ART. IV.—*A Method for Determining the Rate of Tuning-forks ;*
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THE tuning-fork is already employed to a great extent as a measurer of small intervals of time, and this use promises to become more extended as its advantages become more fully appreciated. Any method which will facilitate the operation of finding the rate, or increase its accuracy should therefore merit consideration.

The following plan appears to be open to fewer objections than any now in use, and its accuracy is limited only by the accuracy of the pendulum.

The fork to be rated, for example an U_t , making about 128 double vibrations per second, is compared with an U_t , kept in vibration by electro-magnets and which shall be designated EU_t , and this is compared directly with the seconds pendulum, and then the two forks are once more compared, to make certain that EU_t , has not changed in the interval. Thus U_t , is

determined. The whole operation may be performed in less than ten minutes.

To determine the rate of $E\text{U}_2$, we will suppose that this is within a small fraction of a whole number of vibrations per second, and that this whole number is known; say 128 v. s. A mirror is attached to one prong of $E\text{U}_2$, and in front of this, and at a distance of two or three feet, is placed a Geissler tube. This last is illuminated once every second. The tube itself is continuously illuminated and its image in the mirror presents the appearance of a broad band with well-defined edges. Against this the narrow flash is projected.

Evidently if $E\text{U}_2$ makes an exact whole number of vibrations per second, then this flash will always find the fork in the same phase of vibration, and consequently its image will always appear at the same part of the band. If, however, the fork makes, say 128.1 v. s., then it will occupy successively the positions

0	1	2	3	4	5	6	7	8	9	10
—	—	—	—	—	—	—	—	—	—	—

That is, there would be ten flashes between the positions (0) and (10). If, therefore, ten flashes occur in one "period" then the whole number 128 is to be increased (or diminished) by one-tenth; and in general if a flashes occur in one period then

$E\text{U}_2$ makes $128 \pm \frac{1}{a}$ vibrations per second.

Since $E\text{U}_2$ vibrates continuously the number of periods which may be counted is unlimited, hence a can be found with any desired degree of accuracy.

It was found that the chief difficulty in executing this plan lay in the imperfections of the break-circuit. If thereby the seconds intervals differed as much as 0.002 seconds, then the operation would be impracticable, on account of the irregularities in the position of the flashes. A great many break-circuits were tried, and the one which was finally adopted as being the most reliable and giving the least trouble was the "mercury globule." Even this, however, in its usual form was unsatisfactory. A great improvement was effected by narrowing the globule in the direction of the swing of the pendulum, by placing it in a small tube whose upper end was flattened. If the current from a "bi-chromate" cell is interrupted by this break-circuit the mercury oxidizes rapidly, and the flashes (given by a small induction coil through the Geissler tube) become irregular.

To obviate this difficulty, the circuit of a "gravity" battery was interrupted, and this worked a relay, which in turn inter-

rupted the circuit of the induction coil. The results thus obtained left nothing to be desired.

In order that the rate of EU_t remain constant, it is necessary that the current passing through its electro-magnets be constant. Hence the necessity for constant batteries, which, however, have a comparatively great resistance. Several might be joined in parallel circuit, but it was found that much better results were obtained by using six "gravity" cells in series, and using fine wire magnets. Such a battery will keep the fork in vibration indefinitely with but little alteration in its rate except that due to variations in temperature.

In comparing the forks a convenient plan is to weight the EU_t , so that it gives with the fork to be rated about 9 beats in 10 seconds, and then to time the "coincidences" between the beats and the ticks of the pendulum. By this means the comparison can be much more accurately determined than by counting the beats for sixty seconds. This may also be done by the optical method. It was found that the "gravity" battery which kept the EU_t in vibration could serve at the same time to work the relay.

The whole arrangement is shown in fig. 1, in which, for clearness, all the parts to the right of i have been shown in the plane of the drawing, instead of at right angles.

In practice only the even flashes are counted, the odd ones being disregarded. Consequently, if a be the number of these per period, n , the rate of the fork, and $2N$, the whole number nearest to $2n$, then $n = N \pm \frac{1}{2a}$. To determine which sign is

to be used, EU_t is weighted by a very small piece of wax. If a is greater than before, the sign is +; if a is less, the sign is -.

The electric fork may be dispensed with entirely by the following method. The fork to be rated is placed vertical, so that one edge is in the focus of a microscope provided with

cross-hairs. Behind the fork the Geissler tube is placed horizontal. The appearances observed in the microscope at each flash would then be as follow:—

From these we can deduce the rate as before.

This method would probably give results almost if not quite as accurate as the preceding, and has the advantage of being more direct. The nearest whole number may be found by comparison with one of König's standards, or by the following method.

The fork is, in turn, compared with two pendulums, whose times of vibration are t_1 and t_2 , $t_1 - t_2$ being small.

Let n_1 = number of vibrations of the fork in time t_1 .

Let n_2 = number of vibrations of the fork in time t_2 .

n_1 and n_2 differing from a whole number by less than a small fraction, e_1 .

Let a_1 = number of beats of pendulum (1) per period = $\frac{1}{e_1}$.

Let a_2 = number of beats of pendulum (2) per period = $\frac{1}{e_2}$.

e_1 and e_2 being small fractions less than e_2 .

Let N_1 = whole number nearest to n_1 .

Let N_2 = whole number nearest to n_2 .

Then

$$n_1 + e_1 = N_1$$

$$n_2 + e_2 = N_2$$

$$n_1 - n_2 + e_1 - e_2 = N_1 - N_2 = M, \text{ a whole number.}$$

$$n_1 - n_2 + e_1 - e_2 \text{ is less than } 2(e_1 + e_2).$$

If, therefore, $2(e_1 + e_2)$ is numerically less than $\frac{1}{2}$ then $M = 0$, whence $N_1 = N_2 = N$.

$$\frac{n_1}{n_2} = \frac{N - e_1}{N - e_2} = \frac{t_1}{t_2} \text{ and } N = \frac{c_1 t_2 - c_2 t_1}{t_2 - t_1}.$$

Eight determinations of the rate of an Ut_2 fork, made by the preceding method gave the following results:

Temp. Fahr.	v. s.	v. s. at 60° F.	Diff. from mean.
54.0°	128.134	128.090	+0.003
56.3	128.114	128.087	0.000
58.0	128.102	128.087	0.000
60.0	128.090	128.090	+0.003
62.2	128.077	128.093	+0.006
63.0	128.060	128.082	-0.005
64.5	128.050	128.083	-0.004
73.5	127.984	128.084	-0.003