edge.
2. The three next most effective ones of Mimas occur (1) at the outer confine of the ring-system; (2) at the inner confine of the bright part of it and (3) at a point in the outer ring A not far from where a division has lately been glimpsed at this observatory.
3. The next in order of importance fall (1) at Encke's division and (2) in the midst of the shading on the inner side of ring B; (3) falling inside of everything, near the planet.
4. This marks the point where the shading begins in ring B.
5. This occurs in the midst of the crepe ring.

Enceladus.

1. None.
2. This falls at the inner side of Cassini's division.
3. One of them occurs near Encke's division, the other in the shading of ring B.
4. (1) very near the extreme limit of ring A. A dark

Lowell Observatory, Flagstaff, Ariz., 1909 November.

streak has been glimpsed in that position at this observatory. (2) in the crepe ring.

Importance

5. This occurs near the beginning of the shading of ring B.

Tethys.

1. None.
2. None.
3. At Cassini's division, close to its outer edge.
4. Not far from the beginning of the shading in ring B but inside it.
5. At Encke's division.

Dione.

4. Near Encke's division.

All the ratios of commensurability up to the fifth power of the eccentricities have been examined for each of the four inner satellites. From the way in which they fit into features of the ring-system and furthermore do so in proportion to their force we see that the present form of the system must be due to commensurability of period for the chances of so many coincidences being due to chance is infinitesimally small while on the other hand the principle explains the very beautiful and complicated structure we see.

Percival Lowell.

Maximum von α (Mira) Ceti

beobachtet zu Kasan im Jahre 1909 von dem Studenten der Universität K. May.

[Mitgeteilt von Prof. Dubiago.]

Meine Beobachtungen, ausgeführt nach der Methode Argelanders mit Hilfe eines Opernglases von viermaliger Vergrößerung, erstrecken sich auf einen Zeitraum vom 26. Juli bis zum 4. November 1909. Als Vergleichsterne dienten folgende dem »Atlas stellarum variabilium« P. Hagens entnommene Sterne, wobei ich die von mir ermittelten Stufenwerte beifüge:

	Stufen		Stufen
$m = \mu$ Piscium	0.0	$\vartheta = \vartheta$ Ceti	14.8
$x = \xi$ »	4.1	$\eta = \eta$ »	16.0
$\delta = \delta$ Ceti	6.0	$\gamma = \gamma$ »	17.3
$\xi^2 = \xi^2$ »	7.0	$D = \alpha$ Trianguli	17.6
$\xi^1 = \xi^1$ »	8.0	$\iota = \iota$ Ceti	18.3
$\mu = \mu$ »	8.6	$B = \beta$ Trianguli	23.6
$\zeta = \zeta$ »	8.8	$n = \iota$ Aurigae	28.1
$b = \alpha$ Piscium	9.7	$h = \beta$ »	29.3
$u = \eta$ Eridani	12.3	$a = \alpha$ Piscium	var.?
		$g = \gamma$ Pegasi	var.!

Da sich der Stern $\alpha = \alpha$ Piscium als der Veränderlichkeit verdächtig herausstellte, so wurde für ihn mittels der konstanten Vergleichsterne folgende Skala aufgestellt:

1909	Stufen	1909	Stufen
Aug. 1	15.0	Okt. 17	8.9
2	15.8	20	7.0
3	9.9	23	12.5
		25	13.5

Dagegen erwies sich der Stern $g = \gamma$ Pegasi als entschieden veränderlich (siehe auch Astr. Nachr. 182.163). Für ihn fand ich folgende durch graphische Ausgleichung erhaltene Skala:

1909	Stufen	1909	Stufen	1909	Stufen
Aug. 24	29.0	Sept. 7	23.8	Sept. 17	24.9
25	28.9	9	24.7	18	25.3
29	27.7	10	24.8	21	26.7
30	27.3	14	24.8	21	26.7
Sept. 3	24.5	15	24.8	29	25.6
6	23.3	15	24.8		

Mittels dieser drei Tabellen wurde die Helligkeit von Mira, in Stufen ausgedrückt, aus den Schätzungen berechnet. Diese Stufenwerte wurden endlich nach der Methode der kleinsten Quadrate, unter Zugrundelegung der aus P. Hagens »Atlas« entnommenen photometrischen Größen der Vergleichsterne im Harvard-System, in Sterngrößen verwandelt und so die in der folgenden Übersicht in Spalte 3 aufgeführte Sterngröße von Mira erhalten.

1909	M. Z. Kasan	Gr.	Farbe von α Ceti	Luft-zu-stand	$h\zeta$	$A-D$
Juli 26	13 ^h 52 ^m	5 ^m 11	10 ⁰	2	—	—
29	13 37	4.60	10	1	—	—
31	13 49.5	4.28	—	1	2 ⁰	85 ⁰